Spin accumulation and spin read out without magnetic field

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An idea for construction of two spintronic single-electron nanodevices is presented and supported by a quantum-mechanical simulation of their operation. The first device selects electrons of a given spin orientation and the other performs the spin read out. The operation of proposed devices exploits the spin-dependent deflection of electron trajectories induced by the spin-orbit Rashba coupling and does not require application of an external magnetic field. The operation of the nanodevice requires application of weak voltages applied to the electrodes only.

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I. INTRODUCTION

Many extensive efforts have been conducted for fabrication of a quantum computer based on the semiconductor nanostructures. The quantum bit of information is supposed to be stored in the electron spin confined in a quantum dot. $1-3$ Electrostatic quantum dots $4-8$ are considered particularly promising for quantum logic processing including the storage of single separate electrons and operations on their spins. In most of the devices constructed so far the modification of the electron spin state is induced by absorption of microwave radiation in high magnetic field that energetically separates the spin-up and spin-down states. This is the most direct method for rotation of a single-electron spin. However, the microwave radiation is not suitable for addressing a single spin in a register of several qubits contained within the same nanostructure. The use of the external microwave radiation was avoided in a device of Ref. [8](#page-4-3) in which the spin rotation is accomplished due to the spin-orbit (SO) coupling. This devic[e8](#page-4-3) still requires application of an external magnetic field, which induces a continuous precession of spins of all the confined electrons. The inhomogeneities of this field result in dephasing of the precession of separate spins. A device that could operate without an external magnetic field would be free of this source of decoherence. Recently, we proposed a couple of devices rotating the electron spins with-out the external magnetic field.^{9[,10](#page-4-5)} We introduced an idea and simulated the operation of nanodevices that perform the single-qubit Haddamard, negation and phase change quantum gate operations. The nanodevices exploit the selffocusing of electron wave function due to interaction with the electron gas of the electrodes.¹¹ The interaction allows for formation of a stable electron wave packet that can be put in motion by low voltages applied to the electrodes. The motion of the electron along any desired trajectory combined with the spin-orbit coupling allows for arbitrary spin rotations. In this work we present an idea for construction of spin filters that do without the external magnetic field. A first variant of the nanodevice can be used for selection of an electron of a desired spin orientation for the purpose of the initial state set up. The second variant is suitable for the spin read out on the final state.

II. DEVICE AND NUMERICAL METHOD

The proposed device is based on a planar heterostructure similar to the one previously used in Ref. [9](#page-4-4) with a schematic cross section given in Fig. [1.](#page-0-0) The nanostructure contains a quantum well 10 nm wide sandwiched between two barriers each of 10 nm width. The quantum well is separated from the substrate by a 50 nm thick undoped layer. On top of the upper blocking layer the metal electrodes are deposited. Electron confined in the quantum well forms a charge "cloud" distribution that induces an appearance of positive charge on the lower surface of the metal electrodes. The electric field stemming from the positive induced charge possesses an in-plane component directed to the center of the electron charge distribution. The wave function of the electron that is formed in this way becomes a stable packet that can move within the quantum well with a constant shape. When the self-focusing effect is strong enough (the quantum well is close to the electrodes and the dielectric constant is not too large) the scattering properties of the wave packet become classical, i.e., the electron backscatters or transfers through a potential defect with a 0% or 100% probability.¹¹ The electron with classical scattering properties is still described by the time-dependent Schrödinger equation. The electron spin is also described in a standard way as two-row

FIG. 1. (Color online) Schematics of the considered nanodevice with electrodes deposited on top, the electron wave packet, and the charge induced on the lower surface of the electrode.

single-column vectors. We choose the system of coordinates in which the *y* axis is oriented parallel to the growth direction. The electron motion in this direction is frozen. The electron is free to move in the *x* and *z* directions within the quantum well. Its wave function

$$
\Psi(x,z,t) = \begin{pmatrix} \Psi_1(x,z,t) \\ \Psi_2(x,z,t) \end{pmatrix}
$$
 (1)

depends on two spatial coordinates and time. The time dependence is described by the Schrödinger equation

$$
\Psi(x,z,t+dt) = \Psi(x,z,t-dt) - \frac{2i}{\hbar} \hat{H} \Psi(x,z,t) dt \qquad (2)
$$

with Hamiltonian

$$
\hat{H}(x,z,t) = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) - e \phi(x,y_0,z,t) + H_R, \quad (3)
$$

where y_0 is the center of the quantum well and $\phi(x, y_0, z, t)$ is the electrostatic potential due to the electrodes and the charges induced on them. The potential is found by solution of the Poisson equation in a three-dimensional box that contains the entire nanodevice. The Poisson equation needs to be solved in every time step due to the motion of the wave packet. The description of the method is given in Refs. [9](#page-4-4) and [12.](#page-4-7) The Poisson equation gives the classical potential distribution. Quantum calculations¹³ indicate that this is a good approximation of the actual response potential of the electron gas. The applied approach allows correctly describes the self-focusing mechanism and allows for investigation of the motion of the electron packet. The last term of Hamiltonian (3) (3) (3) accounts for the Rashba spin-orbit interaction¹⁴

$$
H_R = \alpha (p_z \sigma_x - p_x \sigma_z), \tag{4}
$$

where p 's are the momentum operators and σ 's are the Pauli matrices. In the initial moment of each simulation the wave function was assumed as a solution to the time-independent Schrödinger equation with the electron cloud distribution corresponding to the bound state confined under one of the electrodes,

$$
H(x, z, 0)\Psi(x, z, 0) = E\Psi(x, z, 0).
$$
 (5)

In the calculations we adopted Si material parameters with $m=0.19m_0$, the dielectric constant $\epsilon=13$, and the Rashba coupling constant $\alpha = 7.2 \times 10^{-13}$ eV m.

