Anomalous Josephson current through a ferromagnetic trilayer junction

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We studied the anomalous Josephson current appearing at zero phase difference in junctions coupled with a ferromagnetic trilayer which has noncoplanar magnetizations. A $\pi/2$ junction with an equilibrium phase difference $\pi/2$ is obtained under suitable conditions. The equilibrium phase difference and the amplitude of the supercurrent are all tunable by the structure parameters. In addition to calculating the anomalous current using the Bogoliubov–de Gennes equation, we also developed a clear physical picture explaining the anomalous Josephson effect in the structure. We show that the triplet proximity correlation and the phase shift in the anomalous current-phase relation all stem from the spin precession in the first and third ferromagnet layers.

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I. INTRODUCTION

Usually the supercurrent in a Josephson junction vanishes, when the phase difference between the two superconductors is zero, and in the tunneling limit the current-phase relation (CPR) is sinusoidal $I(\varphi)=I_c \sin(\varphi)$.[1] Recently some studies[2–9] found an anomalous Josephson current flow I_a exists even at zero phase difference ($\varphi=0$). The anomalous supercurrent is equivalent to the presence of an additional phase shift φ_0 in the conventional CPR, i.e., $I(\varphi)=I_c \sin(\varphi+\varphi_0)$. In fact, such CPRs have been predicted for Josephson junctions of unconventional superconductors,[10–14] but the experimental verification is still lacking. Recent studies have shown that the anomalous supercurrent can also exist in junctions with conventional *s*-wave BCS superconductors if both spin-orbit interaction (SOI) and a suitably oriented Zeeman field are present in the coupling layer.[2–6] These studies revealed that the anomalous effect in conventional junctions has some intricate physics. More interesting, an anomalous Josephson current can also appear in superconductor (S)-ferromagnet(F) hybrid structure without SOI.[7,8] In Grein's study,[8] a SFS hybrid structure with two spin-active interfaces was considered. The two spin-active interfaces are critical to the triplet proximity effect and the anomalous supercurrent in the structure, but the physics is still unclear.

In this study, we generalize the two spin-active interfaces to two ferromagnetic layers with finite thicknesses and clarify the physical mechanisms responsible for the anomalous supercurrent. In such SFFFS structures, we find that the triplet proximity correlation and the phase shift in the anomalous CPR all stem from the spin precession in the first and third F layers. According to the symmetry analysis,[15] an anomalous supercurrent is possible when the symmetries of the time-reversal operator T and its combination with a spin rotation operator with respect to an arbitrary spin quantum axis $\mathbf{n}$ $\sigma_{\mathbf{n}}T$ are broken at the same time. As a result, the simplest superconductor (S)-ferromagnet (F)-superconductor (S) junction for achieving an anomalous Josephson current requires the F layer to be a ferromagnetic trilayer with noncoplanar magnetizations for breaking the symmetry of the operator $\sigma_{\mathbf{n}}T$. SFFFS junctions where the magnetizations of the three ferromagnetic layers need not be noncoplanar[7,16] and SFS junctions with inhomogeneous magnetization,[17–20] have been studied in order to understand the effects of triplet correlation induced in the F layers. Controllable 0-π transition and spin-triplet supercurrents have been realized experimentally recently.[21,22] In our study, we found that triplet correlation is also an important condition for the anomalous supercurrent.[23–27]

We consider a junction consisting of two conventional *s*-wave superconductors coupled by a ferromagnetic trilayer with noncoplanar magnetizations. For convenience, hereafter we denote the three F layers sequentially by F_1, F_2, F_3. We start with the typical situation where the magnetizations are along the x, y, z axes respectively (i.e., an $SF_xF_yF_zS$ junction), as shown in the upper panel of Fig. 1. This junction is a $\pi/2$ junction with an equilibrium phase difference $\pi/2$ under suitable conditions. The equilibrium phase difference can be tuned by the lengths, the exchange energies, and the magnetization orientations of the F_1 and F_3 layers. And the amplitude of the supercurrent can be tuned by the barriers between the F layers or by the length and the exchange energy of the middle F_2 layer. In this regime the Josephson junction can also act as a supercurrent rectifier.[28,29]

The paper is organized as follows. In Sec. II we present the model and solve the scattering problem for quasiparticles

