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Magnetization dynamics in the inertial regime: Nutation predicted at short time scales

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The dynamical equation for magnetization has been reconsidered by enlarging the phase space of the ferromagnetic degrees of freedom to the angular momentum. The generalized Landau-Lifshitz-Gilbert equation that includes inertial terms, and the corresponding Fokker-Planck equation, are then derived in the framework of mesoscopic nonequilibrium thermodynamics theory. A typical relaxation time τ is introduced describing the relaxation of the magnetization acceleration from the inertial regime toward the precession regime defined by a constant Larmor frequency. For time scales larger than τ, the usual Gilbert equation is recovered. For time scales below τ, nutation and related inertial effects are predicted. The inertial regime offers new opportunities for the implementation of ultrafast magnetization switching in magnetic devices.

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In 1935 Landau and Lifshitz proposed an equation for the kinetic of the magnetization $d\mathbf{M}/dt$ (where $\mathbf{M}$ is of constant modulus), composed of a precession term proportional to $\mathbf{M} \times \mathbf{H}$ and a longitudinal relaxation term proportional to $\mathbf{M} \times (\mathbf{M} \times \mathbf{H})$, that drives the magnetization toward equilibrium along the magnetic field $\mathbf{H}$.[1] Two decades later Gilbert derived the equation that bears his name in which the relaxation toward equilibrium is described by a damping term η (Ref. 2) through the kinetic equation $d\mathbf{M}/dt = \gamma\mathbf{M} \times (\mathbf{H} - \eta d\mathbf{M}/dt)$, with γ as the gyromagnetic ratio. The two equations (Landau-Lifshitz and Gilbert) are mathematically equivalent.

The range of validity of the Landau-Lifshitz-Gilbert (LLG) equation was established one decade later by Brown, with a description of a magnetic moment coupled to a heat bath.[3] The magnetic moment is treated as a Brownian particle described by the slow degrees of freedom ($>10^{-9}$ s) with the angles $\{\theta,\phi\}$. The remaining degrees of freedom of the system relax in a much shorter time scale ($<10^{-11}$ s). The time scale separation between the rapidly relaxing environmental degrees of freedom and the slow magnetic degrees of freedom allows the coupling between the magnetization and its environment to be reduced to single phenomenological damping parameter η, whatever the complexity of the microscopic relaxation involved.[4,5]

However, important experimental advances toward the very short time-resolved response of the magnetization (subpicosecond resolution, i.e., below the limit proposed by Brown) have been reported in the last decade.[6] In parallel, industrial needs for very fast memory storage technologies are approaching the limits imposed by the precessional switching.[7] In these experiments and in the corresponding applications, time scale separation between the degrees of freedom $\{\theta,\phi\}$ and the other degrees of freedom, assumed by Brown,[3] finds its limit.

The purpose of this Rapid Communication is to investigate the dynamics of the magnetization beyond this limit by extending the configuration space (defined by the coordinates of position, i.e., the angles $\{\theta,\phi\}$) to the phase space defined by the dynamical variables, i.e., the angular momentum $\mathbf{L}$.[5,8]

The angular momentum is introduced in the standard derivation of Gilbert's equation[2] on the basis of the well-known gyromagnetic relation $\mathbf{L} = \mathbf{M}/\gamma$.[9] The important point for our purpose is that, within this approach, a crude assumption should be performed in order to obtain the expected equation: Only one component L_3 of the vector $\mathbf{L}$ is different than zero.[2] As a result, the Gilbert equation does not contain any inertial terms. This crude assumption was necessary in order to account for the experiments of damped precession performed so far. Without this assumption, the same model leads straightforwardly to a generalized gyromagnetic relation $\mathbf{L} = \frac{I_1}{M_s^2}\mathbf{M} \times \frac{d\mathbf{M}}{dt} + \frac{\mathbf{M}}{\gamma}$, which contains a dynamical term (I_1 is the first principal moment of inertia).

