Composite fermions without a magnetic field: An application to the metal-insulator transition in a two-dimensional conductor with the long-range Coulomb interaction between electrons

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It is shown that a composite-fermion (CF) paradigm, which works nicely in the fractional quantum Hall effect, can be applied to the case of a low-density two-dimensional (2D) conductor with long-range Coulomb interactions between electrons without external magnetic field. This approach, based on the unitary Chern-Simons (CS) transformation, relates the physics of the metal-to-insulator transition (MIT) in 2D correlated electron system with the MIT in a 2D system of noninteracting CFs subject to the CS gauge field $b=2\Phi_0\rho(\mathbf{r})$ [Φ_0 is the flux quantum, $\rho(\mathbf{r})$ is the electron density]. The MIT in such system is of the same origin as a well-known transitions observed near the peaks in the diagonal resistivity R_{xx} of a 2D electron gas in the integer quantum Hall effect. The calculated longitudinal resistivity changes the sign of the temperature derivative from the metal-like, $dR_{xx}/dT>0$, at $\rho>\rho_c$ to the insulatorlike, $dR_{xx}/dT<0$, at lower densities $\rho<\rho_c$. A separatrix $R_{xx}^S(T)$ demarcating the metal and insulator phases at the critical density ρ_c is temperature independent in the uniform-density approximation $\rho(\mathbf{r})=\rho$. The CFs do not interact in this case, but if $\rho(\mathbf{r})\neq\rho$ a weak interaction between the CFs makes the separatrix a linear function of T. The mechanism of the CF conductivity near the ρ_c is the Mott variable range hopping which, in full agreement with experiments, takes a form $\sigma_{xx}\propto\exp[-AX^{\gamma/2}]$ assuming a scaling with respect to the variable $X=|\rho-\rho_c|/T^{\kappa}$, where γ and $\kappa=1/\gamma$ are the critical indices of the MIT. External perpendicular magnetic field shifts the value of ρ_c at which the MIT occurs due to the partial compensation of the CS gauge field b but does not change the shape of the resistivity curves $R_{xx}(T)$. Reflection symmetry between the R_{xx} and σ_{xx} on the opposite sides of the MIT and other relations of the results obtained with experiments in the high-mobility silicon metal-oxide semiconductor field-effect transistors are discussed.

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I. INTRODUCTION

The discovery of the metal-insulator transition (MIT) in two-dimensional (2D) electron system, a high-mobility silicon metal-oxide semiconductor field-effect transistor (Si MOSFET),[1–6] raised a heated debates because of the conflict with the scaling theory predictions.[7] This theory prohibits a true metallic state and the MIT in 2D systems of noninteracting electrons with disorder. The conflict was resolved after the role of electron-electron correlations in the MIT of 2D conductors has been clarified with the help of a more elaborated two-parametric scaling approach.[8] In the above papers the MIT means that the temperature derivative of the longitudinal resistivity changes its sign from positive (metal-like) $dR_{xx}/dT>0$ at $\rho>\rho_c$ to negative (insulatorlike) $dR_{xx}/dT<0$ at lower densities $\rho<\rho_c$. The critical concentration ρ_c demarcates the metal and insulator phases.

The experimental test of the two-parametric scaling theory[8] done in the work[9] proved that the MIT is governed by the two parameters controlling the strength of disorder and electron-electron correlations, respectively. An empirical two-parametric scaling function was suggested in this paper which fits within the accuracy of a few percents the experimental data.

Theoretically the problem is very difficult and in spite of all efforts the physics of the MIT in 2D conductors remains a puzzle so far. A review of the modern state of theory and experiment in the field of strongly correlated 2D fluids is given in Ref. 10.

On the contrary, the MIT in 2D conductors in quantizing perpendicular external magnetic field is a well-understood phenomenon related to the localization of electrons within the disorder-broaden Landau levels. In these systems only narrow stripes at the center of the Landau levels remain delocalized and provide a metallic conductivity when the chemical potential falls within the stripes. By changing external magnetic field or electron concentration one can shift the chemical potential from the localized (insulating) to extended (metallic) states within the disorder-broaden Landau levels. Such is a qualitative picture of the MIT in the 2D conductor in perpendicular magnetic field. It takes place under the conditions of the quantum Hall effect (QHE) within the transitional plateau-to-plateau regions of the Hall conductivity where narrow stripes of extended states provide sharp peaks in the diagonal conductivity σ_{xx} both in the integer and fractional regimes.[11,12] The fractional QHE (FQHE) is a result of strong electron correlations which become important in the lowest Landau level.[13,14] A concept of the composite fermions (CFs) was introduced in Ref. 15 to explain the FQHE as an integer QHE (IQHE) in a system of these weakly interacting quasiparticles. In what follows we will show that this concept can be applied to the case of strongly interacting 2D particles even without external magnetic field. Based on that idea we explain the MIT in 2D correlated and disordered conductors by mapping it onto the corresponding transition in the QHE systems. In this connection we must note that numerous experiments on the MIT in 2D Si MOSFET conductors with and without external mag-

netic field[1–6] bear many similar features typical for the MIT in the QHE.

It has been found yet in the early paper on the MIT in 2D (Ref. 2) that the temperature dependencies of the in-plane resistivity with and without perpendicular magnetic field are absolutely identical in shape. The only difference is in the shifted (enhanced) values of the electron concentrations ρ at which the corresponding curves have been measured. This observation is very important since external perpendicular quantizing magnetic field in the experiment[2] was so strong that the filling factor was 3/2. In such fields electrons are in the QHE regime and their wave functions dramatically differ from the conventional plane waves of 2D electrons without external magnetic field. The experiment[2] tells that the physics (unknown so far) beyond the MIT in 2D conductors with the strong Coulomb interaction between electrons with and without external magnetic field is actually the same. In particular, the symmetry of the electronic states obviously does not change in the quantizing magnetic field. We will show that this is really the case since the Coulomb interaction between electrons in 2D systems can be described in terms of the Chern-Simons gauge (fictitious, statistical) field which acts on a new quasiparticles, the composite fermions, very much the same as external perpendicular magnetic field acts on the trajectories of 2D electrons.