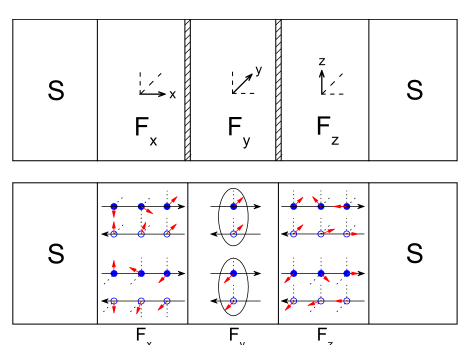

FIG. 1. (Color online) Upper panel: schematic of the $SF_xF_yF_zS$ junction where two barriers are present between the F layers. Lower panel: schematic illustration of the formation of Andreev bound states with triplet correlation in the F_y layer due to spin precession of electrons and holes in the F_x and F_z layers.

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based on the Bogoliubov–de Gennes equation. The Josephson current and Andreev bound states can be obtained from the scattering matrices. In Sec. III we show the numerical results for the anomalous supercurrent and corresponding Andreev bound states and reveal the physics. A conclusion and remarks will be given in Sec. IV.

II. MODEL AND FORMALISM

In the numerical calculation, we consider $SF_1F_2F_3S$ junctions with various lengths and exchange energies for each F layer and various barrier strengths for the two barriers between the F layers. The transport direction is along the x axis. The three F layers have the thicknesses, L_1, L_2, L_3, the exchange energies, h_1, h_2, h_3, and the magnetization orientations, $(\theta_1,\phi_1),(\theta_2,\phi_2),(\theta_3,\phi_3)$ in spherical coordinates. The effective Hamiltonian of the system is given by[30,31]

$$H=\begin{pmatrix} \epsilon_k+h_z & h_{xy}^* & 0 & \Delta(x) \\ h_{xy} & \epsilon_k-h_z & -\Delta(x) & 0 \\ 0 & -\Delta^*(x) & -\epsilon_k-h_z & -h_{xy} \\ \Delta^*(x) & 0 & -h_{xy}^* & -\epsilon_k+h_z \end{pmatrix}, \quad (1)$$

where $\epsilon_k=\frac{\hbar^2}{2m}(k_x^2+k_y^2-k_F^2)+U$ with k_F the Fermi wave number, $U=U_0[\delta(x-L_1)+\delta(x-L_1-L_2)]$ represents the two barriers between the F layers, and $h_z=h\cos\theta$, $h_{xy}=h\sin\theta e^{i\phi}$, with h the strength and (θ,ϕ) the orientation of the exchange field; $\Delta(x)=\Delta[\Theta(-x)e^{i\varphi/2}+\Theta(x-L)e^{-i\varphi/2}]$ describes the pair potential with $L=L_1+L_2+L_3$ and Δ the bulk superconducting gap and $\varphi=\varphi_L-\varphi_R$ the macroscopic phase difference of the two superconductor leads. The temperature dependence of the magnitude of Δ is given by $\Delta(T)=\Delta(0)\tanh(1.74\sqrt{T_c/T-1})$.[32] Since the transversal momentum components are conserved and not important to the total Josephson current, we consider the question in the one-dimensional regime for simplicity. The Bogoliubov–de Gennes equation can be easily solved for each superconductor lead and each F layer, respectively. The scattering problem can be solved by considering the boundary conditions at the interfaces. Each interface gives a scattering matrix. The total scattering matrix of the system can be obtained by the combination of all these scattering matrices of interfaces. From the total scattering matrix, we can obtain the Andreev reflection amplitudes $a_{1\sigma}$ and $a_{2\sigma}$ of the junction where $a_{1\sigma}$ is for the reflection from an electronlike to a holelike quasiparticle and $a_{2\sigma}$ is for the reverse process with σ representing the spin. The stationary Josephson current can be expressed in terms of the Andreev reflection amplitudes by using the temperature Green's function formalism[33]

$$I_e(\varphi)=\frac{e\Delta}{4\hbar}\sum_{\omega_n,\sigma}\frac{k_BT}{\Omega_n}(k_n^++k_n^-)\left(\frac{a_{1\sigma n}}{k_n^+}-\frac{a_{2\sigma n}}{k_n^-}\right), \quad (2)$$

where k_n^+, k_n^-, $a_{1\sigma n}$, and $a_{2\sigma n}$ are obtained from k_s^+, k_s^-, $a_{1\sigma}$, and $a_{2\sigma}$ by analytic continuation $E\rightarrow i\omega_n$. $k_s^{\pm}$ is the wave vector for electron or hole in the superconductors and the Matsubara frequencies are $\omega_n=\pi k_BT(2n+1)$, $n=0,\pm1,\pm2,\cdots$, and $\Omega_n=\sqrt{\omega_n^2+\Delta^2}$.

