

Quantum Zeno effect with a superconducting qubit

Y. Matsuzaki,* S. Saito, K. Kakuyanagi, and K. Semba

NTT Basic Research Laboratories, NTT Corporation, Kanagawa 243-0198, Japan

(Received 24 June 2010; revised manuscript received 6 October 2010; published 23 November 2010)

Detailed schemes are investigated for experimental verification of Quantum Zeno effect with a superconducting qubit. A superconducting qubit is affected by a dephasing noise whose spectrum is $1/f$ and so the decay process of a superconducting qubit shows a naturally nonexponential behavior due to an infinite correlation time of $1/f$ noise. Since projective measurements can easily influence the decay dynamics having such nonexponential feature, a superconducting qubit is a promising system to observe quantum Zeno effect. We have studied how a sequence of projective measurements can change the dephasing process and also we have suggested experimental ways to observe quantum Zeno effect with a superconducting qubit. It would be possible to demonstrate our prediction in the current technology.

DOI: [10.1103/PhysRevB.82.180518](https://doi.org/10.1103/PhysRevB.82.180518)

PACS number(s): 74.20.-z, 85.25.Hv, 03.65.Xp, 03.65.Yz

Quantum Zeno effect (QZE) is one of fascinating phenomena which quantum mechanics predicts. A sequence of projective measurements to an unstable system can suppress the decay process of the state.¹⁻³ This phenomena will be observed if the time interval of projective measurements is sufficiently small and the decay behavior in the time interval is quadratic. Although it was proved that an unstable system shows a quadratic behavior in the initial stage of the decay,⁴ it is difficult to observe such quadratic decay behavior experimentally because the time region to show such quadratic behavior is usually much shorter than typical time resolution of a measurement apparatus in the current technology. After showing the quadratic decay, unstable system shows an exponential decay⁴ and QZE does not occur through projective measurements to a system which decays exponentially. Due to such difficulty, in spite of the many effort to observe the QZE, there was only one experimental demonstration to suppress the decay process of an unstable state.⁵ Note that, except this experiment, all previous demonstration of QZE did not focus on a decoherence process caused by a coupling with environment but focused on a suppression of a unitary evolution having a finite Poincare time such as Rabi oscillation.⁶⁻¹⁰ Such approach to change the behavior of the unitary evolution by measurements are experimentally easy to be demonstrated but is different from the original suggestion of QZE for the decay process of unstable systems¹⁻³ with a decoherence process. Throughout this Rapid Communication, we consider only such QZE to change decoherence behavior.

In this Rapid Communication, we suggest a way to demonstrate QZE for the decay process of unstable system experimentally with a superconducting qubit. A superconducting qubit is one of candidates to realize quantum information processing and, for a superconducting qubit, the quadratic decay has been observed in an experiment,^{11,12} which is necessary condition to observe QZE experimentally. Moreover, a high fidelity single qubit measurement has already been constructed in the current technology.¹³ A superconducting flux qubit has been traditionally measured by superconducting quantum interference device (SQUID).¹⁴ The state of a SQUID is switched from zero-voltage state to a finite voltage state for a particular quantum state of the qubit while no switching occurs for the other state. Such switching transi-

tion produces a macroscopic signal to construct a measurement for a superconducting flux qubit. Also, entirely distinct qubit readout method such as Josephson bifurcation amplifier (JBA) has been demonstrated.^{15,16} The JBA has advantages in its readout speed, high sensitivity, low backaction,¹⁶ and absence of on-chip dissipative process. It is also studied JBA readout mechanism¹⁷ and the projection conditions¹⁸ of the superposition state of a qubit. All these properties are prerequisite in observing the QZE. So a superconducting qubit is a promising system to verify QZE for an unstable state.

