

## Boltzmann approach to dissipation produced by a spin-polarized current

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By using the Boltzmann transport equations, we calculate the dissipation produced by a spin-polarized current. The results can be interpreted in terms of a circuit model which is also commonly used to explain the giant magnetoresistance (GMR) effect. We find that due to the spin-flip scattering of electrons, the GMR effect is associated with extra Joule heating, which is more localized than the spin-diffusion length.

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In spintronic devices the electron's spin degree of freedom, rather than its charge plays an important role.<sup>1</sup> Due to the spin-dependent scattering of electrons, the current flowing through these devices becomes spin polarized. In fact, in some situations, we can even have a "pure" spin current, without any accompanying charge current, by using "nonlocal geometries."<sup>2,3</sup> The dissipation produced by a spin-polarized current is an important issue, and can be a limiting factor in the performance of spintronic devices. Recently, the existence of a dissipationless spin current has been predicted.<sup>4,5</sup> This effect is related to the intrinsic spin Hall effect originating from the spin-orbit coupling in semiconductors. Although there are some reports of experimental observation,<sup>6</sup> the interpretation in terms of dissipationless current is still debated.<sup>7,8</sup> In the present paper, we consider diffusive flow of spin currents and calculate the dissipation (i.e., rate of increase of entropy density) produced by the spin-flip scattering of electrons. Such a diffusive flow of spin current occurs in many devices such as giant magnetoresistance (GMR) sensors, spin transistors,<sup>9</sup> etc. Apart from nonlocal geometries, a pure spin current can also be generated by spin pumping,<sup>10</sup> or application of a temperature gradient to a ferromagnet. In the latter case, the spin current is generated due to the spin-dependent Seebeck coefficient of the ferromagnet.<sup>11-15</sup> The dissipation produced by a spin current has been previously considered by Wegrowe.<sup>16</sup> By using a thermodynamic theory, this author derived an expression for the dissipation. Such a thermodynamic approach has also been used in Refs. 17 and 18. Magnetoelectronic circuit theory has also been used to find out dissipation in magnetic systems.<sup>19</sup> In this paper, we use a microscopic theory based on the semiclassical Boltzmann equation. This theory enables one to evaluate the thermodynamic forces and currents in terms of the scattering probability of electrons. We find that an additional pair of thermodynamic force and current is required to describe the dissipation produced by a spin current.

Consider a conductor subjected to a small electric field and/or temperature gradient. The distribution function of its electrons in position ( $\vec{r}$ )-momentum ( $\vec{k}$ ) space can be written as  $f_s = f_s^0 + (\partial f_s^0 / \partial \varepsilon_s) \phi_s(\vec{r}, \vec{k})$ . Here  $s$  denotes the spin index (+ indicates spin pointing up or and - spin pointing down),  $f_s^0$  denotes the equilibrium Fermi-Dirac (F-D) distribution function, and  $\phi_s$  is the deviation from equilibrium, which in general depends on both the position and momentum of electrons. The equation satisfied by the distribution function

(Boltzmann equation) can be written as<sup>20</sup>

$$\frac{\partial f_s}{\partial t} = \left( \frac{\partial f_s}{\partial t} \right)_{\text{diff+field}} + \left( \frac{\partial f_s}{\partial t} \right)_{\text{coll}}, \quad (1)$$

The diff (diffusion) + field and collision terms appearing in the above equation can be written as<sup>20</sup>

$$\left( \frac{\partial f_s}{\partial t} \right)_{\text{diff+field}} = - \left[ v_{s,z} \frac{\partial f_s}{\partial z} + (-eE) v_{s,z} \frac{\partial f_s}{\partial \varepsilon} \right], \quad (2a)$$

$$\begin{aligned} \left( \frac{\partial f_s}{\partial t} \right)_{\text{coll}} = & \frac{1}{k_B T} \int \frac{d^3 k'_s}{8\pi^3} (\phi_{k,s} - \phi_{k',s}) P_{ks,k'_s}^0 \\ & + \frac{1}{k_B T} \int \frac{d^3 k_{-s}}{8\pi^3} (\phi_{ks} - \phi_{k_{-s}}) P_{ks,k_{-s}}^{sf}. \end{aligned} \quad (2b)$$