The discrete spectrum of the Andreev bound states can be determined by using the condition[34]

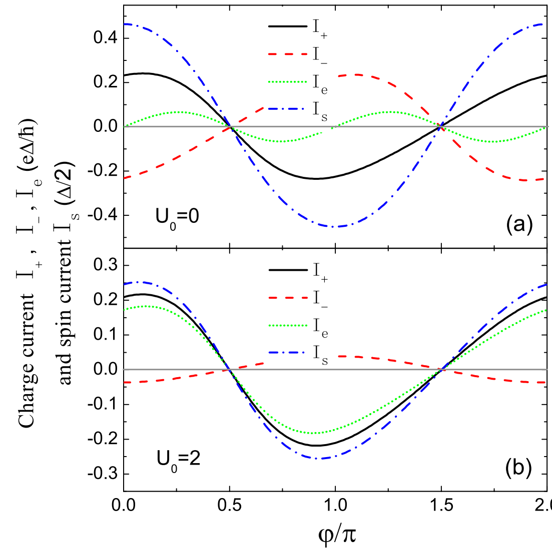

FIG. 2. (Color online) Charge and spin currents I_+, I_-, I_e, I_s versus φ for the $SF_xF_yF_zS$ junction. The barrier strength $U_0=0$ for (a) and $U_0=2$ for (b). The strength of exchange fields $h_1=h_2=h_3=0.05$, and the lengths of F layers $L_1=L_2=L_3=10\pi$. The temperature $T/T_c=0.5$ with T_c being the critical temperature.

$$\det[1-R_2PR_1P]=0, \quad (3)$$

where R_1, R_2, P are 4×4 matrices, P is the propagation matrix of modes in the F_2 layer, and R_1 (R_2) is the reflection matrix of the right-going (left-going) incident waves.

In order to study the spin properties of the Andreev bound states formed at F_2 layer, we can also work out the Green's function $G(x,x',E)$ in F_2 layer which is a 4×4 matrix.[35] Now it is convenient to take the eigenspinors of F_2 layer, i.e., spin parallel and spin antiparallel with respect to the exchange field $\mathbf{h}_2$ as the unit vectors of the spin space. Then the spin current in F_2 layer can be evaluated by[20]

$$I_s(\varphi)=\frac{\hbar^2k_BT}{4mi}\lim_{x'\rightarrow x}\left(\frac{\partial}{\partial x'}-\frac{\partial}{\partial x}\right)\sum_{\omega_n}\mathrm{Tr}\left\{\begin{pmatrix}\sigma_z & 0\\ 0 & \sigma_z\end{pmatrix}G_{\omega_n}(x,x')\right\}$$
$$=\frac{\hbar}{2e}(I_+-I_-), \quad (4)$$

where I_+ (I_-) is the charge currents of electrons with parallel spin (antiparallel spin) and obviously satisfies $I_e=I_++I_-$.

III. RESULTS AND DISCUSSION

We start with the typical noncoplanar magnetization configuration, i.e., the $SF_xF_yF_zS$ junction. For simplicity, we introduce the dimensionless units: the energy $E\rightarrow EE_F$, the wave vector $\mathbf{k}\rightarrow\mathbf{k}k_F$, the coordinate $\mathbf{x}\rightarrow\mathbf{x}/k_F$, and the strength of exchange field $h\rightarrow hE_F$. All physical quantities are expressed in the dimensionless units in the rest of the paper. The superconductors considered are characterized with $\Delta=10^{-3}$ which corresponds to the BCS coherence length at zero temperature $\xi_0=2/\pi\Delta\approx636.6$.