We study a general decay process of unstable system. Although a decay behavior of unstable system has been studied and conditions for quadratic decay have been shown by several authors,^{4,19-21} we introduce a simpler solvable model and we confirm the conditions for the exponential decay and the quadratic decay, respectively. Also, from the analytical solution of the model, we derive a master equation for $1/f$ noise. We consider an interaction Hamiltonian to denote a coupling with an environment such as $H_I = \lambda f(t) \hat{A}$, where $f(t)$ is a classical normalized Gaussian noise, \hat{A} is an operator of the system, and λ denote a coupling constant. Also, we assume nonbiased noise and therefore $\overline{f(t)} = 0$ is satisfied where this overline denotes the average over the ensemble of the noises. In an interaction picture, by solving the Schrodinger equation and taking the average over the ensembles, we obtain

$$\rho_I(t) - \rho_0 = \sum_{n=1}^{\infty} (-i\lambda)^n \int_0^t dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{n-1}} dt_n \times \overline{f(t_1)f(t_2)\cdots f(t_n)} [\hat{A}, [\hat{A}, \dots, [\hat{A}, \rho_0], \dots]], \quad (1)$$

where $\rho_0 = |\psi\rangle\langle\psi|$ is an initial state and $\rho_I(t)$ is a state in the interaction picture. Throughout this Rapid Communication, we restrict ourself to a case that the system Hamiltonian commutes with the operator of $1/f$ noise as $[H_s, \hat{A}] = 0$. Firstly, we consider a case that the correlation time of the noise $\tau_c \equiv \int_0^{\infty} \overline{f(t)f(0)} dt$ is much shorter than the time resolution of experimental apparatus, which is valid condition for the most of unstable systems. Since the correlation

time of the noise is short, we obtain $\int_0^t \int_0^{t'} f(t') f(t'') dt' dt'' = \int_0^t d\tau \int_0^\tau f(\tau) f(0) \approx t\tau_c$. Also, since the noise $f(t)$ is Gaussian, $f(t_1)f(t_2), \dots, f(t_n)$ can be decomposed of a product of two-point correlation $f(t_i)f(t_j)$, and so we obtain

$$\rho_f(t) \approx \sum_{A,A',\nu,\nu'} |A\nu\rangle\langle A\nu| \rho_0 |A'\nu'\rangle\langle A'\nu'| e^{-\lambda^2 \tau_c |A-A'|^2 t}, \quad (2)$$

where $|A\nu\rangle$ is an eigenstate of the operator \hat{A} and ν denote a degeneracy. So a dynamical fidelity $F \equiv \langle \psi | e^{iH_s t} \rho(t) e^{-iH_s t} | \psi \rangle$, a distance between the state $\rho(t)$ and a state $e^{-iH_s t} \rho_0 e^{iH_s t}$, becomes a sum of exponential decays.

$$F \approx \sum_{A,A',\nu,\nu'} |\langle A\nu | \psi \rangle|^2 |\langle A'\nu' | \psi \rangle|^2 e^{-\lambda^2 \tau_c |A-A'|^2 t}. \quad (3)$$

Secondly, when the correlation time of the noise is much longer than the time resolution of the apparatus such as $1/f$ noise having an infinite correlation time, we obtain $\int_0^t \int_0^{t'} f(t') f(t'') dt' dt'' \approx \frac{1}{2} t^2$. So, by taking average over the ensemble of noise in Eq. (1), we obtain

$$\rho_f(t) \approx \sum_{A,A',\nu,\nu'} |A\nu\rangle\langle A\nu| \rho_0 |A'\nu'\rangle\langle A'\nu'| e^{-(1/2)\lambda^2 |A-A'|^2 t^2}. \quad (4)$$

So we obtain a master equation for $1/f$ noise as $\frac{d\rho(t)}{dt} = -\lambda^2 t [\hat{A}, [\hat{A}, \rho(t)]]$. The behavior of the dynamical fidelity becomes quadratic in the early stage of the decay ($t \ll \frac{1}{\lambda}$) as

$$F \approx \sum_{A,A',\nu,\nu'} |\langle A\nu | \psi \rangle|^2 |\langle A'\nu' | \psi \rangle|^2 e^{-(1/2)\lambda^2 |A-A'|^2 t^2} \approx 1 - \frac{1}{2} \lambda^2 t^2 \sum_{A,A',\nu,\nu'} |A-A'|^2 |\langle A\nu | \psi \rangle|^2 |\langle A'\nu' | \psi \rangle|^2. \quad (5)$$

These results show that an unstable system has an exponential decay for $t \gg \tau_c$ while a quadratic decay occurs for $t \ll \tau_c$.