The purpose of this paper is to demonstrate a fundamental relationship between the MITs in the QHE systems and 2D strongly correlated system such as Si MOSFET conductors within the CF approach. This approach makes possible a mapping of the MIT in 2D conductors without magnetic field on the phase transition between delocalized and localized states in the 2D QHE systems subject to the statistical gauge field of the composite fermions. We will show that all basic experimental observations can be explained within this framework.

The CF is a quasiparticle composed of an electron with $2p$ magnetic flux quanta $\Phi_0=2\pi\hbar c/e$ attached ($p=1,2,3,\ldots$ is an integer, $\Phi_0=2.068\times10^{-15}$ Wb).[15] In the FQHE regime the 2D electrons in a strong magnetic field condense at the lowest Landau level so that their kinetic energy becomes smaller than the energy of the e-e Coulomb repulsion which determines completely the physics of the FQHE. Within the CF approach to the FQHE (Refs. 14 and 15) it was shown that a system of 2D strongly interacting electrons can be replaced by a gas of weakly interacting CFs moving in an effective magnetic field

$$B^*=B\pm 2p\Phi_0\rho, \tag{1}$$

where B is the external magnetic field, ρ is the electron density, and $\pm$ corresponds to two possible orientations of the flux attached (along or opposite to the direction of the field B).

The Landau quantization of a gas of noninteracting CFs yields discrete Landau levels with the filling factor $\nu^*=\rho\Phi_0/B^*$. The electron filling factor $\nu=\rho\Phi_0/B$ can be written in the form $\nu=\nu^*/(2p\nu^*\pm1)$, as one can see from Eq. (1). At integer values of the CFs filling factor $\nu^*=n$ the IQHE regime develops so that the Hall conductivity σ_{xy} jumps between the neighboring plateaus while the diagonal conductivity σ_{xx} has peaks as a function of B^* at the values where σ_{xy} jumps. These jumps and peaks in the 2D system of the CFs are mapped on the corresponding electron conductivities at fractional values of the filling factor $\nu=n/(2pn\pm1)$ called the principal sequence. That explains the FQHE in 2D correlated electron system as an IQHE of noninteracting composite fermions. In the case $p=1$ two flux quanta are attached to each electron so that, if the gauge field is antiparallel to the external field, Eq. (1) reads as $B^*=B(1-2\nu)$. At $\nu=1/2$ the external magnetic field is compensated completely by the fictitious field of the CFs making the effective field $B^*=0$. The value $\nu=1/2$ is a special point in the FQHE regime which is well confirmed by experiments. A theory of the FQHE at fillings $\nu=1/(2n\pm1)$ was developed first in terms of the Laughlin's wave function[13,14] and then a concept of the CF was introduced[15,16] which explained the FQHE states in the principle sequence. The Chern-Simons (CS) gauge field plays a crucial role in the theory of CFs which appears as vortices in the incompressible electron liquid attached to the electrons and carrying $2p$ flux quanta. A special experimental tests proved that the CFs at $\nu=1/2$ are real quasiparticles rather than a convenient theoretical construction.[17,18]

A natural question arises what happens with the composite fermions if the external magnetic field will be switched off? Theoretically, the composite fermions in 2D conductors appear as a result of the Coulomb interaction between electrons in the FQHE regime.[14,15] The key point is the CS unitary transformation which replaces strongly correlated 2D electrons by a 2D gas of weakly interacting composite fermions moving in the gauge magnetic field $b=2p\Phi_0\rho$. The value of this field for the typical experimental electron densities in the Si MOSFET $\rho\approx10^{11}$ cm^{-2} gives an estimate $b\approx4$ T for CFs with two fluxes ($p=1$). Such values of the CS gauge magnetic field in Si MOSFET assume the QHE regime and Landau quantization effects in the CFs dynamics even without external magnetic field. In the next section we will show how this unusual physics originates from the Coulomb interaction of electrons in 2D conductors.

II. CHERN-SIMONS TRANSFORMATION AND THE BASIC EQUATIONS

The driving force of the MIT in 2D IQHE systems is the filling factor of the disorder-broaden Landau levels which fixes the chemical potential either at extended or localized states depending on the strength of magnetic field and electron concentration. In the FHQE regime the Coulomb e-e correlations become important and the MIT takes place very much the same as in the IQHE but with the replacement of electrons by the CFs moving in the field B^*. The CFs appear in the FQHE physics as a result of the strong long-range Coulomb interactions. We will show below that the external magnetic field is not necessary for the creation of the CFs in 2D conductors with strong long-range Coulomb interactions. That gives a new look at the MIT in 2D strongly correlated electron systems such as Si MOSFET within the composite-fermion approach.

A quantitative measure of the effective e-e interaction in 2D conductors is the ratio of the Coulomb energy to the

Fermi energy, known as a dimensionless Wigner-Seitz radius $r_s=(\sqrt{\pi}\rho a_B)^{-1}$, where $a_B=\varepsilon\hbar^2/me^2$ is the effective Bohr radius. The Coulomb interaction is taken in the form $v(r)=e^2/\varepsilon r$ with ε standing for a dielectric constant. It is believed that in the limit $r_s\to\infty$ and in the absence of disorder the ground state of a 2D conductor is the Wigner crystal. Disorder, finite values of the correlation parameter r_s, and temperature prevent the Wigner crystallization but electrons might remain a liquid. An example is the GaAs-based QHE systems in which electrons comprise an incompressible electron liquid within the plateau regions.[13,14] The value of the correlation parameter r_s in the 2D Si MOSFET high-mobility conductors is evaluated as $r_s\geq 10$ which is less than the critical value $r_s\approx 38$ at which numerical calculations[19] predict a transition to the Wigner crystal state. Therefore electrons in these systems are strongly correlated and the MIT is driven by the electron concentration ρ.[1–6]

A decrease in the electron concentration enhances the parameter r_s driving thereby interacting 2D electrons toward the insulating Wigner-crystal state. In reality, disorder localizes the electrons long before the Wigner crystallization at some critical value, $\rho=\rho_c$, where a quantum phase transition into the insulating state occurs. The critical concentration ρ_c separates the metal and the insulator phases in which the derivative dR_{xx}/dT has different signs.

As will be shown below, the fact that *e-e* correlations are responsible both for the MIT and for formation of the CFs in 2D conductors assumes a fundamental relationship between these two phenomena. Such relationship can be established through the unitary Chern-Simons transformation which maps a system of the 2D strongly correlated electrons on the 2D gas of the weakly interacting CFs placed in a gauge magnetic field. This approach yields a possibility to describe the MIT as a quantum phase transition in the QHE system.