Figure 2 shows the charge and spin currents I_+, I_-, I_e, I_s as functions of the phase difference φ for the $SF_xF_yF_zS$ junc-

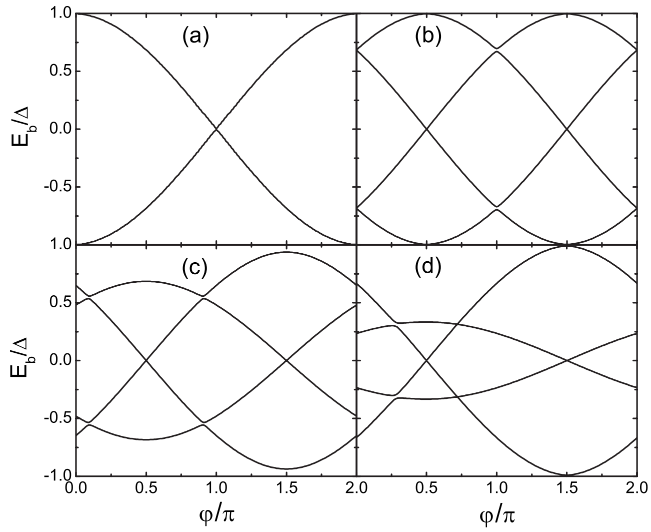

FIG. 3. The energy levels of the Andreev bound states E_b. The strength of exchange fields $h_1=h_2=h_3=0$ for (a) and $h_1=h_2=h_3=0.05$ for (b), (c), (d). The barrier strength $U_0=0$ for (b), $U_0=1$ for (c), and $U_0=2$ for (d). The other parameters are the same as those in Fig. 2.

tion. The corresponding Andreev bound states are shown in Fig. 3. It is interesting to note that when there is a barrier between the F layers, ($U_0=2$), there is a significant anomalous Josephson current. When there is no barrier $U_0=0$, the anomalous Josephson current is nearly zero. This interesting dependence on the barrier strength U_0 will be explained below in terms of the spin characteristics of the Andreev bound states in the F_y layer.

First, it is useful to point out a large spin current exists in the F_y layer, implying that the superconductivity correlation is mainly triplet in the F_y layer. This is easily understood by considering the formation of an Andreev bound state in the F_y layer. A right-going electron with spin parallel to the y axis $(1,i)^T$ from the F_y layer will have its spin precessing about the z axis in the F_z layer before it reaches the right superconductor. After the Andreev reflection from the right superconductor, a hole with reverse spin goes left and its spin continues to precess. The one-way angle of precession is approximately $(k_+-k_-)L_3 \approx h_3L_3$ where k_+ (k_-) is the wave vector of up-spin (down-spin) quasiparticle. Thus if the condition $h_3L_3=n\pi+\pi/2$ (n is an integer) is satisfied, the reflected hole from the right superconductor will have its spin parallel to the incident electron's spin in the F_y layer. An Andreev bound state is formed, after this reflected hole travels through the F_y and F_x layers and Andreev reflected from the left superconductor and changes to an electron to move right to finish a cycle. If the spin rotation angle in the F_x layer satisfies the same condition $h_1L_1=n\pi+\pi/2$. The electrons and holes have identical spins (parallel to the y axis) in the F_y layer and the Andreev bound state formed has complete triplet correlation in the F_y layer, as schematically shown in the lower panel of Fig. 1. Triplet correlation can exist in other different type of magnetic inhomogeneity too. Bergeret *et al.*[17] have studied S/F/S junctions with spiral magnetization in the F layer and found spin-triplet correlation there. In the present model, we found two Andreev bound states below the Fermi level with complete triplet correlation; one is "spin-up" (with respect to the y axis), which carries the current I_+, and one is "spin-down," which carries the current I_-, as shown in Fig. 3. In the short junction limit, the Josephson current is totally carried by the Andreev bound states.[36]