However, as shown below, the generalized gyromagnetic relation is not deterministic and the transient nature of the dynamical term is fundamental: The parameter I_1 should be related to a finite relaxation time τ_1. The generalized gyromagnetic relation and the corresponding extension of Gilbert's equation are a consequence of out-of-equilibrium angular momentum, i.e., valid at very short time scales only. The generalized relation is then no longer in contradiction with experiments. Typically, in the logic of Barnett's or the Einstein–de Haas magnetomechanical measurements,[9] the inertial behavior of the magnetization introduced here is equivalent to the time variation of the mechanical angular momentum that would be measured if subnanosecond transients were measurable. The new nonzero components of the angular momentum are defined following the classical definition of the effective *Ampère currents* with a current distribution that is no longer confined in the plane perpendicular to L_3, but distributed on an ellipsoid of revolution.[10]

For the sake of simplicity, we assume the existence of an intermediate time domain for which the magnetization is still uniform and the angular momentum is already out of equilibrium. From the experimental point of view, this hypothesis should guarantee that the dynamics of the other internal degrees of freedom (optical phonons, electronic relaxations, etc.) would not mask the inertial response of the magnetization generated by a THz frequency excitation (e.g., far-infrared spectroscopy) or subpicosecond time resolved excitation (pump-probe optics).[7,11] Note that the existence of inertial terms in the dynamics of the magnetization opens the way to deterministic ultrafast magnetization switching strategies, beyond the limitations of the precessional regime.[12]

We derive below the generalized Gilbert equation and the corresponding Fokker-Planck equation for a uniform magnetic moment. The derivation is performed in the

framework of mesoscopic nonequilibrium thermodynamics (MNET).[13–16]

It is convenient to model the dynamics of a magnetic moment $\mathbf{m} = M_s\mathbf{e}$ (submitted to an applied magnetic field $\mathbf{H} = -\frac{1}{M_s}\frac{\partial V^F}{\partial \mathbf{e}}$ and coupled to a heat bath) with a statistical ensemble composed by noninteracting identical uniform magnetic moments found in the same given conditions (ergodic property). Here, $\mathbf{e}$, M_s, and V^F are, respectively, the radial unit vector of angles $\{\theta,\phi\}$, the magnetization at saturation, and the ferromagnetic potential energy. The ensemble of magnetic moments of constant modulus M_s defines a sphere surface Σ and the number of magnetic moments oriented within $(\mathbf{e},\mathbf{e}+d\mathbf{e})$ defines the density $n(\mathbf{e})$ of magnetic moments over Σ. The magnetization $\mathbf{M}$ is the mean value of $\mathbf{m}$. In contrast to the standard derivation of Gilbert that introduces the angular momentum, we have shown in previous works that associating two degrees of freedom $\{\theta,\phi\}$ with a magnetic moment is sufficient to derive both the Gilbert equation and the corresponding rotational Fokker-Planck equation (the gyromagnetic relation does not play any role in this derivation).[16,17]

Extending the configuration space to the angular momentum $\mathbf{L}$,[10] the space Σ is extended from a two-dimensional space to a space of *a priori* five dimensions $\{\theta,\phi,\mathbf{L}\}$. A distribution function $f(\mathbf{e},\mathbf{L})$ of magnetic moments with the magnetization orientation within $(\mathbf{e},\mathbf{e}+d\mathbf{e})$ and the angular momentum within $(\mathbf{L},\mathbf{L}+d\mathbf{L})$ should then be defined, where f is assumed to vanish for infinite values of $\mathbf{L}$ as $\lim_{\mathbf{L}\to\pm\infty} f(\mathbf{e},\mathbf{L}) = 0$. The angular momentum $\mathbf{L}$ associated with a magnetic moment is either changed by an applied torque $\mathbf{N} = \mathbf{m}\times\mathbf{H}$ as $(\frac{d\mathbf{L}}{dt})_s = \mathbf{N}$, or by the interaction with the heat bath. When considering the statistical ensemble, the interaction with the bath is modeled through a phase-space flux $\mathbf{J_L}$ (defined below) which vanishes for large values of $\mathbf{L}$: $\lim_{\mathbf{L}\to\pm\infty}\mathbf{J}_L = 0$.