Although a concept of the CFs was introduced into the physics of 2D systems in the context of the FQHE it has been recognized later that the external magnetic field is not a necessary ingredient of the theory. As was shown in Ref. 20 the Chern-Simons gauge-field approach, developed earlier in Refs. 21 and 22 for description the FQHE, can be applied to 2D electron systems without external magnetic field. The CS gauge transformation actually produces a mapping of the kinetic energy of the 2D electrons on the Landau Hamiltonian for the new fermion particles of the same charge but interacting with the perpendicular gauge (statistical) magnetic field. Such a fermion-to-fermion transformation changes the nomenclature of the eigenstates in the kinetic energy of the Hamiltonian from the electronic plane waves to the Landau orbitals of the composite fermions. The latter is possible because in 2D systems not only the momentum within the plane but as well the angular momentum perpendicular to the plane is a quantum integral of motion. Another important point is that the CS transformation attaches an even number of gauge magnetic field flux quanta to each electron which means an appearance of new quasiparticles—the CFs.

Consider, following Refs. 20 and 21, a system of 2D spinless electrons with the Hamiltonian

$$H=\int d^2\mathbf{r}\frac{1}{2m}\left|\left(-i\hbar\nabla+\frac{e}{c}\mathbf{A}\right)\psi(\mathbf{r})\right|^2+V+U, \tag{2}$$

where

$$V=\frac{1}{2}\int d^2\mathbf{r}d^2\mathbf{r}'v(\mathbf{r},\mathbf{r}'):\rho(\mathbf{r})\rho(\mathbf{r}'): \tag{3}$$

is the energy of the screened *e-e* Coulomb interaction and

$$U=\int d^2\mathbf{r}U(\mathbf{r})\rho(\mathbf{r}). \tag{4}$$

$U(\mathbf{r})$ is the disorder potential, $\rho(\mathbf{r})=\psi(\mathbf{r})^+\psi(\mathbf{r})$ is the density of electrons in point $\mathbf{r}$, and colons in Eq. (3) represent normal ordering of the creation and annihilation electron operators. In general, due to the intrinsic disorder of the QHE systems, we assume below that the screened *e-e* Coulomb interaction $v(\mathbf{r},\mathbf{r}')$ is a function of the two independent arguments $\mathbf{r}$ and $\mathbf{r}'$ rather than $|\mathbf{r}-\mathbf{r}'|$.

The key point of the composite-fermion approach is the Chern-Simons unitary transformation

$$\psi_*(\mathbf{r})=\psi(\mathbf{r})\exp\left[i2p\int d^2\mathbf{r}'\rho(\mathbf{r}')\arg(\mathbf{r}-\mathbf{r}')\right], \tag{5}$$

where $\arg(\mathbf{r}-\mathbf{r}')$ is the angle between $(\mathbf{r}-\mathbf{r}')$ and the X axis.

The density operator is an invariant of the CS transformation $\rho(\mathbf{r})=\psi(\mathbf{r})^+\psi(\mathbf{r})=\psi_*(\mathbf{r})^+\psi_*(\mathbf{r})$ and so are the Coulomb [Eq. (3)] and disorder [Eq. (4)] terms but the kinetic energy changes. Therefore, after the CS unitary transformation Hamiltonian (2) takes the form

$$H_*=\int d^2\mathbf{r}\frac{1}{2m}\left|\left(-i\hbar\nabla+\frac{e}{c}\mathbf{A}-\frac{e}{c}\mathbf{a}\right)\psi_*(\mathbf{r})\right|^2+V+U. \tag{6}$$

The gauge-field operator

$$\mathbf{a}=2p\Phi_0\int d^2\mathbf{r}'\frac{\hat{z}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\rho(\mathbf{r}') \tag{7}$$

appears as a result of the CS transformation ($\hat{z}$ is the unit vector directed perpendicular to the 2D plane). In difference to the vector potential of an external magnetic field, $\mathbf{A}$, the quantity $\mathbf{a}$ is an operator which means that the kinetic energy term in Eq. (6) contains interactions between the new composite-fermion particles. At first glance this makes any manipulation with Hamiltonian (6) nearly intractable in general. Fortunately, as is well established in the CF theory, the mean-field approximation is a rather good approach to the problem which picks up the major physics and maps a system of 2D correlated electrons of the FQHE regime onto the nearly noninteracting gas of CFs in the IQHE regime.[20,21] Following this standard approximation we first replace the operator $\rho(\mathbf{r})$ by its expectation value $\rho(\mathbf{r})=\Sigma_i\delta(\mathbf{r}-\mathbf{r_i})$, which yields

$$\nabla\times\mathbf{a}=2\mathbf{p}\Phi_0\sum_{\mathbf{i}}\delta(\mathbf{r}-\mathbf{r_i}). \tag{8}$$

One can see from this equation that a gauge field $\mathbf{a}$ is produced by the ensemble of δ solenoids, each carrying $2p$ flux quanta attached to every electron. These new particles are nothing but the composite fermions.

Further standard approximation is a replacement of the $\rho(\mathbf{r})$ in Eq. (8) by the constant average value ρ which means

that the gauge magnetic field $\mathbf{b}(\mathbf{r})=\nabla\times\mathbf{a}(\mathbf{r})$ is a constant too $\mathbf{b}(\mathbf{r})=b\hat{z}$, where $b=2p\Phi_0\rho$. In the case $\rho(\mathbf{r})=\rho$ the Coulomb (V) and disorder (U) terms in Eq. (6) become constants (finite, for samples of finite area S) and can be discarded. Hamiltonian (6) in this approximation describes a motion of the free CFs in the uniform magnetic field $B^*=B-b$ [Eq. (1)]. Note that nothing changes in the above derivation in the absence of the external magnetic field $\mathbf{A}$. We arrive therefore at the following picture. In the first approximation (which neglects the electron-density fluctuations) a system of 2D electrons with *e-e* Coulomb interaction can be mapped through a unitary CS transformation on a 2D system of noninteracting CFs placed into a uniform gauge field of the strength $b=2p\Phi_0\rho$ directed perpendicular to the plane. The nontrivial point is that the CFs belong to the Landau states even without external magnetic field. That explains an enigmatic experimental result of the paper[2] in which a strong perpendicular magnetic field B did not change the shape of the temperature dependencies of resistivity in the MIT compared to the case $B=0$. The shift in the electron concentrations which was found for the identical curves follows directly from the relation $B^*=B-2p\Phi_0\rho$ (see the Appendix for details).