Besides complete triplet correlation in the F_y layer, another interesting feature noted in Fig. 2 is that I_+ has a phase shift of $\pi/2$ while I_- has a phase shift of $-\pi/2$ compared with the conventional CPR. So, these two currents move in opposite directions. Now we follow the Andreev reflection processes occurring in the formation of the bound states to find out the phase shift. For simplicity, we assume $h_1=h_2=h_3=h$; thus, the wave vectors of spin-up (+) and spin-down (−) electrons (holes) with energy E at each F layer are $k_\pm^{e(h)}=\sqrt{k_F^2+\rho_{e(h)}E\mp h}$ with $\rho_{e(h)}=+(-)1$. In the short junction limit and the limit $E \ll h \ll E_F$, we have $k_\pm^e \approx k_\pm^h \approx k_\pm = k_F \mp \frac{h}{2}$. We start with a right-going spin-up electron at the position $x=L_1+0$, the wave function can be written as $(1,i,0,0)^T$. The electron moves right and acquires a phase $e^{ik_+L_2}$ when it arrives at the interface $x=L_1+L_2$. To simplify the discussion we focus on the Andreev reflections at the F/S interfaces and ignore the normal reflections at the barriers which affect only the amplitude of the supercurrent but not the phase shift. When the electron travels through the F_z layer, its spin precesses. The state becomes $(e^{ik_+L_3},ie^{ik_-L_3},0,0)^Te^{ik_+L_2}$ when the electron arrives at the interface $x=L_1+L_2+L_3$. Then, the electron is reflected as a hole with reverse spin and the hole wave function is $(0,0,-ie^{ik_-L_3},e^{ik_+L_3})^Te^{ik_+L_2}\frac{v}{u}e^{i\varphi/2}$ where $u=\sqrt{(1+\Omega/E)/2}$, $v=\sqrt{(1-\Omega/E)/2}$ with $\Omega=\sqrt{E^2-\Delta^2}$. The algebraic derivation is not shown here for space limitation and the approximation $k_\pm^e \approx k_\pm^h \approx k_\pm^s \approx k_F$ has been used in the derivation where k_+^s (k_-^s) is the wave vector of electronlike (holelike) quasiparticle in the superconductors. The Andreev-reflected hole moves left and has its spin rotated in the F_z layer again and then goes back to the F_y layer $x=L_1+L_2-0$. Now the wave function becomes $(0,0,-ie^{ihL_3},e^{-ihL_3})^Te^{ik_+L_2}\frac{v}{u}e^{i\varphi/2}=(0,0,1,-i)^Te^{ik_+L_2}\frac{v}{u}e^{i\varphi/2}$ where the condition $hL_3=\pi/2$ has been used. The wave function describes a spin-up hole with respect to the y direction. Then the hole goes left through the F_y layer and acquires a phase $e^{-ik_+L_2}$. So the wave function becomes $(0,0,1,-i)^T\frac{v}{u}e^{i\varphi/2}$ when the hole arrives at the interface $x=L_1$. Consequently, the hole has its spin precessed in the F_x layer and moves left to the interface $x=0$ with the wave function $\frac{1}{2}[(1-i)(0,0,1,1)^Te^{-ik_+L_1}+(1+i)(0,0,1,-1)^Te^{-ik_-L_1}]\frac{v}{u}e^{i\varphi/2}$. The hole is Andreev-reflected as an electron with reverse spin described by $\frac{1}{2}[(1-i)(1,-1,0,0)^Te^{-ik_+L_1}-(1+i)(1,1,0,0)^Te^{-ik_-L_1}](\frac{v}{u})^2e^{i\varphi}$. Then the electron goes through the F_x layer again and back to the starting position $x=L_1+0$ to finish a cycle. The final wave function is $(1,i,0,0)^Te^{i\pi/2}(\frac{v}{u})^2e^{i\varphi}$ where $hL_1=\pi/2$ is used. Comparing with the initial wave function $(1,i,0,0)^T$, we can see the phase shift of the spin-up Andreev bound state is indeed $\pi/2$ when considering a conventional CPR. In the same way, we can find out the phase shift of the spin-down Andreev bound state is $-\pi/2$. In this round-trip cycle of the quasiparticle, we can clearly see that the phase shifts in Andreev bound states come from the spin precession of electron and hole in the F_x and F_z layers.