In writing Eq. (2a) we assumed that both the electric field ( $E$ ) and temperature gradient are along the  $z$  direction. We also assume that all the magnetizations are collinear, either parallel or antiparallel. The charge of the electron is taken as  $-e$ .  $v_{s,z}$  denotes the  $z$  component of the electron velocity and  $P_{ks,k'_s}^0 d^3 k'_s / (2\pi)^3$  denotes the transition probability per unit time in equilibrium conditions that an electron with wave vector  $k_s$  is scattered to a final state  $k'_s$  in the volume element  $d^3 k'_s / (2\pi)^3$ . Similarly,  $P_{ks,k_{-s}}^{sf} d^3 k_{-s} / (2\pi)^3$  denotes the transition probability per unit time in equilibrium conditions that an electron with wave vector  $k_s$  is scattered to a final state  $k_{-s}$  in the volume element  $d^3 k_{-s} / (2\pi)^3$ . All the scatterings are assumed to be elastic. The change in the distribution function  $\phi_s$  can be expressed in terms of the change in chemical potential ( $\delta\mu_s$ ), temperature ( $\delta T$ ), and displacement of the constant energy surface ( $h_s$ ). The first-order change in  $\phi_s$  can be written as

$$\phi_s(k, z) = -\delta\mu_s(z) - [\varepsilon_s(k) - \mu_0] \delta T(z) / T_0 + h_s(k, z), \quad (3)$$

where  $\mu_0$  and  $T_0$  are the chemical potential and temperature in equilibrium. The first two terms on the right-hand side (RHS) of the above equation represent the isotropic change of the F-D distribution in  $k$  space, whereas the  $h_s$  term represents a displacement in  $k$  space, such that the average of  $h_s$  over a constant energy surface vanishes.<sup>21</sup> The change in temperature ( $\delta T$ ) is assumed to be the same for up and down spin channels. Equation (3) is valid to first order in  $\delta\mu_s$ ,  $\delta T$ , and  $h_s$ , but is sufficient for calculating the entropy production, which is second order in these quantities.

From the distribution function, we can calculate various quantities such as the energy density, entropy density, and also energy current density, heat current density, entropy current density, etc. All these quantities are assumed to be additive in the two-channel model used here. The spin-density and spin-current density are defined as  $n_s = (-e) \int (d^3k/8\pi^3) f_{k,s}$  and  $J_s = (-e) \int (d^3k/8\pi^3) v_z f_{k,s}$ . It should be noted that, according to this definition, the spin density has units of charge density. Similarly, the energy density and energy current density are defined as  $u_s = \int (d^3k/8\pi^3) \varepsilon_{k,s} f_{k,s}$  and  $J_{u,s} = \int (d^3k/8\pi^3) v_z \varepsilon_{k,s} f_{k,s}$ . From Eqs. (1)–(3), we can write down the continuity equations for the spin and total energy densities as<sup>22</sup>

$$(\partial n_s / \partial t) + \partial J_s / \partial z = 2s G_{\text{mix}} (\Delta \mu / e), \quad s = \pm 1, \quad (4a)$$

$$(\partial u / \partial t) + (\partial J_u / \partial z) = E J. \quad (4b)$$

where the chemical potential splitting and the quantity  $G_{\text{mix}}$  are defined as  $\Delta \mu = (\delta \mu_+ - \delta \mu_-) / 2$  and

$$G_{\text{mix}} = \frac{e^2}{k_B T} \int \frac{d^3 k_s}{8\pi^3} \frac{d^3 k_{-s}}{8\pi^3} P_{ks,k-s}^{sf}. \quad (5)$$