The kinetic-energy expression that Gilbert associated with the magnetization[2] is written as $\mathcal{K} = \mathbf{LL} : \bar{\bar{I}}^{-1}/2$, where the magnetic inertial tensor $\bar{\bar{I}}$ is related to the magnetic moment. It is assumed that $\bar{\bar{I}}$ keeps the symmetry of the magnetic moment, i.e., is axial symmetric of symmetry axis $\mathbf{e}$: $\bar{\bar{I}} = I_1(\bar{\bar{U}} - \mathbf{ee}) + I_3\mathbf{ee}$, with $\bar{\bar{U}}$ the dyadic unit (where $I_1 = I_2$ and I_3 are the diagonal coefficients of the inertial tensor).

In the space-fixed reference frame denoted by the subscript s, the conservation law for the number of axial symmetric moments $f(\mathbf{e},\mathbf{L})$ reads[18]

$$\frac{\partial}{\partial t} f(\mathbf{e},\mathbf{L}_s) = -\left\{\frac{\partial\,(f\dot{\mathbf{e}})}{\partial \mathbf{e}}\right\}_{\mathbf{L}_s} - \mathbf{N}_s\cdot\frac{\partial f}{\partial \mathbf{L}_s} - \frac{\partial \mathbf{J_L}}{\partial \mathbf{L_s}}, \qquad (1)$$

where the derivatives with respect to the angles are made while holding the Cartesian components of $\mathbf{L}_s$ constant:

$$\left\{\frac{\partial(f\dot{\mathbf{e}})}{\partial \mathbf{e}}\right\}_{\mathbf{L}_s} = \frac{1}{\sin\theta}\left\{\frac{\partial(f\sin\theta\,\dot{\theta})}{\partial\theta}\right\}_{\mathbf{L}_s} + \left\{\frac{\partial(f\dot{\phi})}{\partial\phi}\right\}_{\mathbf{L}_s}. \qquad (2)$$

The density $n(\mathbf{e})$ of magnetic moments in the space Σ is recovered by integrating over the angular momentum degree of freedom $n(\mathbf{e}) = \int f(\mathbf{e},\mathbf{L})\,d^3\mathbf{L}$. The conservation law for the magnetic moments in the Σ space is hence deduced from Eq. (1):

$$\frac{\partial n}{\partial t} = \int \frac{\partial f}{\partial t}\,d^3\mathbf{L}_s = -\frac{\partial}{\partial \mathbf{e}}\cdot\int f\dot{\mathbf{e}}\,d^3\mathbf{L}_s = -\frac{\partial(n\dot{\mathbf{e}})}{\partial \mathbf{e}}. \qquad (3)$$

Beyond this, the conservation law for the mean value of the angular momentum $\langle\mathbf{L}_s\rangle$ is also derived[18,19]:

$$\begin{aligned}\frac{\partial\, n\langle\mathbf{L}_s\rangle}{\partial t} &= \int \frac{\partial f}{\partial t}\,\mathbf{L}_s\,d^3\mathbf{L}_s \\ &\overset{(1)}{=} -\frac{\partial}{\partial \mathbf{e}}\cdot\int f\dot{\mathbf{e}}\,\mathbf{L}_s\,d^3\mathbf{L}_s + n\,\mathbf{N}_s(\mathbf{e}) + \int \mathbf{J_L}\,d^3\mathbf{L}_s, \\ n\,\frac{d\langle\mathbf{L}_s\rangle}{dt} &= -\frac{\partial}{\partial \mathbf{e}}\cdot(\mathbf{e}\times\bar{\bar{P}}_s) + n\,\mathbf{N}_s(\mathbf{e}) + \int \mathbf{J_L}\,d^3\mathbf{L}_s, \qquad (4)\end{aligned}$$

where the *magnetic pressure* tensor is defined as $\bar{\bar{P}}_s = \bar{\bar{I}}^{-1}\int(\mathbf{L}-\langle\mathbf{L}_s\rangle)(\mathbf{L}-\langle\mathbf{L}_s\rangle)\,f\,d^3\mathbf{L}_s$. The conservation equation (4) states that the rate of change of the average angular momentum $\langle\mathbf{L}_s\rangle$ is due to three contributions: an applied torque $\mathbf{N}_s$, an average interaction with the bath $\int\mathbf{J_L}d^3\mathbf{L}_s$ (i.e., damping), and a torque due to pressure (i.e., rotational diffusion).