III. DIAGONAL IN-PLANE CONDUCTIVITY OF THE COMPOSITE FERMIONS

To explain the experiments[1–6] we have to calculate the diagonal conductivity, σ_{xx}, of the strongly correlated 2D electron system. In view of the unitary equivalence of Hamiltonians (2) and (6) for electrons and the CFs this conductivity equals to the $\sigma_{xx}^{\mathrm{CF}}$, the conductivity of the CFs, moving in the CS gauge magnetic field $\mathbf{b}(\mathbf{r})=\nabla\times\mathbf{a}(\mathbf{r})$.

In the uniform-density approximation, $\rho(\mathbf{r})=\rho$, CFs do not interact. Small deviations from the uniform approximation make the CFs a weakly interacting particles. The effects related with this interaction can be treated perturbatively and, as the analysis shows,[23] they only add some details but do not change the physics and results of the free CF approach.

We start calculations of the diagonal conductivity $\sigma_{xx}=\sigma_{xx}^{\mathrm{CF}}$ by putting $\rho(\mathbf{r})=\rho+\delta\rho(\mathbf{r})$ into Eqs. (3) and (4) which yields $U=U_0+\delta U$, $V=V_0+\delta V$. The constants U_0 and V_0 [given by Eqs. (3) and (4) with $\rho(\mathbf{r})=\rho$] can be discarded and remaining terms are

$$\delta U=\int d^2\mathbf{r}U_{eff}(\mathbf{r})\delta\rho(\mathbf{r}), \tag{9}$$

$$U_{eff}=U(\mathbf{r})+\rho\int d^2\mathbf{r}'v(\mathbf{r},\mathbf{r}'), \tag{10}$$

$$\delta V=\frac{1}{2}\int d^2\mathbf{r}d^2\mathbf{r}'\,\delta\rho(\mathbf{r})v(\mathbf{r},\mathbf{r}')\delta\rho(\mathbf{r}'). \tag{11}$$

One can see from Eqs. (9) and (10) that the screened Coulomb interaction changes effective disorder potential and thereby can influence the localization-to-delocalization transition in 2D conductor. Within the renormalization-group approach this point was considered in Ref. 8.

Assuming then that the inhomogeneity is small, $|\delta\rho(\mathbf{r})|\ll\rho$, we will neglect the interaction term $\delta V\ll\delta U$ and a small correction $\delta b=|2p\Phi_0\delta\rho(\mathbf{r})|$ to the gauge field $B^*\gg\delta b$. The role of the remaining weak correlations between the CFs will be discussed later.

Then following the standard CF approach, we reduce the problem of the conductivity calculation in a 2D strongly correlated electron system to calculations of the conductivity in a system of 2D noninteracting CFs moving in a fictitious uniform gauge field and weak disorder potential. The latter problem is identical to calculations of the diagonal conductivity σ_{xx} in the 2D electron conductor subjected to perpendicular magnetic field b. For typical concentrations of electrons in the Si MOSFET $\rho\approx10^{11}$ cm^{-2} this field can be estimated as $b\approx4$ T. The 2D electrons in the Si MOSFET in such perpendicular magnetic fields are in the QHE regime.[6,10,24]

Therefore, the problem is reduced to the calculations of conductivity for 2D gas of the composite fermions in perpendicular magnetic field under the conditions of the QHE. The main difficulty in the calculations of σ_{xx} in the QHE regime is related to the localization. As is well known, all states within the disorder-broadened Landau levels are localized except those belonging to the narrow strips of extended states in the middle responsible for the peaks in diagonal conductivity located at the plateau-to-plateau transitions in the Hall conductivity. Correspondingly, the neighboring insulating phases on both sides of the peak in $\sigma_{xx}(B)$ are separated by metallic strips as follows from the phase diagram of the Si MOSFET.[6,10,24] At fields (or electron concentrations) where the critical points are placed the derivatives dR_{xx}/dT change sign and the MITs occur.

An analytic expression for the σ_{xx} in 2D quantum Hall conductor taking into account the localization and mobility edges effects within the disorder-broaden Landau bands was obtained in Refs. 25 and 26. It explains unusual quantum magnetic oscillations with the MIT in the background magnetoresistance in quasi-2D organic superconductor,[27] irregular peak-splitting effect in the σ_{xx} caused by the fractal Hofstadter butterfly substructure of the Landau levels[28] in experiments with the artificial lateral superlattices.[29]

At low temperatures, $T\ll\hbar/\tau$, (τ is the scattering time) the conductivity σ_{xx} in the QHE regime nearby the critical field B_c takes the form[25,26]

$$\sigma_{xx}(B,T)=\sigma_\tau(0)\exp(-\sqrt{T_0/T}). \tag{12}$$

The Mott exponent in this equation reflects the variable-range-hopping mechanism of conductivity within the localized states of disorder-broaden Landau levels.[30,31] Since $T_0\propto1/\xi\propto|B-B_c|^\gamma$ this factor as a function of the magnetic field and temperature reads as follows:

$$\exp(-\sqrt{T_0/T})=\exp[-A(|B-B_c|/T^\kappa)^{\gamma/2}]. \tag{13}$$

(ξ is the correlation length, γ and $\kappa=1/\gamma$ are the critical indices, B_c is the critical field at which the MIT occurs, A is a constant)

$$\sigma_\tau(0) = \frac{e^2 N_L \tau \langle v_x^2 \rangle}{\hbar \Omega}. \tag{14}$$

The average of the velocity squared, is given by[26]

$$\langle v_x^2 \rangle = \frac{a^2}{2\hbar^2} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} d\varepsilon g(\varepsilon) |t_{\varepsilon,\varepsilon}|^2. \tag{15}$$

The other notations are: $N_L = \Phi / S\Phi_0$ is the electron density at the Landau level, Φ is the flux through a sample, $|t_{\varepsilon,\varepsilon}|$ is the hopping matrix element between the Landau orbitals, a is an average distance of hopping, $\Omega = eB/mc$ is the cyclotron frequency, m is electron mass, and c is the speed of light.