$G_{\text{mix}}$  is always non-negative and has units of conductance/area. Equation (4b) for the total energy density is exact whereas Eq. (4a) for the spin density is valid to first order in  $\delta \mu_s$ ,  $\delta T$ , and  $h_s$ . This accuracy is sufficient for the present paper. The heat current density and rate of heat generation are defined as<sup>23</sup>  $J_{q,s} = J_{u,s} + (\mu_s / e) J_s$  and  $\partial q_s / \partial t = \partial u_s / \partial t + (\mu_s / e) \partial n_s / \partial t$ . The continuity equation for the total heat after some algebraic simplifications can be written as

$$\frac{\partial q}{\partial t} + \frac{\partial J_q}{\partial z} = J_+ \frac{1}{e} \frac{\partial \bar{\mu}_+}{\partial z} + J_- \frac{1}{e} \frac{\partial \bar{\mu}_-}{\partial z} + \frac{4(\Delta \mu)^2}{e^2} G_{\text{mix}}. \quad (6)$$

$\bar{\mu}_s$  in this equation represents the electrochemical potential, i.e.,  $\bar{\mu}_s = \mu_s - eV$ . Equation (6) is valid to second order in  $\delta \mu_s$ ,  $\delta T$ , and  $h_s$ .

Further, the entropy density of the electrons and entropy current density can be defined as<sup>20</sup>

$$S_s = -k_B \int \frac{d^3 k_s}{8\pi^3} [f_s \ln f_s + (1 - f_s) \ln(1 - f_s)], \quad (7)$$

$$J_{\text{ent},s} = -k_B \int \frac{d^3 k_s}{8\pi^3} v_{s,z} [f_s \ln f_s + (1 - f_s) \ln(1 - f_s)].$$

The above definition of the entropy density is exact in equilibrium, and we assume it also holds for small deviations from equilibrium. We can show that the relationship between heat current density and entropy current density,  $J_{q,s} = T J_{\text{ent},s}$ , is valid up to the second order in  $\delta \mu_s$  and  $\delta T$ .

The dissipation in the system can be identified by writing the continuity equation for the total entropy density ( $S = S_+ + S_-$ ) in the form<sup>24</sup>  $(\partial S / \partial t) + (\partial J_{\text{ent}} / \partial z) = \sigma$ . The RHS of this equation ( $\sigma$ ) is identified as the rate of entropy density production or dissipation.

From the expression for the entropy density given in Eq. (7), it can be shown that<sup>20</sup>

$$T_0 \frac{\partial S_s}{\partial t} = \int \frac{d^3 k_s}{8\pi^3} (\varepsilon_{k,s} - \mu_0 + \phi_{k,s}) \times \left[ \left( \frac{\partial f_s}{\partial t} \right)_{\text{diff+field}} + \left( \frac{\partial f_s}{\partial t} \right)_{\text{coll}} \right], \quad (8)$$

Thus  $\partial S_s / \partial t$  can be written as a sum of contributions from the diffusion + field term and the collision term. We first evaluate the contribution from the diffusion + field term up to second order in  $\delta \mu_s$  and  $\delta T$ . After some algebraic simplifications we get<sup>25</sup>

$$\left( \frac{\partial S_s}{\partial t} \right)_{\text{diff+field}} = -\frac{\partial}{\partial z} \left( \frac{J_{q,s}}{T} \right). \quad (9)$$

Replacing  $J_{q,s} / T$  by  $J_{\text{ent},s}$ , the continuity equation for entropy density becomes  $(\partial S_s / \partial t) + (\partial J_{\text{ent},s} / \partial z) = (\partial S_s / \partial t)_{\text{coll}}$ . Adding the two channels, we get  $(\partial S / \partial t) + (\partial J_{\text{ent}} / \partial z) = (\partial S / \partial t)_{\text{coll}}$ . We can thus identify  $(\partial S / \partial t)_{\text{coll}}$  as dissipation. In effect, we have shown that this continuity equation for entropy density holds even if we have a spatial gradient of the  $h_s$  term in the Boltzmann equation [see Eq. (2a)]. (Note that such a spatial gradient of  $h_s$  accounts for the spatial distribution of spin accumulation.) From Eq. (8), we can see that the contribution from the first two terms (involving  $\varepsilon_{k,s} - \mu_0$ ) to  $(\partial S / \partial t)_{\text{coll}}$  vanishes due to energy and total number conservation during collisions. The contribution from the second term (considering only spin-flip collisions) after adding the two spin channels and using the symmetry property of the scattering probability ( $P_{ks,k-s}^{sf} = P_{k-s,ks}^{sf}$ ) can be expressed as<sup>20</sup>

$$\left( \frac{\partial S}{\partial t} \right)_{\text{coll}}^{s-f} = \frac{1}{k_B T_0^2} \int \frac{d^3 k_s}{8\pi^3} \frac{d^3 k_{-s}}{8\pi^3} (\phi_{k_s} - \phi_{k_{-s}})^2 P_{ks,k-s}^{s-f}.$$