The expression for $\mathbf{J_L}$ is deduced from the entropy production expression $\sigma(\mathbf{e})$.[13,14,19] Defining the ferromagnetic chemical potential $\mu(\mathbf{e},\mathbf{L})$, the power $T\sigma(\mathbf{e})$ dissipated by the magnetic system is the product of the generalized flux by the generalized force:

$$T\sigma(\mathbf{e}) = -\int \mathbf{J_L}\cdot\frac{\partial\mu}{\partial \mathbf{L}_s}\,d^3\mathbf{L}_s, \qquad (5)$$

where the chemical potential takes the canonical form,[13,15]

$$\mu(\mathbf{e},\mathbf{L}) = kT\ln[f(\mathbf{L},\mathbf{e})] + \mathcal{K}(\mathbf{L}) + V^F(\mathbf{e}). \qquad (6)$$

The application of the second law of thermodynamics together with the local equilibrium hypothesis in the $(\mathbf{e},\mathbf{L})$ space lead us to introduce the Onsager matrix $\bar{\bar{\mathcal{L}}}$ such that $\mathbf{J_L} = -\bar{\bar{\mathcal{L}}}\cdot\frac{\partial\mu}{\partial \mathbf{L}_s}$. As the Onsager coefficients are a reflection of the system's symmetry,[14] the relaxation tensor defined as $\bar{\bar{\tau}}^{-1} = \frac{1}{f}\bar{\bar{\mathcal{L}}}\bar{\bar{I}}^{-1}$ is also axial symmetric: $\bar{\bar{\tau}}^{-1} = \tau_1^{-1}(\bar{\bar{U}} - \mathbf{ee}) + \tau_3^{-1}\mathbf{ee}$ (where $\tau_1 = \tau_2$ and τ_3 are the diagonal coefficients), and is related to damping. Moreover, as $\mathbf{e}$ is an axis of symmetry for the ferromagnetic potential $V^F(\theta,\phi)$, the relaxation tensor $\bar{\bar{\tau}}^{-1}$ is not expected to have any components in the $\mathbf{e}$ direction,[18] leading to $\tau_3^{-1} = 0$.

The dynamic equation (4) can be rewritten as

$$\frac{d\langle\mathbf{L_s}\rangle}{dt} = \mathbf{N_s} - \bar{\bar{\tau}}_s^{-1}\cdot\langle\mathbf{L_s}\rangle - \frac{1}{n}\frac{\partial}{\partial \mathbf{e}}\cdot(\mathbf{e}\times\bar{\bar{P}}_s). \qquad (7)$$

As the inertial tensor $\bar{\bar{I}}$ and the relaxation tensor $\bar{\bar{\tau}}^{-1}$ are time independent in the rotating frame (or the magnetization frame), a simpler expression of Eq. (7) can be obtained in this frame. After introducing the average angular velocity $\boldsymbol{\Omega}$ such that $\langle\mathbf{L}\rangle = \bar{\bar{I}}\cdot\boldsymbol{\Omega}$, Eq. (7) rewrites as

$$\frac{d\boldsymbol{\Omega}_r}{dt} = \bar{\bar{I}}_r^{-1}\cdot\left[\mathbf{N_r} - \frac{1}{n}\frac{\partial}{\partial \mathbf{e}}\cdot(\mathbf{e}\times\bar{\bar{P}})\right] - \bar{\bar{\tau}}_{\rm rot}^{-1}\cdot\boldsymbol{\Omega}_r. \qquad (8)$$

The rotating frame is denoted by the subscript r and $\overline{\overline{\tau}}_{\rm rot}^{-1} = \overline{\overline{\tau}}_{r}^{-1} - (\frac{I_3}{I_1} - 1)\Omega_3\, \mathbf{e} \times \overline{\overline{U}}$, or

$$\overline{\overline{\tau}}_{\rm rot}^{-1} = (\tau_1\, \alpha^*)^{-1} \begin{pmatrix} \alpha^* & 1 & 0 \\ -1 & \alpha^* & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{9}$$

where $\alpha^* = \alpha\,(I_3/I_1 - 1)^{-1}$ with $\alpha = (\Omega_3\,\tau_1)^{-1}$.