The integral in Eq. (15) is taken within the narrow stripe of delocalized states at the center of the disorder-broaden Landau bands. The tails in the density of states within these bands $g(\varepsilon)$ can be wide but the width of the delocalized states stripe is small $(\varepsilon_M - \varepsilon_m) \ll \hbar/\tau$. It is believed that at least one state in the middle of Landau bands is delocalized. Variations in the magnetic field change the population of the Landau bands and the MIT in the QHE takes place when the chemical potential μ crosses the mobility edge separating extended and localized states. On the technical side the smallness of the width of extended states simplifies equations and make possible to write the conductivity in a general form of Eq. (12) without a specific choice of the density of states $g(\varepsilon)$.[25,26]

The Hall and diagonal conductivities in the QHE regime are related through empiric semicircle rule[32] which for transition between the neighboring plateaus with the Hall conductivities σ_1 and σ_2 takes the form

$$(\sigma_{xx})^2 + \left(\sigma_{xy} - \frac{\sigma_1 + \sigma_2}{2}\right)^2 = \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2. \tag{16}$$

The Hall conductivities in this equation can take integer and/or fractional values in units of e^2/h.[32–34] Physically the semicircle rule reflects the fact that in transitional region the critical regime is represented by the random mixture of the two quantum Hall liquids in approximately equal proportion with the local conductivities σ_1 and σ_2.[32] Although a semicircle rule is not a universal it works well at high magnetic fields when a few Landau levels are occupied.[11] It was also established in this paper that the MITs between adjacent quantum Hall states are equivalent to the MIT in the lowest Landau level. In particular, an equivalence of the transitions (2-1) and (1-0) was demonstrated in Ref. 11. At typical electron densities of experiments in the Si MOSFET, $\rho \approx 10^{11}$ cm^{-2}, the gauge field scales as $b \approx 4$ T. At the corresponding region of the phase diagram in Fig. 2 of Ref. 6 and Fig. 9 of Ref. 10 the metallic strips belonging to the lower Landau levels merge into a metallic pool at the border with the insulating state $\sigma_{xy}=0$. An increase in the ρ (say, from $\rho = 7.12 \times 10^{10}$ cm^{-2} to $\rho = 13.7 \times 10^{10}$ cm^{-2}, as in Fig. 3 of Ref. 2) results in a transition from the insulating state to the metallic pool.

In that region of fields one can use the semicircle rule and Eq. (12) for calculations of the Hall conductivity. The temperature behavior of the resistivity $R_{xx}(T)$ then can be found from the equation

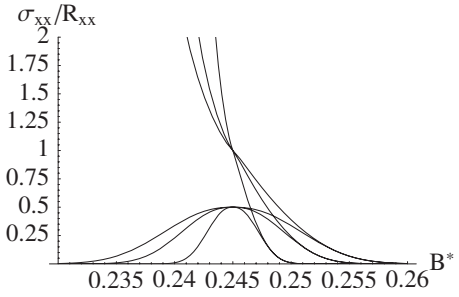

FIG. 1. The conductivity σ_{xx}, according to the Eq. (12), and the resistivity R_{xx}, according to the Eq. (17), as a function of the effective parameter $B^* = |B - b|$ given by Eq. (1). A fixed point $B^* = B_c$ in the magnetoresistivity is the critical point of the MIT ($B_c = 0.245$ in adopted dimensionless units) separating the metallic ($B^* > B_c$) and insulating ($B^* < B_c$) branches of the $R_{xx}(B^*)$. The curves are normalized, respectively, on the values $\sigma_{xx}(0)$ and $R_{xx}(0)$. Three couples of curves correspond to the temperatures $T = 0.1, 0.2, 0.3$ in conventional units and $A = 1$. The conductivity σ_{xx} has a domelike shape which is wider for higher temperature.

$$R_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}. \tag{17}$$

The result of numerical calculations for transition from the insulating to metallic state (0-1) is shown in Fig. 1 in which σ_{xx} and R_{xx} are plotted as a function of the effective magnetic field of the CFs $B^* = |B - b|$ at three different temperatures. One can see in Fig. 1 a fixed point $B^* = B_c$ in the resistivity $R_{xx}(B^*)$ at which all curves cross independently of the temperature. For $B^* > B_c$ the temperature behavior of the $R_{xx}(T)$ is metalliclike with the positive derivative $dR_{xx}(T)/dT > 0$. At fields $B^* < B_c$ the insulatinglike behavior holds with the negative derivative $dR_{xx}(T)/dT < 0$. This picture is in a good agreement with the experimental plots of Refs. 2, 3, and 6. The temperature dependence of the $R_{xx}(T)$ for several constant values of the effective field $B^* = |B - b|$ near the critical point is shown in Fig. 2. The plots like this are typical for the MITs observed in the QHE systems nearby the Hall plateau-to-plateau transitions.[35]

It is important to note here that in full agreement with the experiments of the paper[2] the shape of the metal-to-insulator transition in Figs. 1 and 2 does not depend on the value of the perpendicular magnetic field which only shifts the effective field $B^* = |B - b|$.

In the absence of external magnetic field $B^* = 2p\Phi_0\rho$, and the MIT in Figs. 1 and 2 is governed by the electron density only. In that case a transition from the metal-like type of the resistivity $[dR_{xx}(T)/dT > 0]$ at $\rho > \rho_c = B_c/2p\Phi_0$ to the insulatorlike behavior $[dR_{xx}(T)/dT < 0]$ at $\rho < \rho_c$ shown in Fig. 2 is typical for the numerous experiments on the MIT in 2D conductors.[1–6] The temperature-independent separatrix $R_{xx}^S(T)$ demarcating the insulating and metallic curves in Fig. 2 corresponds exactly the critical value of the electron concentration $\rho = \rho_c$.