This shows that the total dissipation arising from the spin-flip scattering is always non-negative. (This is basically Boltzmann's  $H$  theorem for electrons.<sup>20</sup>)

We are interested in calculating the dissipation in steady state. In steady state,

$$\left( \frac{\partial S}{\partial t} \right)_{\text{coll}} = -\left( \frac{\partial S}{\partial t} \right)_{\text{diff+field}} = \frac{1}{T} \frac{\partial J_q}{\partial z} + J_q \frac{\partial}{\partial z} \left( \frac{1}{T} \right). \quad (10)$$

Using the continuity equation for the heat current density [Eq. (6)], the steady state dissipation can be expressed as

$$\left( \frac{\partial S}{\partial t} \right)_{\text{coll}} = \frac{J_+}{T e} \frac{\partial \bar{\mu}_+}{\partial z} + \frac{J_-}{T e} \frac{\partial \bar{\mu}_-}{\partial z} + \frac{4(\Delta \mu)^2}{T e^2} G_{\text{mix}} + J_q \frac{\partial}{\partial z} \left( \frac{1}{T} \right). \quad (11)$$

Using the continuity equation for spin density [Eq. (4a)],  $J = J_+ + J_-$  with  $\partial J / \partial z = 0$ , and defining the spin current as  $J_{\text{spin}} = J_+ - J_-$ , Eq. (11) can be written in alternative forms:

$$\left( \frac{\partial S}{\partial t} \right)_{\text{coll}} = \frac{J_+}{T e} \frac{\partial \bar{\mu}_+}{\partial z} + \frac{J_-}{T e} \frac{\partial \bar{\mu}_-}{\partial z} + \frac{\partial J_{\text{spin}}}{\partial z} \frac{\Delta \mu}{T e} + J_q \frac{\partial}{\partial z} \frac{1}{T} \quad (12a)$$

$$= \frac{J}{T e} \frac{\partial \bar{\mu}}{\partial z} + \frac{1}{T} \frac{\partial}{\partial z} \left( \frac{\Delta \mu}{e} J_{\text{spin}} \right) + J_q \frac{\partial}{\partial z} \frac{1}{T} \quad (12b)$$

$$= \frac{J}{T e} \frac{\partial \bar{\mu}}{\partial z} + \frac{J_{\text{spin}}}{T e} \frac{\partial \Delta \mu}{\partial z} + \frac{\partial J_{\text{spin}}}{\partial z} \frac{\Delta \mu}{T e} + J_q \frac{\partial}{\partial z} \frac{1}{T}. \quad (12c)$$

The entropy density production in a system can always be written as a product of thermodynamic forces and thermodynamic currents.<sup>24</sup> From Eq. (12a) we can identify  $(J_+, \frac{1}{Te} \frac{\partial \bar{\mu}_+}{\partial z})$ ,  $(J_-, \frac{1}{Te} \frac{\partial \bar{\mu}_-}{\partial z})$ ,  $(\frac{\partial J_{\text{spin}}}{\partial z}, \frac{\Delta \mu}{Te})$ , and  $(J_q, \frac{\partial}{\partial z} \frac{1}{T})$  as thermodynamic current-force pairs. The terms 1, 2, and 4 on the RHS of Eq. (12a) are dot products of two vectors (in position space), whereas the third term is a product of two scalars.  $[(\partial J_{\text{spin}}/\partial z)$  in the general case should be replaced by  $\nabla_r \cdot J_{\text{spin}}$ , which is a scalar in position space.] This pair of thermodynamic force and current was considered as the affinity and speed of a chemical reaction in Ref. 16. In general we have the following linear relationship between the currents and forces:

$$\begin{pmatrix} J_+ \\ J_- \\ J_q \\ \frac{\partial J_{\text{spin}}}{\partial z} \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} & L_{13} & 0 \\ L_{21} & L_{22} & L_{23} & 0 \\ L_{31} & L_{32} & L_{33} & 0 \\ 0 & 0 & 0 & 4TG_{\text{mix}} \end{pmatrix} \begin{pmatrix} \frac{1}{Te} \frac{\partial \bar{\mu}_+}{\partial z} \\ \frac{1}{Te} \frac{\partial \bar{\mu}_-}{\partial z} \\ \frac{\partial(1/T)}{\partial z} \\ \frac{\Delta \mu}{Te} \end{pmatrix}. \quad (13)$$

By Onsager's relation,<sup>20</sup> this matrix should be positive definite and symmetrical. The identification of thermodynamic forces and currents is not unique. For example, we can also use Eq. (12c) instead of Eq. (11) to define them.<sup>16</sup> The conductance  $G_{\text{mix}}$  is always non-negative as can be seen from its definition in Eq. (5). Also, from the continuity equation (4a) for the spin density, the relation  $\partial J_{\text{spin}}/\partial z = 4G_{\text{mix}} \Delta \mu$  is valid to first order, and thus the matrix elements in the last row and last column in Eq. (10) are zero. This is also expected from the fact that the last pair of thermodynamic current and force  $(\frac{\partial J_{\text{spin}}}{\partial z}, \frac{\Delta \mu}{Te})$  is scalar, while the other pairs are vectors.

By using the relationship between  $P$  and the scattering probability  $W$ ,  $P_{ks,k-s}^{sf} = f_{ks}^0(1 - f_{k-s}^0)W_{ks,k-s}^{sf}$ ,  $G_{\text{mix}}$  (in the case of metals) can be written in terms of the relaxation time as  $G_{\text{mix}} \approx e^2 \frac{g_s}{\tau_s^{sf}}$ , where  $g_s$  is the density of states at the Fermi energy and  $\tau_s^{sf}$  is the spin-flip time at the Fermi energy. The energy-dependent spin-flip time is defined here as  $\frac{1}{\tau_s^{sf}(\epsilon_s)} = \int \frac{d^3k_{k-s}}{8\pi^3} W_{ks,k-s}^{sf}$ . Further, by using the definition of the spin-diffusion lengths for the + and - channels (defined in Ref. 21 for metals by using the relaxation time approximation),  $G_{\text{mix}}$  can be written as  $G_{\text{mix}} = \sigma_+/l_+^2 = \sigma_-/l_-^2$ , where  $\sigma_s$  denotes the conductivity and  $l_s$  the spin-diffusion length.

We now give a simple interpretation of the result we have obtained. We consider a case where the temperature is uniform, and we make the relaxation time approximation for the collision term.<sup>21</sup> In this case, we need to solve the following four equations:  $J_+ = (\sigma_+/e)(\partial \bar{\mu}_+/\partial z)$ ,  $J_- = (\sigma_-/e)(\partial \bar{\mu}_-/\partial z)$ ,  $(\partial J_{\text{spin}}/\partial z) = 4G_{\text{mix}} \Delta \mu/e$ , and  $\partial J/\partial z = 0$ . These equations can be represented as the continuous circuit model shown in Fig. 1. It can be seen that the first and second terms in Eq. (11) correspond to the dissipation in the + and - spin channel resistances, respectively, and the third term corresponds to the dissipation in the mixing conductance branch.

In Eqs. (12b) and (12c), we have separated the dissipation arising from the charge and spin currents. It can be seen that even if we have a pure spin current with no charge current, there is still dissipation. This dissipation occurs due to the spin-flip scattering. Thinking in terms of the circuit model (Fig. 1), the

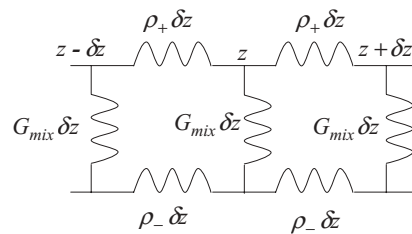


FIG. 1. Continuous circuit representation for the equations of spin-polarized current transport in the two-channel model.  $\rho^+$  and  $\rho^-$  are the resistivities of up and down spin channels.  $G_{\text{mix}}$  is the conductance per unit area through which the current can pass from one channel to other. The total dissipation is given by the sum of the dissipations in each branch.