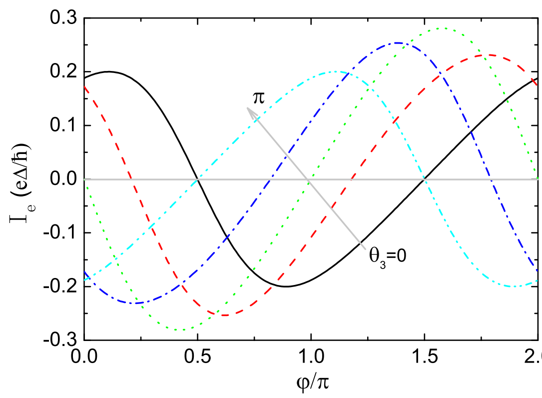

FIG. 4. (Color online) I_e versus φ for the $SF_xF_yF_zS$ junction with different θ_3: varying θ_3 from 0 to π with a step $\pi/4$, $\phi_3=0$. The other parameters are chosen as: $U_0=2$, $h_1=h_2=h_3=0.1$, $L_1=L_2=L_3=5\pi$, $T/T_c=0.5$.

If we neglect the second-harmonic term in the CPR, the charge current carried by the two Andreev bound states can be written as[37]

$$I_+ \approx I_+^0 \sin\left(\varphi + \frac{\pi}{2}\right), \quad I_- \approx I_-^0 \sin\left(\varphi - \frac{\pi}{2}\right), \qquad (5)$$

where I_+^0 (I_-^0) is the amplitude of the spin-up (spin-down) charge current. When the barriers are absent, the normal scattering at the two F/F interfaces can be ignored and we can have $I_+^0 \approx I_-^0$. As a result, the total charge current $I_e = I_+ + I_-$ is very small and only the second-harmonic term remains, as shown in Fig. 2(a). At zero phase difference, the charge current is very small and the spin current in the F_y layer is almost a pure spin current.

When the barriers are present, the normal scattering at the barriers reduces the amplitudes of the two charge currents I_+^0 and I_-^0. The transmission probability through the double delta function barriers of electrons or holes depends on the wave vector of the particle in the F_y layer and reaches the maximum when resonance transmission occurs. Here in the F_y layer, the spin-up Andreev bound state couples a spin-up electron with a spin-up hole which have the same wave vector $k_+ \approx k_F - \frac{h}{2}$ while the spin-down Andreev bound state couples a spin-down electron with a spin-down hole which have the same wave vector $k_- \approx k_F + \frac{h}{2}$. The difference in the wave vector between the two Andreev bound states leads to the difference in the transmissions through the F_y layer. Consequently, we can make a large difference between I_+^0 and I_-^0 as shown in Fig. 2(b) by using two barriers as well as suitable exchange field strength and length of the F_y layer. In this way, an anomalous Josephson current appears at zero phase difference. The CPR of the junction has a phase shift of $\pm\pi/2$ in comparison with the conventional CPR where the sign of the phase shift depends on the relative magnitude of I_+^0 and I_-^0.

Since the phase shift of the anomalous CPR stems from the spin precession of electrons and holes in the F_x and F_z layers, we can modulate the phase shift by tuning the parameters of these two layers. If the conditions $\phi_1=\phi_3=0$ and $h_1L_1=h_3L_3=(n+1/2)\pi$ are satisfied, the complete equal-spin triplet correlation in the F_y layer is maintained. Now the phase shifts of the spin-up and spin-down Andreev bound states are $\pm(\pi+\theta_3-\theta_1)$ according to the above discussion. Figure 4 shows the tuning of the equilibrium phase difference by varying θ_3. Bergeret *et al.*[17] have found that the relative orientation of the two magnetizations in S/F/I/F/S junctions can change the critical current. For the particular structure considered in Fig. 4, which is different from theirs, the orientation of the magnetization in the third layer has no strong effect on the critical current. However, for other values of h_2, the orientation can also modify the critical current. On the other hand, the amplitude or even the sign of the supercurrent can be changed by the barrier strength, the exchange field strength or the length of the F_y layer, as shown in Fig. 5. The dependence of the supercurrent on the barrier strength is because of the condition of resonance transmission through double delta barriers $\sin(2kL_2) = -4U/(U^2+4)$ with k the wave vector of particles. Figure 6 shows the anomalous supercurrent at zero phase difference as functions of h_2 and L_2 for the $SF_xF_yF_zS$ junction. It is noted that the dependence on h_2 exhibits a period of $2\pi/L_2=0.2$ which confirms the occurrence of resonance transmission of electrons and holes through the F_y layer. And the dependence on L_2 exhibits two periodic behavior. One period is nearly π and the other is 10π. Because the wave vector of spin-up electrons and holes is $k_+ \approx k_F - \frac{h_2}{2}$ while the wave vector of spin-down electrons and holes is $k_- \approx k_F + \frac{h_2}{2}$ in the F_y layer, the