A. Scaling and the metal-to-insulator transition

The Mott-type conductivity given by Eqs. (12)–(15) depends on the variable $|B^* - B_c|/T^\kappa$ very much the same as in

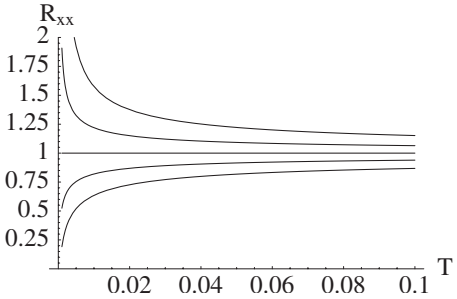

FIG. 2. The resistivity R_{xx} as a function of temperature obtained under the same conditions as in Fig. 1 for the five values of the effective magnetic field $B^*=|B_\perp-b|$: 0.225, 0.235, 0.245, 0.255, and 0.265 (from top to bottom). The horizontal separatrix between the insulating and metallic types of the $R_{xx}(T)$ reflects the fact that interactions between the composite fermions are absent in the adopted uniform approximation $\rho(\mathbf{r})=\rho$. In agreement with the experiment (Ref. 2) the shape of the $R_{xx}(T)$ does not changed if $B_\perp=0$. The only effect is a shift in the electron density at which $R_{xx}(T)$ is observed. See text for details.

Ref. 31 in which the universal scaling in the IQHE systems with respect to this variable was studied. In the absence of external magnetic field $T_0 \propto 1/\xi \propto |b-B_c|^\gamma \propto |\rho-\rho_c|^\gamma$. Using this relation we can write Eq. (12) in the form which has been established experimentally in Ref. 2

$$\sigma_{xx}(\rho,T)=\sigma_\tau(0)\exp[-A(|\rho-\rho_c|/T^\kappa)^{\gamma/2}], \qquad (18)$$

where A is a constant and $\kappa=1/\gamma$. The critical index γ in Si MOSFET equals to 1.6 (Refs. 1 and 2) which means a scaling with the exponent $\kappa=0.625$.

The Mott conductivity [Eq. (18)] is an exponential function of the argument $|\rho-\rho_c|/T^\kappa$ which assumes a universal scaling with respect to this variable provided that index κ is the same (universal) for different samples and materials. Experiments show that this is not the case, in general. Different theoretical and experimental aspects related to the problems of scaling in 2D correlated disordered conductors can be found in the papers.[9,12,36–38]

The reflection symmetry near the critical point has been established empirically in the MOSFETs which tells that the normalized conductivity as a function of the $|\rho-\rho_c|$ on either side of the transition is equal to its inverse on the other side.[39] This rule follows directly from Eqs. (17) and (18). On the insulating side $\sigma_{xy}=0$ and

$$R_{xx}(\rho,T)=1/\sigma_{xx}(\rho,T)\propto \exp[A(|\rho-\rho_c|/T^\kappa)^{\gamma/2}]. \qquad (19)$$

This is exactly the empiric formula (5) of the paper.[2] On the metallic side $\sigma_{xy}=e^2/h$ and $R_{xx}(T)\propto(h/e^2)\sigma_{xx}(T)$.

B. Separatrix and the *e-e* correlations

The separatrix dividing metallic and insulating curves $R_{xx}(T)$ at the critical value of the effective field $B^*=0.245$ in Fig. 2 is a horizontal line. Such type of the temperature-independent separatrix has been found in many experimental papers.[6,35,40] In general the separatrix is temperature dependent, usually close to the linear function in shape.[3–6]

A temperature dependence of the separatrix follows from the two parameter scaling near the critical point in which one variable is related to disorder and another to the *e-e* interaction.[8] This idea was checked experimentally in Ref. 9.

Below we develop this idea further and put forward semiphenomenological arguments which relate a linear temperature dependence of the separatrix with the residual interactions between the composite fermions. In this connection we note that the result plotted in Fig. 2 was obtained without taking account of the weak residual interaction between the CFs given by Eq. (11). Therefore, we can make conjecture that a temperature-independent horizontal separatrix is a consequence of the fact that the residual interactions between the CFs are negligibly small as is really the case in many instances. In our approach this is true if $\delta V \ll \delta U$. The value of the δU depends on the effective disorder potential $U_{eff}(\mathbf{r})$ [Eq. (10)] which varies with the changes in the electron density ρ and disorder potential $U(\mathbf{r})$. Therefore, in case the above inequality is not too strong the interaction term δV should be taken into account.

The effect produced by the small interactions between the CFs on the FQHE has been discussed in the literature (see paper,[23] comment,[41] and Refs. 41–43). It was established that they change only some details of the phenomenon compared to the case of noninteracting CFs but preserve the principal features of the FQHE. This is an important point since the MIT in our approach is related to the CFs physics. The effect of interactions between electrons on the conductivity of 2D Fermi liquid was studied in Ref. 44. It was shown in this paper under rather general assumptions concerning the shape of the *e-e* interaction function $V(\mathbf{r})$ that correlations renormalize the in-plane diagonal conductivity of the free electrons, $\sigma(0)$, by the temperature-dependent factor $\sigma=\sigma(0)(1-A_0T)$. The physics behind this linear in temperature correction to the conductivity is a coherent scattering of electrons by the 2D Friedel oscillations.

The constant A_0 can be either positive or negative depending on the specific form of the interaction $V(\mathbf{r})$. It was shown in Ref. 44 that a linear temperature dependence of the resistivity holds down to rather small values of the parameter $T\tau/\hbar \approx 0.05$. Below this value the logarithmically driven Altshuller-Aronov corrections to conductivity,[45] caused by the weak localization shift the conductivity from the linear temperature dependence.

Experiments on the temperature behavior of the conductivity done on different 2D Si MOSFET samples confirmed a linear dependence within a broad interval of values of the parameter $T\tau/\hbar$ ranging from the diffusive $T\tau/\hbar \ll 1$ to the ballistic $T\tau/\hbar > 1$ regimes.[6,46] Qualitatively, the linear in temperature correction to the conductivity comes from the thermal smearing of the Fermi distribution which results in additional terms in the scattering time and conductivity proportional to the Fourier transform of interaction function and temperature.[44] The composite fermions also have a sharp Fermi distribution at low temperatures which assumes a similar scattering physics with the same linear in temperature correction to the conductivity in full analogy with interacting electrons.