The three components of Eq. (8) read

$$\begin{cases} \dot{\Omega}_1 = -\frac{\Omega_1}{\tau_1} - \left(\frac{I_3}{I_1} - 1\right)\Omega_3\Omega_2 - \frac{M_s H_2}{I_1} - \left[\frac{1}{I_1 n}\frac{\partial(\mathbf{e}\times\overline{\overline{P}})}{\partial \mathbf{e}}\right]_1 \\ \dot{\Omega}_2 = -\frac{\Omega_2}{\tau_1} + \left(\frac{I_3}{I_1} - 1\right)\Omega_3\Omega_1 + \frac{M_s H_1}{I_1} - \left[\frac{1}{I_1 n}\frac{\partial(\mathbf{e}\times\overline{\overline{P}})}{\partial \mathbf{e}}\right]_2 \\ \dot{\Omega}_3 = -\tau_3^{-1}\Omega_3 = 0. \end{cases} \tag{10}$$

According to the last equation, the quantity $L_3 = I_3\Omega_3$ is a constant of motion, so that the well-known gyromagnetic ratio γ can be defined as the ratio $\gamma = \frac{M_s}{\langle L_3\rangle}$ (this is precisely the hypothesis performed by Gilbert[2]).

On the other hand, the generalized gyromagnetic relation discussed in the introduction is a formal property: Since the modulus of $\mathbf{M}$ is conserved, the relation $\frac{d\mathbf{M}}{dt} = \mathbf{\Omega} \times \mathbf{M}$ holds. Cross-multiplying by $\mathbf{M}$ and using the above definition of γ leads to the identity $\mathbf{\Omega} = \frac{\mathbf{M}}{M_s^2} \times \frac{d\mathbf{M}}{dt} + \frac{\mathbf{M}}{I_3\gamma}$ (from which the generalized gyromagnetic relation for $\langle \mathbf{L}\rangle$ is obtained).

Furthermore, the averaged dynamic Eq. (10) introduces a characteristic time scale τ_1, which separates the behavior of the magnetic system of particles in two regimes: the diffusion regime or the long time scale limit $t \gg \tau_1$, and the inertial regime $t \approx \tau_1$.

In a diffusive regime, i.e., for $t \gg \tau_1$, the inertial terms $\frac{d\Omega_1}{dt}$ and $\frac{d\Omega_2}{dt}$ are negligible with respect to $\frac{\Omega_1}{\tau_1}$ and $\frac{\Omega_2}{\tau_1}$. Equation (8) then rewrites as the Gilbert equation with an inertial correction performed on the previously defined gyromagnetic coefficient $\gamma^* = \frac{\gamma}{1 - I_1/I_3}$:

$$\frac{d\mathbf{M}}{dt} = \gamma^*\, \mathbf{M} \times \left(\mathbf{H}_{\rm eff} - \eta \frac{d\mathbf{M}}{dt}\right). \tag{11}$$

The Gilbert damping coefficient η is now defined as $\eta = \frac{I_1}{\tau_1 M_s^2}$ (so that $\alpha^* = \gamma^* \eta M_s$ is the corresponding dimensionless coefficient), and $\mathbf{H}_{\rm eff}$ is an effective field that includes the diffusion term.