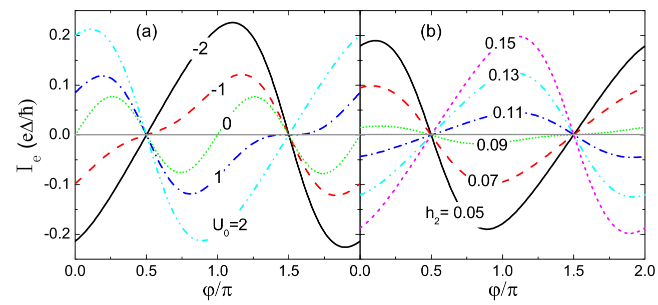

FIG. 5. (Color online) I_e versus φ for the $SF_xF_yF_zS$ junction with different U_0 and h_2: (a) varying U_0 from −2 to 2 with a step 1, $h_2=0.1$, $L_2=5\pi$; (b) varying h_2 from 0.05 to 0.15 with a step 0.02, $L_2=10\pi$, $U_0=2$. The other parameters are chosen as: $h_1=h_3=0.1$, $L_1=L_3=5\pi$, $T/T_c=0.5$.

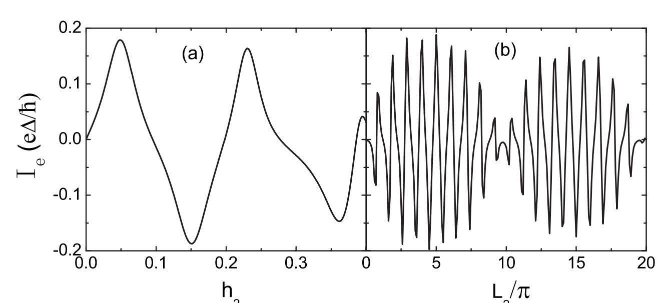

FIG. 6. The anomalous supercurrent at zero phase difference $I_e(\varphi=0)$ for the $SF_xF_yF_zS$ junction as functions of: (a) h_2 with $L_2=10\pi$, and (b) L_2 with $h_2=0.1$. The other parameters are chosen as: $h_1=h_3=0.1$, $L_1=L_3=5\pi$, $U_0=2$, $T/T_c=0.5$.

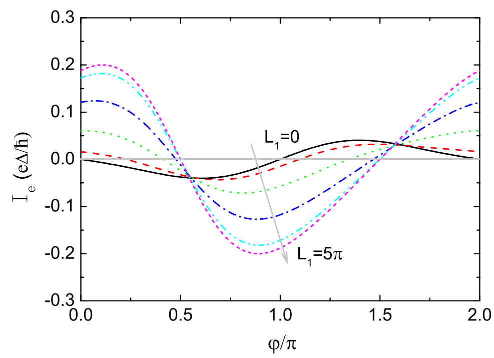

FIG. 7. (Color online) I_e versus φ for the $SF_xF_yF_zS$ junction with different L_1: varying L_1 from 0 to 5π with a step π, $L_3=L_1$. The other parameters are chosen as: $U_0=2$, $h_1=h_2=h_3=0.1$, $L_2=5\pi$, $T/T_c=0.5$.

period π stems from $2\pi/2k_F=\pi$ with $k_F=1$. The other period 10π stems from $2\pi/2h_2=10\pi$ which is related to the difference of wave vectors $k_+-k_-=-h_2$.

If anyone of the two conditions $\phi_1=\phi_3=0$ and $h_1L_1=h_3L_3=(n+1/2)\pi$ is not satisfied, the pure equal-spin triplet correlation in the F_y layer is changed. For example, if we vary the length or the exchange field strength of the F_x and F_z layers, the spin precession angle of electron and hole in a round trip in the F_x and F_z layers is not π any more. The reflected hole will have both the same spin component and the opposite spin component to the incident electron in the F_y layer. Now the correlation is the mixing of singlet and triplet. But only the triplet correlation can contribute to the anomalous Josephson current, so the anomalous supercurrent is reduced with increasing singlet component. Figure 7 shows that both the amplitude of the supercurrent and the equilibrium phase difference is tuned by the length of the F_x and F_z layers. To study the characteristics of Cooper pairs in the F_y layer in detail, the pair function can be defined by the anomalous Green function and be decomposed into four components[23,25]