Thus, on a phenomenological level one can expect a linear temperature correction to the conductivity at low tem-

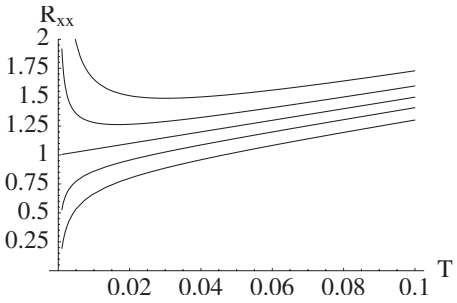

FIG. 3. The resistivity R_{xx} as a function of temperature according to Eq. (20). The difference with Fig. 2 is that the additional factor $1-CT$ is taken into account which is due to the weak residual interaction of the composite fermions. The interaction parameter is taken equal to $C=-5$.

peratures caused by the weak interactions between the CFs. This effect can be included qualitatively by the substitution

$$R_{xx}(T) \to R_{xx}(T)(1 - CT). \tag{20}$$

Two factors contributing into the total resistivity $R_{xx}(T)$ come from the disorder and Coulomb interactions, respectively. They are written in Eq. (20) in a fashion similar to the phenomenology of the two-parametric scaling analysis of the MIT given in Refs. 8 and 9. Equation (20) is in a good agreement with experimental observations of the $R_{xx}(T)$ in 2D Si-MOSFET.[1–6,10,46] The value of the constant C and its sign depend on the strength and shape of residual interactions between the CFs. The diagrams for $R_{xx}(T)$ with positive and negative signs of the constant C are shown in Figs. 3 and 4. Both diagrams are typical not only for the 2D Si MOSFET but rather for a broader class of different 2D conductors with the strong correlations between the charge carriers. An example of the case $C>0$ is given in the paper[47] in which a linear temperature dependence of the resistivity in 2D correlated Si systems was studied near the metal-to-insulator transition. The case $C<0$, shown in Fig. 3, is typical for the underdoped high-T_c cuprates, which are known to be 2D strongly correlated hole conductors.[48]

C. MIT in perpendicular magnetic field

Numerous experiments[1–6] display a strong effect of external magnetic field on the MIT in a 2D conductors. It was found first that even in a strong quantizing perpendicular magnetic field $B_\perp$ the MIT as a function of temperature preserves the same shape as in the case $B_\perp=0$. The only difference is that the same curves $R_{xx}(T)$ correspond to shifted values of the electron concentrations.[2] This puzzling experimental fact is in full agreement with our approach in which the effective magnetic field $B^*=|B_\perp-b|$ is the driving force of the MIT. The curves $R_{xx}(B^*,T)$ in Figs. 1–3 with and without perpendicular magnetic field are identical in the same sample with the only difference that they correspond to the shifted values of electron concentration ρ.

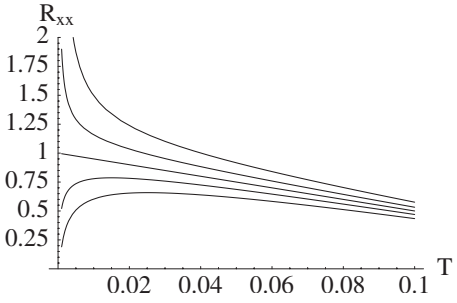

FIG. 4. The same as in Fig. 3 but with a different value of the interaction parameter $C=5$.

Without external magnetic field the effective field B^* equals to $B^*=|2p\Phi_0\rho|$. In perpendicular magnetic field $B_\perp \neq 0$ the effective field is shifted $B^*=|B_\perp-2p\Phi_0\rho|$ but the curves in Figs. 1–4 remain intact in agreement with experiments. The shift in electron concentrations at which the same curves holds is $\Delta\rho=B_\perp/2p\Phi_0$. For $B_\perp\approx 1$ T the estimated value of this shift, $\Delta\rho\approx 10^{10}$ cm^{-2}, is in a good agreement with the corresponding observations in 2D Si MOSFETs.[2]

IV. RESULTS AND DISCUSSION

In conclusion, in this paper an approach to the theory of the metal-to-insulator transition in 2D correlated electron systems such as Si MOSFET is proposed based on the composite-fermion paradigm. The physics behind this approach is as follows. The correlations between electrons in 2D are strongly depend on the phase of their wave functions. By the appropriate unitary Chern-Simons transformation the phase may be chosen in such a way that new quasiparticles (the composite fermions) would be free or weakly interacting (depending on the interplay between the disorder and quasiparticle correlations). The price for the freedom of the CFs is their complex dynamics in the gauge magnetic field which they produce collectively by the fluxes attached to each CF. If the electron density is uniform throughout the sample, which is really often the case, the gauge field is uniform as well and the CFs dynamics is reduced to the Landau problem of a charged particle moving in external magnetic field. The estimate of this field for typical electron densities $\rho \approx 10^{11}$ cm^{-2} yields $b=2\Phi_0\rho$ on the order of a few tesla which assumes a QHE regime for the CFs. The conductivity of the 2D correlated electrons in this approach equals to the diagonal conductivity of a 2D gas of the CFs subject to the perpendicular quantizing uniform gauge magnetic field at which the QHE holds. This conductivity has sharp peaks each time the chemical potential crosses the narrow metallic-like stripes of delocalized states within the disorder-broaden Landau levels. The MITs occur at the boundaries of these metallic stripes. Therefore, within the CF approach the MIT in a 2D conductor with the Coulomb interaction between electrons is mapped on the MIT in a well-studied system of free 2D fermions in external magnetic field in the QHE regime.

The key point of the approach is the Chern-Simons unitary transformation [Eq. (5)] which maps 2D strongly correlated electrons with Hamiltonian (2) on a system of weakly correlated composite fermions—electrons each capturing an even number $2p$ $(p=1,2,3,\ldots)$ of the gauge-field flux quanta as described by Eqs. (6)–(8).

In 2D AlGaAs heterostructures the Coulomb correlations between electrons become important only at high magnetic fields when the parameter $r_s \gg 1$ and the FQHE regime with the CFs physics develops in the lowest Landau level. In the Si-MOSFETs the parameter $r_s \geq 10$ and electron correlations are strong even without external quantizing magnetic field. That makes a principal difference with the AlGaAs heterostructures. Correspondingly, the CFs quasiparticles appear in Si MOSFET without quantizing magnetic field and that may be the reason why the FQHE has not been observed in these conductors so far.

The CS gauge field within the uniform-density approximation acts on the orbital motion of the CFs in the same fashion as an external magnetic field does in the QHE regime. In the case $\rho(\mathbf{r})=\rho$ CFs do not interact. Correspondingly, the MIT in 2D strongly correlated electron system can be described in a full analogy with the MITs which hold near the plateau-to-plateau transitions in conventional QHE systems.