At the diffusive limit, the magnetic moments follow a distribution function $f(\mathbf{e},\mathbf{L})$ close to a Maxwellian centered on the average angular momentum $\langle \mathbf{L}\rangle$.[8] This leads to a diagonal form for the pressure tensor: $\overline{\overline{P}} = nkT/\overline{\overline{U}}$ and $\mathbf{H}_{\rm eff} = \mathbf{H} - \frac{kT}{n}\frac{1}{M_s}\frac{\partial n}{\partial \mathbf{e}}$.[16,17] Equation (11) contains the density $n(\mathbf{e})$. Inserting Eq. (11) into the conservation law (3) leads to the rotational Fokker-Planck equation of $n(\mathbf{e})$, derived by Brown:[3] $\frac{\partial n}{\partial t} = \frac{\partial(n\mathbf{e}\times\mathbf{\Omega})}{\partial \mathbf{e}}$.

For sufficiently short time scales $t \approx \tau_1$, the inertial terms cannot be neglected and the Gilbert approximation is no longer valid. The dynamic equation (10) takes the following generalized form:

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \left[\mathbf{H}_{\rm eff} - \eta\left(\frac{d\mathbf{M}}{dt} + \tau_1 \frac{d^2\mathbf{M}}{dt^2}\right)\right]. \tag{12}$$

The corresponding generalized rotational Fokker-Planck equation for the statistical distribution f is obtained with replacing $\mathbf{J_L}$ by the Onsager relation derived earlier into the conservation

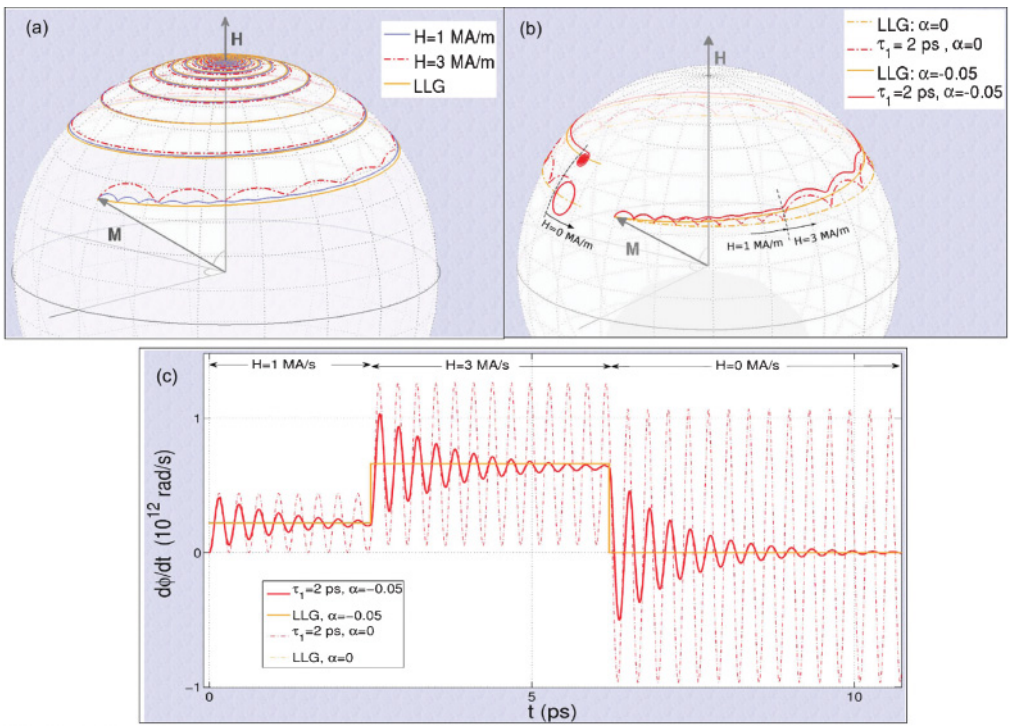

FIG. 1. (Color online) Numerical solution of Eq. (12) with $\tau_1 = 2$ ps. (a) Trajectories at two different fields with $|\alpha| = 0.05$ (dashed line) and curves deduced from the LLG equation (lower line). (b) Trajectory of the magnetization with changing suddenly the effective fields from $H = 1$ MA/m to $H = 3$ MA/m and $H = 0$, with damping (continuous lines) and without (dotted lines). (c) Time derivative of the azimuth angle ϕ plotted as a function of time for the trajectory of (b).