dissipation occurs in the mixing conductance branch as well as in the + and - spin channel resistances. The change in  $\bar{\mu}_s$  with distance due to spin-flip scattering gives rise to dissipation in the + and - spin channel resistances. If we are interested only in transporting spin current in a spintronic device, it is better to use a pure spin current to avoid the dissipation arising from the charge current. The dissipation produced by the pure spin current can be measured experimentally by using nonlocal geometry, e.g., by measuring the rise in the temperature by local thermometry or scanning thermal microscopy. Materials with long spin-diffusion lengths could be more suitable for these experiments.

We now apply these results to an instructive example involving two semi-infinite metallic ferromagnets with opposite magnetizations as shown in Fig. 2. The nonmagnetic layer separating these two ferromagnets is assumed to be very thin compared to the spin-diffusion length, and hence can

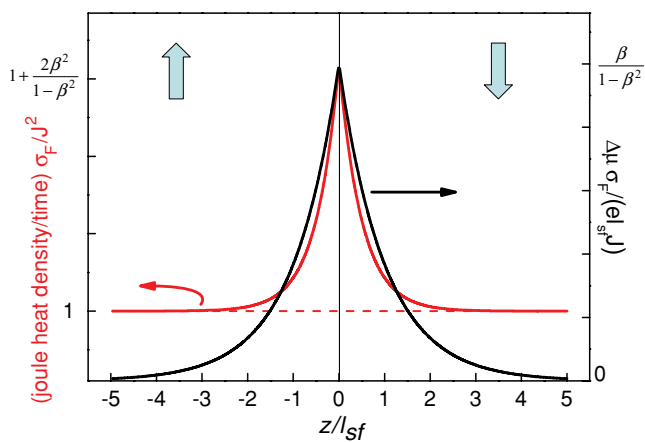


FIG. 2. (Color online) Two ferromagnets with opposite magnetizations with interface at  $z = 0$ . A current density  $J$  is flowing perpendicular to the interface, from left to right. The black curve shows the variation of chemical potential splitting ( $\Delta \mu$ ), which gives rise to spin-coupled interface resistance related to the GMR effect. The red curve shows the Joule heat density produced per unit time by the current. The Joule heat density per unit time is increased over its background value (shown by the dotted red line) near the interface due to the dissipation in the spin-coupled interface resistance. The Joule heat density per unit time produced due to the spin-coupled interface resistance is more localized than the variation of  $\Delta \mu$ .

be neglected. A charge current density  $J$  flows from left to right, and the temperature is uniform. We can solve for the spin-dependent electrochemical potentials and spin-current densities by assuming that they are continuous at the interface between the two ferromagnets.<sup>21</sup> (The interface resistance is neglected here.) The chemical potential splitting decays on the length scale of the spin-diffusion length as shown by the black curve in Fig. 2. This variation of  $\Delta\mu$  gives rise to a “spin-coupled interface resistance” equal to  $\frac{2l_{sf}}{\sigma_F} \frac{\beta^2}{(1-\beta^2)}$ , where  $\beta = (\sigma_\uparrow - \sigma_\downarrow)/(\sigma_\uparrow + \sigma_\downarrow)$ ,  $1/l_{sf}^2 = (1/l_\uparrow^2) + (1/l_\downarrow^2)$ , and  $\uparrow, \downarrow$  denote the majority and minority spin channels respectively.<sup>21,26</sup> (The units of spin-coupled interface resistance defined above are resistance  $\times$  area.) The application of Eq. (12b) to calculate the total Joule heat per unit area per unit time produced between two planes far away from the interface gives the expected result:  $\int_{-\infty}^{\infty} T(\partial S/\partial t)_{\text{coll}} dz = (J/e)[\bar{\mu}(\infty) - \bar{\mu}(-\infty)]$ , since the chemical potential splitting is 0 at  $\pm\infty$ . Further, the position dependence of the Joule heat density per unit time, obtained from Eq. (12b), can be written as

$$T \left( \frac{\partial S}{\partial t} \right)_{\text{coll}} = \frac{J^2}{\sigma_F} \left[ 1 + \frac{2\beta^2}{1-\beta^2} \exp(-2|z|/l_{sf}) \right]. \quad (14)$$