The analysis of the phase diagram shows that without external magnetic field the MIT in Si MOSFET holds at the border between the Hall insulator $\sigma_{xy}=0$ and metallic strip in the middle of the lowest Landau band. The MIT is driven by the enhancement of the electron density along the line parallel to the metallic strip $\nu=1/2$ at the phase diagram in the vicinity of $\rho \approx 10^{11}$ cm^{-2}. The conductivity of the CFs near the MIT in the QHE regime is governed by the Mott variable range hopping mechanism and has a simple exponential form given by Eqs. (12)–(14). Without external magnetic field the conductivity takes the form in Eq. (18) which was established empirically in Ref. 2 and displays a scaling with respect to the variable $|\rho-\rho_c|/T^{\kappa}$ as well as reflecting symmetry with the resistivity [Eq. (19)] on the insulating side of the MIT.[2,24]

The basic results of the calculations are summarized in Figs. 1–4. The resistivity $R_{xx}(B^*)$ as a function of the effective magnetic field $B^*=|B-b|$ near the critical point B_c is shown in Fig. 1 for different temperatures. Different types of the temperature behavior of the resistivity $R_{xx}(T)$ in Figs. 2–4 are related to the cases of the free and weakly correlated CFs. The temperature-independent horizontal separatrix in Fig. 2 corresponds to the case of noninteracting CFs. Small residual interactions between the CFs makes separatrix a linear function of temperature as shown in Figs. 3 and 4. All three types of the temperature behavior of the resistivity shown in Figs. 2–4 have been observed experimentally.

In perpendicular magnetic field the MIT depends on the effective field $B^*=|B_{\perp}-b|$ which, in agreement with the experiment,[2] results in that the shape of the curves in Figs. 2–4 remains the same in external field as without it. The only difference is the shift of electron concentrations at which the same curves are observed (see the Appendix for more detail).

So far CFs have been studied experimentally only in semiconducting heterojunctions at strong magnetic fields in the FQHE regime. It was shown above that in 2D strongly correlated systems such as Si MOSFET they can exist without external magnetic field. I hope that this paper will stimulate the experimental search for the CFs in the Si MOSFET and other 2D strongly correlated systems.

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APPENDIX: THE IDENTITY OF THE RESISTIVITY VS TEMPERATURE PLOTS WITH AND WITHOUT QUANTIZING MAGNETIC FIELD

An observation made in Ref. 2 that the temperature behavior of resistivities in the Si MOSFET with and without quantizing perpendicular magnetic field is identical and crucial for the understanding the nature of the MIT in correlated 2D conductors. The important point of this experiment is that electron density ρ and perpendicular magnetic field B have been varied along the metallic strip with the fixed filling factor $\nu=\rho\Phi_0/B$ equal to $\nu=3/2$ as shown by the arrow at the schematic phase diagram of Ref. 2. Under such condition of experiment there is only one critical point at the phase diagram where the MIT occurs which is located at the boundary between the insulating state $\sigma_{xy}=0$ and metallic strip $\nu=3/2$.

Without external magnetic field the effective field is given by $B^*=|2p\Phi_0\rho|$ and corresponding effective filling factor $\nu^*=\rho\Phi_0/B^*=1/2p$. Since at the schematic phase diagram of Ref. 2 and experimental phase diagrams in Fig. 2 of Ref. 6 and Fig. 9 of Ref. 10 there are no metallic strips below the one at $\nu=1/2$ the only option is $p=1$. Under this condition the MIT occur at the boundary between the insulating state $\sigma_{xy}=0$ and metallic strip $\nu=1/2$ on the same scenario as in case $\nu=3/2$. In the Si MOSFET this boundary has a very nontrivial oscillating shape at the ρ-B plane in the low-field region ($\rho \approx 10^{11}$ cm^{-2} and $B<4$ T).[6] In this region a fan of metallic strips merge into a pool at the border with the insulating state where the above MITs take place. The basic features of such MIT plotted in Figs. 1–4 are in good agreement with experiments on the Si MOSFET. According to these experiments and Eqs. (12)–(20) the resistivity of the Si MOSFET is an universal function of the variable $|B^*-B_c|/T^{\kappa}$. Equating this variable taken at $B=0$ and $\nu=3/2$ we arrive at the condition at which the corresponding resistivity plots vs temperature, $R_{xx}(T)$, with and without external magnetic field become identical, as in experiment of Ref. 2

$$(2p_{\nu}-1/\nu)\Phi_0\rho_{\nu}-B^*_{c\nu}=2p\Phi_0\rho-B^*_c. \tag{A1}$$

Here we take into account the experimental fact that critical index κ in both transitions remains the same. Subscript ν marks the quantities related to the filling factor 3/2. We also assume that, in general, $p_{\nu}\neq p$. The equation relating p_{ν} and p follows from Eq. (A1)

$$p_{\nu}=p(1-\Delta\rho/\rho_{\nu})+1/2\nu-\Delta B_c/2\rho_{\nu}\Phi_0, \tag{A2}$$

where $\Delta\rho=\rho_{\nu}-\rho$ and $\Delta B_c=B^*_c-B^*_{c\nu}$ ($B^*_{c\nu}<B^*_c$). The metallic

strips have finite widths which can be estimated numerically from the phase diagrams in Fig. 2 of Ref. 6 and Fig. 9 of Ref. 10. This yields ΔB_c of the order 1 T. The quantity $2\rho_\nu\Phi_0$ in Fig. 8 of Ref. 2 vary together with ρ_ν between 2.95 and 4.72 T. For the upper curves in Fig. 8 of Ref. 2 we have $\Delta\rho = (7.13-6.68)\times 10^{10}$ cm^{-2} which yields $\Delta\rho/\rho_\nu \approx 0.063$. Under such conditions and in view that the upper bound of the ratio $\Delta B_c/2\Phi_0\rho_\nu$ is close to the value 1/3 (i.e., $1/2\nu$ taken at $\nu=3/2$) Eq. (A2) can be satisfied by the choice $p_\nu=p=1$. In that case the observed in Ref. 2 identity of resistivities require a fulfillment (at least approximate in real experiments) of the condition $\Delta B_c = 2\rho_\nu\Phi_0(1/3-\Delta\rho/\rho_\nu)$. Further experimental and theoretical works are necessary to clarify this issue.

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