Let us summarize the QZE. Usually, to observe QZE, survival probability is chosen as a measure for the decay. However, we use a dynamical fidelity to observe the QZE rather than a survival probability to take into account of the effect of a system Hamiltonian. We consider a sequence of projective measurements $\hat{P}(k) = e^{-iH_s k \tau} |\psi\rangle\langle \psi| e^{iH_s k \tau}$ with $\tau = \frac{t}{N}$ and $k=1, 2, \dots, N$ to an unstable state where N denotes the number of the measurements performed during the time t . For noises whose correlation time is short, a dynamical fidelity without measurements becomes a sum of exponential decay terms such as $F(t) = \sum_{j=1}^m c_j e^{-\Gamma_j t}$. The success probability to project the unstable state into the target states becomes $P(N) = (\sum_{j=1}^m c_j e^{-\Gamma_j \tau})^N \approx 1 - t \sum_{j=1}^m c_j \Gamma_j$, and so the success probability decreases linearly as the time increases. On the other hand, if the dynamical fidelity has a quadratic decay without projective measurements such as $F = e^{-\Gamma^2 t^2}$, we obtain the success probability to project the unstable state into the state $e^{-iH_s t} |\psi\rangle$ becomes as following: $P(N) = [1 - \Gamma^2 \tau^2 + O(\tau^4)]^N \approx 1 - \Gamma^2 \frac{t^2}{N}$. So, by increasing the number of the measurements, the success probability goes to unity, and this means

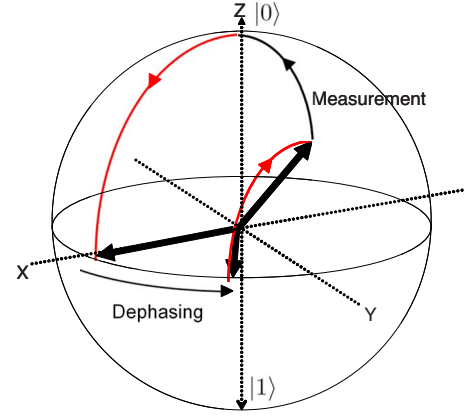


FIG. 1. (Color online) A schematic of quantum states in a Bloch sphere to show how QZE is observed with a superconducting qubit. An initial state is prepared in $|+\rangle$, and the state has an unknown rotation around z axis due to a dephasing. To construct a measurement $|0\rangle\langle 0|$, one performs a $\frac{\pi}{2}$ rotation U_y around y axis, performs a measurement $|0\rangle\langle 0|$, and performs a $\frac{\pi}{2}$ rotation U_y^\dagger . If a measurement interval is much smaller than a dephasing time, this measurement $|+\rangle\langle +|$ recovers a state into the initial state with almost unity success probability.

that the time evolution of this state is confined into $e^{-iH_s t} |\psi\rangle$, which is a purely unitary evolution without noises, and so one can observe the QZE.

It is known that a superconducting qubit is mainly affected by two decoherence sources, a dephasing whose spectrum is $1/f$ and a relaxation whose spectrum is white. The $1/f$ noise causes a quadratic decay to the quantum states as we have shown. Moreover, such quadratic decay has already been observed experimentally.^{11,12} On the other hand, since the relaxation process from an excited state $|1\rangle$ to a ground state $|0\rangle$, where a high frequency is cutoff, the correlation time of the environment is extremely small and so only an exponential decay can be observed for a relaxation process in the current technology. Therefore, when the dephasing is relevant and the relaxation is negligible, it should be possible to observe QZE with a superconducting qubit as following. Firstly, one prepares an initial state $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, which is an eigenstate of $\hat{\sigma}_x$. Secondly, in a time interval $\tau = \frac{t}{N}$, one continues to perform projective measurements $|+\rangle\langle +| = \frac{1}{2}(\hat{1} + \hat{\sigma}_x)$ to the superconducting qubit where N is the number of the measurement performed. For simplicity, let us make an assumption that an effect of a system Hamiltonian is negligible compared with the dephasing effect. (Since this assumption could be unrealistic for a superconducting qubit, we will relax this condition and discuss more rigorous case later.) Note that we perform a selective measurement here to consider only a case to project the state into $|+\rangle\langle +|$ and, if the state is projected into the other state, we discard the state as a failure case. As a result, due to the quadratic decay behavior caused by $1/f$ noise, the success probability to project the state into a target state N times goes to a unity as the number of the measurements becomes larger, and therefore one can observe QZE (Fig. 1). Note that a direct measurement of $\hat{\sigma}_x$ with a superconducting qubit has not been constructed yet experimentally. So, in order to know a measurement result of