$$\sum_{\omega_n>0} G^{eh}_{\omega_n}(x,x) = i\sum_{\nu=0}^{3} f_\nu(x)\sigma_\nu\sigma_2, \tag{6}$$

where $G^{eh}_{\omega_n}$ is the anomalous electron-hole correlation function, σ_0 is the unit matrix and $\sigma_\nu(\nu=1,2,3)$ are three Pauli matrices. In Eq. (6), the frequency summation is only made over positive frequencies because the triplet pair functions are odd functions of frequency. f_0 (f_3) is the pairing function of spin-singlet (spin-triplet) pairs with spin structure of $[|\uparrow\downarrow\rangle-(+)|\downarrow\uparrow\rangle]/\sqrt{2}$. The pairing functions of $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ pairs are given by $f_{\uparrow\uparrow}=if_2-f_1$ and $f_{\downarrow\downarrow}=if_2+f_1$, respectively. Figure 8(a) shows the absolute values of pairing functions at the center of the F_y layer as functions of the length of the F_x and F_z layers for the $SF_xF_yF_zS$ junction. The equal-spin pair functions and opposite-spin pair functions oscillate with the length of the F_x and F_z layers which determines the angle of spin precession of quasiparticles in these two layers. Compared with the anomalous supercurrent shown in Fig. 8(b), we can see that the anomalous supercurrent is nearly proportional to the equal-spin triplet correlations.

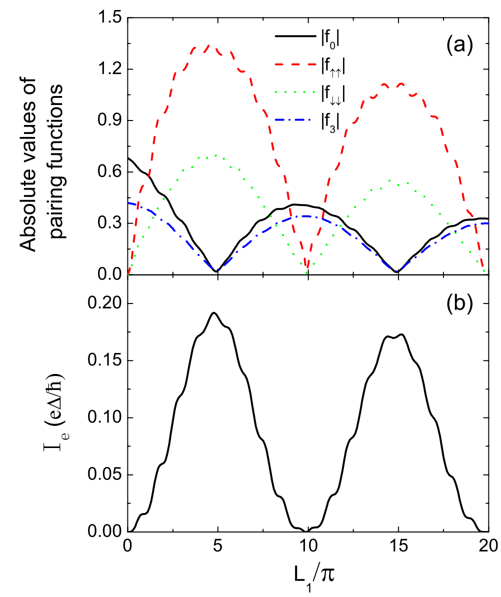

FIG. 8. (Color online) The absolute values of pair functions in the F_y layer (a) and the anomalous supercurrent at zero phase difference (b) as functions of the length of the F_x and F_z layers for the $SF_xF_yF_zS$ junction. $L_3=L_1$. The other parameters are the same as in Fig. 7.

IV. CONCLUSION

In summary, we predict a tunable anomalous Josephson effect in $SF_1F_2F_3S$ junction where the three F layers have noncoplanar magnetizations. The superconducting correlation can be completely triplet in the F_2 layer due to the spin precession of electrons and holes in the F_1 and F_3 layers. If the condition $h_1L_1=h_3L_3=(n+1/2)\pi$ is satisfied, an electron incident to the left (right) superconductor will precess its spin by $\frac{\pi}{2}$ in the F_1 (F_3) layer before it arrives at the supercondcutor and the Andreev-reflected hole proceeds to precess the spin by $\frac{\pi}{2}$ when it goes back to the F_2 layer. Thus the Andreev-reflected hole will have the same spin with the incident electron and the complete triplet correlation arises in the F_2 layer. The two spin-resolved Andreev bound states carry two spin-polarized supercurrents which have opposite phase shifts and different amplitude thus leading to an anomalous Josephson current. And the phase shift in the anomalous current-phase relation is also a result of the spin precession of electron and hole in the F_1 and F_3 layers. The equilibrium phase difference of the anomalous supercurrent can be tuned by the lengths, the exchange energies, and the magnetization orientations of the F_1 and F_3 layers. And the amplitude of the supercurrent can be tuned by the barriers between the F layers or by the length and the exchange energy of the F_2 layer.

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