law (1) and rewriting the law in the rotating frame[19]:

$$\frac{\partial f_{(\mathbf{e},\mathbf{L}_r)}}{\partial t} = \left\{ \frac{\partial \left(f\,\mathbf{e} \times \overline{\overline{I}}_r^{-1}\mathbf{L}_r \right)}{\partial \mathbf{e}} \right\}_{\mathbf{L}_r} \tag{13}$$

$$+ \frac{\partial}{\partial \mathbf{L}_r} \cdot \left[f\overline{\overline{\tau}}_{\text{rot}}^{-1} \cdot \mathbf{L}_r - f\mathbf{N}_r + kT\overline{\overline{\tau}}_r^{-1}\overline{\overline{I}}_r \cdot \frac{\partial f}{\partial \mathbf{L}_r} \right]. \tag{14}$$

At short time scales $t \approx \tau_1$ and due to the inertial effect, the usual precessional behavior is enriched by a *nutation* effect. The simplest way to understand nutation is to imagine that the effective field is switched off suddenly with zero damping: The precession stops suddenly because the Larmor frequency $\omega_L = \gamma^* \mathbf{H}$ drops to zero at the same time. However, in the absence of inertial terms, the magnetic moment also stops at this position within an arbitrarily short time scale. But if the kinetic energy is different from zero the movement cannot be stopped suddenly, the precession (around the magnetic field) stops but the magnetic moment starts to rotate around the angular momentum vector in order to conserve the energy—the precession is transformed into nutation.

Figure 1 shows the numerical solution of Eq. (12) (neglecting thermal fluctuations) with a field along the z axis and for a parameter τ_1 fixed to 2 ps with $|\alpha| = 0.05$. The trajectories are plotted on the sphere Σ. The usual trajectory deduced from the LLG equation is also plotted for comparison. The motion of the magnetic moment displays the familiar curve due to Larmor precession, with superimposed loops generated by the nutation effect. Figure 1(b) shows a trajectory starting without initial velocity under an effective field of 1 MA/m, changed suddenly to 3 MA/m and once again down to zero. Four curves are represented: two for Eq. (12) with $|\alpha| = 0.05$ (continuous line) and $\alpha = 0$ (dashed line), and two for the usual LLG equation with and without damping. At the end of the motion (left), the field is set to zero and the precession is destroyed, with the nutation effect describing a circle (without damping) or a spiral (with damping). Note that the profile of the nutation loops depends on the initial conditions (the cusp presented in Fig. 1 instead of loops is due to zero initial velocity). Figure 1(c) shows the time derivative of the angle ϕ as a function of time for the trajectory displayed in Fig. 1(b). The horizontal lines represent the constant Larmor frequencies, and the oscillations are due to nutation (for $|\alpha| = 0.05$ and $\alpha = 0$).

In conclusion, we have shown that extending the phase space of the magnetization to the degrees of freedom of the angular momentum leads to an out-of-equilibrium generalization of the gyromagnetic relation, and to a generalization of the Landau-Lifshitz-Gilbert equation that contains inertial terms. It is predicted that inertial effects should be observed at sufficiently short time scales (typically below the picosecond), e.g., by measuring nutation loops superimposed to the usual precession motion of a magnetic moment. The inertial regime at short time scales would also offer possibilities for new experiments and devices based on ultrafast magnetization switching.

[1] L. Landau and E. Lifshitz, Phys. Z. Sowjetunion **8**, 153 (1935).

[2] T. L. Gilbert, IEEE Trans. Mag. **40**, 3443 (2004); Phys. Rev. **100**, 1243 (1955) (abstract only).

[3] W. F. Brown, Jr., Phys. Rev. **130**, 1677 (1963).

[4] W. T. Coffey, Yu. P. Kalmykov, and J. T. Waldron, *The Langevin Equation*, World Scientific Series in Contemporary Chemical Physics No. 11 (World Scientific, Singapore, 1996).

[5] E. Fick and S. Sauermann, *The Quantum Statistics of Dynamic Processes*, Springer Series in Solid-States Sciences No. 86 (Springer, New York, 1990).