$\hat{\sigma}_x$ in the current technology, one has to perform a $\frac{\pi}{2}$ rotation around y axis before and after performing a projective measurement about $\hat{\sigma}_z$. However, recently, a coupling about $\hat{\sigma}_x$ between a superconducting qubit and a flux bias control line has been demonstrated,^{22,23} which shows a possibility to realize a direct measurement of $\hat{\sigma}_x$ in the near future. Since it is not necessary to perform preliminary rotations around y axis, this direct measurement of $\hat{\sigma}_x$ has advantage in its readout speed.

In the above discussion, the effect of the relaxation and system Hamiltonian is not taken into account. Since they are not always negligible in a superconducting qubit, it is necessary to investigate whether one can observe QZE or not under the influence of them. When considering the effect of dephasing and relaxation whose spectrum are $1/f$ and white, respectively, we use a master equation as following:

$$\begin{aligned} \frac{d\rho_f(t)}{dt} = & -\frac{1}{2}\Gamma_1(\hat{\sigma}_+\hat{\sigma}_-\rho_f(t) + \rho_f(t)\hat{\sigma}_+\hat{\sigma}_- - 2\hat{\sigma}_-\rho_f(t)\hat{\sigma}_+) \\ & -\frac{1}{2}(\Gamma_2)^2 t[\hat{\sigma}_z, [\hat{\sigma}_z, \rho_f(t)]], \end{aligned} \quad (6)$$

where Γ_1 and Γ_2 denote a decoherence rate of relaxation and dephasing, respectively. In this master equation, the first part is a Lindblad-type master equation to denote a relaxation, and the second part denotes a dephasing whose spectrum is $1/f$ coming from the fluctuation of ϵ . Also, we assume that a system Hamiltonian is $H_s = \frac{1}{2}\epsilon\hat{\sigma}_z + \frac{1}{2}\Delta\hat{\sigma}_x \approx \frac{1}{2}\epsilon\hat{\sigma}_z$ for $\epsilon \gg \Delta$ because we have derived a master equation for $1/f$ noise only when the system Hamiltonian commutes with the noise operator of $1/f$ fluctuation. We find an analytical solution of this equation, and when the initial state is $|\psi\rangle = |+\rangle$, we obtain

$$\begin{aligned} \rho(t) = & e^{-iH_s t} \left[\frac{1}{2}e^{-\Gamma_1 t} |1\rangle\langle 1| + \frac{1}{2}e^{-(1/2)\Gamma_1 t - (\Gamma_2)^2 t^2} |0\rangle\langle 1| \right. \\ & \left. + \frac{1}{2}e^{-(1/2)\Gamma_1 t - (\Gamma_2)^2 t^2} |1\rangle\langle 0| + \left(1 - \frac{1}{2}e^{-\Gamma_1 t}\right) |0\rangle\langle 0| \right] e^{iH_s t}. \end{aligned} \quad (7)$$

Note that, while the $1/f$ noise causes a quadratic dephasing, the relaxation causes an exponential decay, which cannot be suppressed by projective measurements. Here, we consider the effect of system Hamiltonian, and so we perform a projective measurement to the state $e^{-iH_s t} |+\rangle$. Since there always exists a time-dependent single qubit rotation U_t to satisfy $U_t e^{-iH_s t} |+\rangle = |0\rangle$, this measurement can be realized by performing the single qubit rotation before and after a measurement of $\hat{\sigma}_z$. Note that this single qubit rotation U_t can be performed in a few nanoseconds by using a resonant microwave.²⁴ In this Rapid Communication, we call the entire process including U_t as ‘‘measurement’’ for simplicity. The success probability $P(N)$ to project the state into the target state is calculated as

$$P(N) = \left(\frac{1}{2} + \frac{1}{2}e^{-(\pi/2T_1) - (\tau^2/(T_2)^2)} \right)^N, \quad (8)$$

where $T_1 = (\Gamma_1)^{-1}$ and $T_2 = (\Gamma_2)^{-1}$ denote a relaxation time and a dephasing time, respectively. So we