III. RESULTS AND DISCUSSION

The trajectory of an electron that is put in motion within a quantum well, in which the Rashba coupling is present, depends on the direction of its spin. The results of the simulation for various initial states of the spin are given in Fig. [2.](#page-1-1) For the initial condition we took the ground state of the electron confined in an induced quantum dot under electrode e_1 . The simulation is started by raising the potential of e_2 electrode by 0.1 mV, which extracts the electron from under electrode e_1 to under electrode e_2 . The electron acquires an initial velocity that is parallel to the *x* axis. The electron

FIG. 2. (Color online) Electron trajectories for opposite orientations of the spin. The solid lines—black, dark gray (blue online), and light gray (red online) correspond to the spin orientation parallel to x , y , and xy (a bisector of the angle formed by x and y axes). The dotted lines correspond to spin state containing a 95% of the spin parallel to the *z* axis and 5% of the state parallel to the *x*, *y*, and *xy* axes. The straight green line shows the electron trajectory for the initial spin orientation parallel or antiparallel to the *z* axis.

whose spin is parallel or antiparallel to the *z* axis moves along a straight line that is parallel to the *x* axis. This trajec-tory is marked in Fig. [2](#page-1-1) by the light gray straight line (green online). The Rashba coupling induces rotation of the electron spin moving along the *x* axis around the *z* axis, hence for both considered spin orientations they remain unchanged as the electron moves. However, when the electron wave function contains a contribution of any other spin component the electron trajectory is no longer a straight line. This effect can be used to filter out the electrons with spins that are not parallel to the *z* axis. This operation can be performed using the nanodevice presented in Fig. [3.](#page-1-2) The electron is initially confined under electrode e_1 in the lowest energy state for a given spin orientation. Then, the voltage on electrodes e_1 and e_2 e_2 is lowered by 0.1 mV and we start the iteration of Eq. (2) . The electron is ejected under electrode e_3 and acquires a velocity parallel to the *x* axis. The width of e_3 electrode and the distance to the lateral electrodes e_4 and e_5 is adjusted in a way that the center of the packet is localized under electrode e_3 and the tails of the packet reach the lateral e_4 and e_5 electrodes. On electrodes e_4 and e_5 we put a voltage 0.1 mV higher than the one applied to the to e_3 . Hence the straight motion is only weakly stabilized and any deviation of the electron direction leads to its extraction to one of the lateral electrodes. In consequence only electrons with spin parallel or antiparallel to the *z* axis cross the entire length of the e_3 electrode. Even a small admixture of the spin that is neither parallel not antiparallel to the *z* axis leads to the electron escape to the area below the lateral electrodes (e_4, e_5) .

FIG. 3. (Color online) The system of electrodes in the nanodevice that filters out the electrons with spin that is not parallel to the *z* axis. Electron trajectories of various initial spin orientations are marked as in Fig. [2.](#page-1-1)

FIG. 4. (Color online) Nanodevice for the spin accumulation. The system of electrodes (gray color) and electron trajectories of spin up (down) marked in gray (red online) [dark gray (blue online)]. The trajectory is defined by mean positions of the electron packet.

We use the idea of the spin filter of Fig. [3](#page-1-2) for a larger nanodevice dedicated to spin accumulation (see Fig. [4](#page-2-0)). The spin filter is placed at the lower left corner of the plot. Only the electrons of spins parallel or antiparallel to the *z* axis pass through this filter and get to the area below the large electrode e_6 . The proposed device separates the electrons with the spin oriented up from the spin-down electrons. The electron trajectory turns by 90° upon reflection on a 45° cut of the e_6 electrode for $x \approx 1300$ nm. After the electrons trajectory is changed, the spin-up and spin-down electrons follow different trajectories: the spin-down electron is deflected to the left (blue curve in Fig. 4) and the spin-up electron to the right (red curve in Fig. [4](#page-2-0)). Figures $5(a)$ $5(a)$ and $5(b)$ shows the time dependence of the mean values of the electron positions $x(t) = \langle x \rangle$, $z(t) = \langle z \rangle$ (solid lines), and the mean value of the spin components $s_{x,y,z}(t) = \langle s_{x,y,z} \rangle$. (dashed lines) for the spin oriented initially "up" and "down," respectively. The $z(t)$ is the same for both initial spin orientations, the $x(t)$ curves overlap only at the first part of the trajectory when the electron moves ideally parallel to the *x* axis. The electron spin undergoes precession when it follows a curved trajectory. Thus one needs to allow the electron to cross a distance equal to SO length (for the applied material parameters λ_{SO} = 1750 nm) after which the initial electron spin orientation is restored and the trajectory becomes parallel to the *z* axis again (see Figs. 4 and 5). Then, the electron is reflected for the second time from a properly cut top edge of the e_6 electrode. It starts to move parallel to the *x* axis and the spin precession is terminated. The electrons with spin oriented initially up (down) get under electrode e_8 (e_7) . The electrons can be stored therein or taken away to other locations within the nanodevice.