Thus the Joule heat density per unit time is the sum of a constant and a term decaying on the length scale of half the spin-diffusion length, as shown by the red curve in Fig. 2. It can be seen that the integral of the last term in Eq. (14), from  $z = -\infty$  to  $z = \infty$ , is equal to the product of  $J^2$  and the spin-coupled interface resistance. Thus the second term in Eq. (14) can be associated with the dissipation produced by the spin-coupled interface resistance. Interestingly, this dissipation is

more localized than the variation of the chemical potential splitting.

We now consider the effect of the spin-dependent interface resistance between the ferromagnet (FM) and nonmagnet (NM). This gives rise to a discontinuity in the spin-dependent electrochemical potentials, as  $\delta\bar{\mu}_s \equiv \bar{\mu}_s(z = z_0^+) - \bar{\mu}_s(z = z_0^-) = er_s J_s(z = z_0)$ , where  $r_s$  is the spin-dependent interface resistance.<sup>21</sup> We assume that the spin current density is continuous at the interface. The discontinuity in  $\bar{\mu}_s$  will give rise to a localized Joule heat per unit area per unit time of  $(J_+/e)\delta(\bar{\mu}_+) + (J_-/e)\delta(\bar{\mu}_-)$  at the interface. This expression can be written as  $(J/e)\delta(\bar{\mu}) + (J_{\text{spin}}/e)\delta(\Delta\mu)$ . This expression gives the correct Joule heat produced at the interface as can be seen as follows: Consider an interface at  $z = 0$  between a semi-infinite NM and FM. The total Joule heat per unit area per unit time should be given by the product  $(J/e)[\bar{\mu}(\infty) - \bar{\mu}(-\infty)]$ . From Eq. (12b), the Joule heat per unit area per unit time produced in the NM is given by  $(J/e)[\bar{\mu}(0^-) - \bar{\mu}(-\infty)] + J_{\text{spin}}(0)\Delta\mu(0^-)/e$ , whereas in the FM it is given by  $(J/e)[\bar{\mu}(\infty) - \bar{\mu}(0^+)] + J_{\text{spin}}(0)\Delta\mu(0^-)/e$ . Thus the Joule heat per unit area per unit time at the interface should be given by  $(J/e)\delta(\bar{\mu}) + J_{\text{spin}}(0)\delta(\Delta\mu)/e$ .

In summary, we have calculated the dissipation produced due to spin-flip scattering by using the semiclassical Boltzmann equation. We found that even a pure spin current without any charge current gives rise to dissipation. We applied the results to obtain the Joule heat density produced by current flowing through an interface between two ferromagnets. Within the Valet-Fert model,<sup>21</sup> these results can be interpreted in terms of a simple circuit model. The Joule heat density per unit time produced near the interface is more localized than the variation of the chemical potential splitting.

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<sup>22</sup>To derive Eq. (4), we set the integral  $\int \frac{d^3k_s}{8\pi^3} \frac{d^3k_{-s}}{8\pi^3} h_s P_{k_s, k_{-s}}^{sf}$  to 0.

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<sup>24</sup>D. Kondepudi and I. Prigogine, *Modern Thermodynamics: From Heat Engines to Dissipative Structures*, (Wiley, New York, 1998).

<sup>25</sup> $h_s$  represents the displacement of the Fermi sphere, thus we set integrals of the type  $\int \frac{d^3k}{4\pi^3} h_s^2 v_z \frac{\partial f_s^0}{\partial \epsilon}$  to 0 while deriving Eq. (9).

<sup>26</sup>We define  $\sigma_\uparrow = (\sigma_F/2)(1 + \beta)$  and  $\sigma_\downarrow = (\sigma_F/2)(1 - \beta)$ .