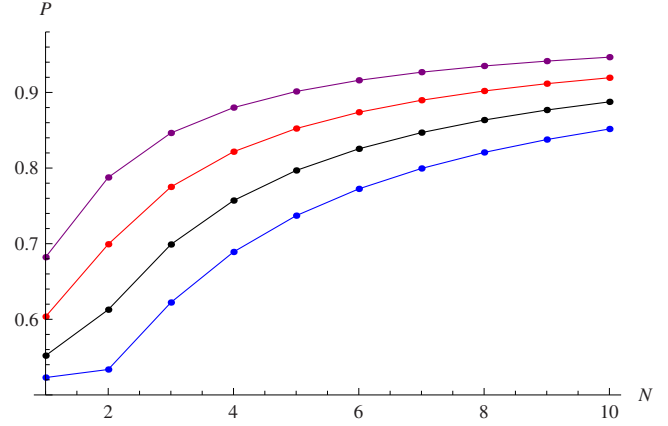


FIG. 2. (Color online) A success probability to perform projective measurements into a target state under the effect of dephasing and relaxation is plotted. The horizontal axis and the vertical axis denote the success probability and the number of measurements, respectively. The lowest line is for $t=35$ ns, and the other lines are for $t=30$ ns, 25 ns, and 20 ns, respectively. As one increases the number of measurements, the success probability increases. Here, we assume a relaxation time $T_1=1$ μ s and a dephasing time $T_2=20$ ns, respectively.

obtain $P(N) = \left(\frac{1}{2}e^{-(\tau^2/2(T_2)^2)} + \frac{1}{2}e^{-(\pi/2T_1) - (\tau^2/2(T_2)^2)} \right)^N e^{-(\tau^2/2N(T_2)^2)} \approx \left(1 - \frac{\tau}{4T_1}\right) e^{-(\tau^2/2N(T_2)^2)}$ for $\frac{\tau}{T_1}, \frac{\tau}{T_2} \ll 1$. So, as long as the T_1 is much larger than T_2 , one can observe that the success probability increases as one increases the number of the projective measurements (see Fig. 2). Note that we assume a Hamiltonian as $H_s \approx \frac{1}{2}\epsilon\hat{\sigma}_z$, far from the optimal point for a superconducting flux qubit and so a coherence time T_2 of this qubit becomes as small as tens of nanoseconds. In the current technology, it takes tens of nanoseconds to perform JBA (Ref. 16) and so one has to use a switching measurement to utilize a SQUID to be performed in a few nanoseconds. The state of a SQUID remains a zero voltage when the state of a qubit is $|0\rangle$ while a SQUID makes a transition to a finite voltage state to produce a macroscopic signal for $|1\rangle$. One of the problems of the SQUID measurements is that a transition to a finite voltage state destroys quantum states of the qubit and following measurements are not possible after the transition. However, as long as the state is $|0\rangle$, the state of a SQUID remains a zero-voltage state and so sequential measurements are possible. Since one postselects a case that all measurement results are $|0\rangle$ while one discards the other case as a failure, the SQUID can be utilized to observe QZE with the selective measurements.

Importantly, it is also possible to observe QZE at the optimal point where T_2 can be as large as μ s. A recent demonstration of coupling about $\hat{\sigma}_x$ between a superconducting qubit and a flux bias control line shows a possibility to have a relevant $1/f$ noise caused by a fluctuation of Δ due to a replacement of a Josephson junction with a SQUID,^{22,23} and the noise operator from the $1/f$ fluctuation becomes $\hat{\sigma}_x$ to commute a system Hamiltonian at the optimal point as $H = \Delta\hat{\sigma}_x$. So, by replacing the notation from $\hat{\sigma}_z$ to $\hat{\sigma}_x$ and from $|+\rangle$ to $|0\rangle$, one can apply our analysis in this paper to a case observing QZE at the optimal point. (For example, in this replaced notation, an initial state should be prepared in

$|0\rangle$ and frequent measurements in the zy plane will be performed.) Moreover, since T_2 at the optimal point is much longer than a necessary time to perform JBA, a sequence of measurements is possible for all measurement results. This motivates us to study a verification of QZE without postselection as following.