Figure [6](#page-2-2) contains a schematic drawing of a nanodevice which is supposed to read the electron spin after completion of a quantum computation. Since the quantum algorithms account for limitations set for a possible measurement outcome due to the quantum nature of the system, our device is designed to answer the question: "is the electron spin oriented up?" The measurement is of a projective type, which

FIG. 5. (Color online) Average position and spin components of the electron following the trajectories depicted in Fig. [4.](#page-2-0) Solid lines shows the mean values of the position of the electron packet in *z* direction (black) and in x direction [gray (red online)], dashed lines shows the average components of the spin (a) for the electron spin oriented initially up and (b) for initial spin down.

means that in case of positive answer the spin state is unaffected. Otherwise the state may be changed.

The electron whose spin state we want to test is localized under electrode e_1 (Fig. [6](#page-2-2)). We lower the voltage applied to electrode e_2 , the electron is ejected out under electrode e_3 and moves beneath in the *z* direction till it reaches the ample

FIG. 6. (Color online) Nanodevice for the spin read out and trajectories of electrons with spins initially oriented up $\lceil \text{gray} \rceil$ (red online) curve] and down [dark gray (blue online)].

FIG. 7. (Color online) Same as Fig. $5(a)$ $5(a)$ for the spin-up electron following the red trajectory in the spin read out device of Fig. [6.](#page-2-2)

electrode *e*4, where the direction of its motion is changed by 90 $^{\circ}$, this time by reflection off the edge of electrode e_5 set at 45° angle, to which a (repulsive) voltage is applied by 1.5 mV lower than the one applied to *e*4. After the direction of electron motion is changed to *z*, the electron trajectory is no longer a straight line. If the electron spin was initially parallel to the z axis (oriented up) it chooses the trajectory that turns right and is marked in Fig. [6](#page-2-2) by the red color. For this specific case in Fig. [7](#page-3-0) we plotted the mean values of position and spin components in function of time. As long as the electron moves parallel to the *x* axis the spin is constant s_z $=\hbar/2$, $s_x = s_y = 0$. After the direction of motion is changed the spin starts to precess: s_z is changed and nonzero s_x , s_y components appear. After passing a distance of λ_{SO} in the *z* direction s_z component returns to its initial value and the others components vanish. The device is designed in such a way that at this very moment the electron hits the top edge of the electrode that is in this place perpendicular to the electron velocity. Upon reflection the electron does not follow the precedent trajectory when moving in the −*z* direction but is deflected to the left. After passing a distance of λ_{SO} in the −*z* direction the electron is reflected from the edge of *e*⁴ electrode and change its direction to −*x*. The electron goes under electrode e_5 whose voltage was in the meantime set equal to the voltage of *e*⁴ electrode, hence the region under $e₅$ is transparent for the electron motion. The electron is finally recaptured under e_1 electrode. On its way back the electron spin rotates by a full angle, its initial state is restored and the spin can be recycled for the quantum computation. For the electron spin oriented initially antiparallel to the *z* axis, the electron after reflection by the e_5 electrode is de-flected to the left (see the blue trajectory in Fig. [6](#page-2-2)). After passing the λ_{SO} distance in the *z* direction the electron is reflected by the edge of *e*⁴ electrode and starts to move in the −*x* direction. Eventually, the electron gets beneath electrode e_6 and it is taken away to under electrode e_7 where it is

FIG. 8. (Color online) Same as Fig. $5(b)$ $5(b)$ for the spin-down electron following the blue trajectory in the spin read out device of Fig. [6.](#page-2-2)

trapped. The presence of the electron under e_7 electrode can be verified by any method, including the ones that destroy its spin, since the mere presence of the electron in this location implies a negative answer for the question that was asked. The mean values of position and spin components in function of time are plotted in Fig. [8.](#page-3-1) In Fig. [6](#page-2-2) on the electron trajectories we marked the electron positions in subsequent moments in time t_0 − t_4 . The duration of the entire cycle is 0.43 ns, which is lower enough than the decoherence time in $Si.¹⁵$

IV. SUMMARY AND CONCLUSIONS

We have proposed and simulated operation of two nanodevices. One of them can be used to extract from the electron gas single electrons of a well-defined spin state or to spin accumulation, i.e., storage of electrons of opposite spin orientations in two separate regions of the nanodevice. The second device serves for the spin read out. It performs a projective measurement which for a positive answer leaves the spin in the identified state. The spin read out has the "interaction free measurement" character since the electron whose state could be perturbed implies a negative answer to the question asked. The nanodevices work without the external magnetic field. Its operation is controlled by low dc voltages applied to gates.

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