[6] E. Beaurepaire, J. C. Merle, A. Daunois, and J. Y. Bigot, Phys. Rev. Lett. **76**, 4250 (1996); J. Hohlfeld, E. Matthias, R. Knorren, and K. H. Bennemann, *ibid.* **78**, 4861 (1997); A. Scholl, L. Baumgarten, R. Jacquemin, and W. Eberhardt, *ibid.* **79**, 5146 (1997); M. Aeschlimann, M. Bauer, S. Pawlik, W. Weber, R. Burgermeister, D. Oberli, and H. C. Siegmann, *ibid.* **79**, 5158 (1997); C. H. Back, D. Weller, J. Heidmann, D. Mauri, D. Guarisco, E. L. Garwin, and H. C. Siegmann, *ibid.* **81**, 3251 (1998); H. S. Rhie *et al.*, *ibid.* **90**, 247201 (2003); A. V. Kimel *et al.*, Nature (London) **435**, 655 (2005); C. D. Stanciu, F. Hansteen, A. V. Kimel, A. Kirilyuk, A. Tsukamoto, A. Itoh, and T. Rasing, Phys. Rev. Lett. **99**, 047601 (2007).

[7] I. Tudosa, C. Stamm, A. B. Kashuba, F. King, H. C. Siegmann, J. Steohr, G. Ju, B. Lu, and D. Weller, Nature (London) **428**, 831 (2004).

[8] J. M. Rubí and A. Pérez-Madrid, Physica A **264**, 492 (1999).

[9] The gyromagnetic relation $\mathbf{M} = \gamma\mathbf{L}$ is established through magnetomechanical measurements (in isolated systems), as shown by S. J. Barnett [see Rev. Mod. Phys. **7**, 129 (1935)] and A. Einstein and W. J. de Haas [Verh. d. D. Phys. Ges. **17**, 152 (1915)].

[10] This definition gives the characteristic frequency of the inertial regime. The principal moments of inertia are $I_3 = ma_0^2$ and $I_1 = I_2 = ma_0^2\sin^2\theta$, where m is the mass of the electron and a_0 is the Bohr radius. On the other hand, the corresponding magnetization M_s is given by the Bohr magneton μ_B (i.e., $L_3 = \mu_B/\gamma$). The angular velocity Ω_3 is then given by $\Omega_3 = \mu_B/(I_3\gamma) \approx 10^{16}$ Hz.

[11] J.-Y. Bigot, M. Vomir, and E. Beaurepaire, Nature Phys. **5**, 515 (2009); B. Koopmans, G. Malinowski, F. Dalla Longa, D. Steiauf, M. Fähnle, T. Roth, M. Cinchetti, and M. Äschlimann, Nat. Mater. **9**, 259 (2009); K. H. Bennemann, Ann. Phys. (NY) **18**, 480 (2009).

[12] A. V. Kimel, B. A. Ivanov, R. V. Pisarev, P. A. Usachev, A. Kirilyuk, and Th. Rasing, Nature Phys. **5**, 727 (2009).

[13] D. Reguera, J. M. G. Vilar, and J. M. Rubí, J. Phys. Chem. B **109**, 21502 (2005).

[14] P. Mazur, Physica A **261**, 451 (1998).

[15] S. R. De Groot and P. Mazur, *Non-equilibrium Thermodynamics* (North-Holland, Amsterdam, 1962).

[16] J.-E. Wegrowe, Phys. Rev. B **62**, 1067 (2000); J.-E. Wegrowe, S. M. Santos, M.-C. Ciornei, H. J. Drouhin, and J. M. Rubí, *ibid.* **77**, 174408 (2008); J.-E. Wegrowe, Solid State Commun. **150**, 519 (2010).

[17] J.-E. Wegrowe, M. C. Ciornei, and H.-J. Drouhin, J. Phys.: Condens. Matter **19**, 165213 (2007).

[18] D. W. Condiff and J. S. Dahler, J. Chem. Phys. **44**, 3988 (1966).

[19] M.-C. Ciornei, Ph.D. thesis, Ecole Polytechnique, Palaiseau, France, 2010.