Finally, we discuss how to observe QZE without postselection of measurement results, which can be realized by JBA. We perform frequent nonselective measurements in the xy plane to the state which was initially prepared in $|+\rangle$. Such nonselective measurements to a single qubit is modeled as $\hat{\mathcal{E}}(\rho) = |\phi_+\rangle\langle\phi_+|\rho|\phi_+\rangle\langle\phi_+| + |\phi_-\rangle\langle\phi_-|\rho|\phi_-\rangle\langle\phi_-|$, where $|\phi_+\rangle = e^{-iH_s t}|+\rangle$ and $|\phi_-\rangle = e^{-iH_s t}|-\rangle$ are orthogonal with each other. So, when performing this nonselective measurement with a time interval $\tau = \frac{t}{N}$ under the influence of dephasing and relaxation, we obtain

$$\rho(N, t) = e^{-iH_s t} \left(\frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} e^{-(t/2T_1) - (t^2/N(T_2)^2)} |0\rangle\langle 1| + \frac{1}{2} e^{-(t/2T_1) - (t^2/N(T_2)^2)} |1\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1| \right) e^{iH_s t}, \quad (9)$$

where we use a result in Eq. (7). Since we consider a state just after performing a measurement in the xy plane (not along z axis), the population of a ground state becomes equivalent as the population of an excited state. Note that a nondiagonal term is decayed by the white noise and $1/f$ noise, and only the decay from $1/f$ noise is suppressed by the measurements. In Fig. 3, we show this decay behavior of the nondiagonal term. A possible experimental way to remove out the effect of the white noise is measuring $\langle 0|\rho(N, t)|1\rangle$ and $\langle 0|\rho(1, t)|1\rangle$ separately by performing a tomography, and plotting the value of $\langle 0|\rho(N, t)|1\rangle / \langle 0|\rho(1, t)|1\rangle = e^{-t^2/N(T_2)^2}$ for a fixed time t . As a result one can observe the suppression of the dephasing caused by $1/f$ noise through measurements.

In conclusion, we have studied detailed schemes for experimental verification of QZE to a decay process with a superconducting qubit. Since a superconducting qubit is af-

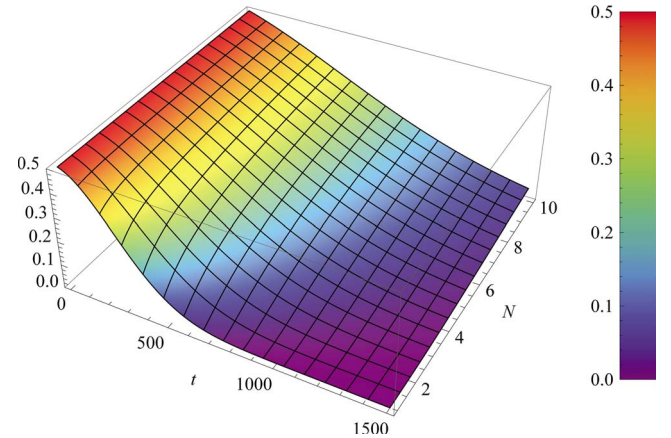


FIG. 3. (Color online) A behavior of a phase term $|\langle 0|\rho|1\rangle|$ under the effect of nonselective measurements realized by the JBA is plotted. This shows a suppression of the decay by measurements. Here t and N denote the time(ns) and the number of measurements, respectively. We assume a relaxation time $T_1 = 1 \mu\text{s}$ and a dephasing time $T_2 = 400 \text{ ns}$, respectively. These conditions can be realized at the optimal point where the system Hamiltonian is $H_s = \Delta \hat{\sigma}_x$ and Δ has a $1/f$ fluctuation due to a replacement of a Josephson junction with a SQUID.

ected by the dephasing with a $1/f$ spectrum, the dynamics show a quadratic decay which is suitable for an experimental demonstration for QZE while the relaxation process has an exponential decay to cause unwanted noise for QZE. We have suggested a way to observe QZE even under an influence of relaxation. Our prediction is feasible in the current technology.

Authors thank H. Nakano and S. Pascazio for valuable discussions on QZE. This work was done during Y.M.'s short stay at NTT corporation and was also supported in part by Funding Program for World-Leading Innovative R&D on Science and Technology (FIRST), Scientific Research of Specially Promoted Research under Grant No. 18001002 by MEXT, and Grant-in-Aid for Scientific Research (A) under Grant No. 22241025 by JSPS.

*matsuzaki@asone.c.u-tokyo.ac.jp

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