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Optimal topological spin pump

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We study the recently introduced $\mathbb{Z}_2$ pump consisting of a family of one-dimensional bulk insulators with time-reversal restriction on the pumping cycle. We find that the scattering matrices of these pumps are dichotomized by a topological index. We show that the class of pumps characterized by a nontrivial topological index allows, in contrast to its topologically trivial counterpart, for the noiseless pumping of quantized spin, even in the absence of spin conservation. This distinction sheds light on the $\mathbb{Z}_2$ classification of two-dimensional time-reversal invariant insulators.

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I. INTRODUCTION

The idea to pump spin through a mesoscopic device at zero bias by cyclic variation in two system parameters is very appealing due to its promise of precise and reversible flow control.[1] *Optimal pumps* which operate noiseless, transferring quantized spin in a cycle are particularly relevant for potential applications.[2] In this Rapid Communication, we propose the concept of an optimal topological spin pump, composed of a bulk insulator. We show that the ability to pump quantized noiseless spin is a hallmark of the nontrivial topological invariant characterizing a quantum spin-Hall system.

The discovery of the quantum Hall effect introduced an interesting new way to classify different states of matter. Unlike more familiar phases, a quantum Hall state does not break any symmetry and cannot be described by a local-order parameter. Rather, it differs from a regular two-dimensional insulator by topological invariants, known as Chern numbers, which reflect the global structure of its ground-state wave function.[3,4] This topological classification of states of matter found recently an exciting extension to two-dimensional time-reversal invariant bulk insulators.[5–7] In accordance with the Chern numbers used to classify quantum Hall systems,[6] these systems can be characterized by a $\mathbb{Z}_2$ topological index, based on the properties of their bulk ground state. The group of insulators described by a nontrivial $\mathbb{Z}_2$ index are known as quantum spin-Hall systems.

In analogy to the quantum Hall state, it is possible to gain insight into this topological classification by studying a pump formed by placing the two-dimensional system on a cylinder threaded by a magnetic flux.[8,9] The resulting system can be mapped onto a set of one-dimensional time-dependent Hamiltonians by identifying the magnetic flux with time. This mapping defines a $\mathbb{Z}_2$ pump.[7] Nonetheless, in the absence of spin rotation invariance, the physical meaning of the pumped $\mathbb{Z}_2$ charge remains elusive. In this Rapid Communication, we study the scattering matrix of the open one-dimensional pump constructed in this way. We find that mapping the two-dimensional insulating system onto a one-dimensional pump establishes a $\mathbb{Z}_2$ classification of the scattering matrix. The resulting one-dimensional pumps are dichotomized. The family of scattering matrices belonging to the topological nontrivial class allows, in contrast to its topological trivial counterpart, for the noiseless pumping of a quantized spin, even in the absence of spin conservation. We illustrate these ideas by two examples.

II. $\mathbb{Z}_2$ INDEX

We consider a family of one-dimensional Hamiltonians of noninteracting electrons with a bulk energy gap that depend continuously on a cyclic pumping parameter t, $H(t+T)=H(t)$, and satisfy

$$H(-t) = \sigma_y H^T(t) \sigma_y, \tag{1}$$

where σ_i are the Pauli matrices. These systems can be viewed as a mapping of the set of two-dimensional time-reversal invariant insulators placed on a cylinder, where t corresponds to a magnetic flux threading the cylinder. Indeed, Eq. (1) implies time-reversal invariance of the corresponding two-dimensional system, as is evident upon identification $(k_x,t)\rightarrow(k_x,k_y)$. In the context of pumping, Eq. (1) ensures the existence of two time-reversal invariant moments (TRIMs) $t_1=0$ and $t_2=T/2$, at which $H(t_i)=\sigma_y H^T(t_i)\sigma_y$, where $i=1,2$. Upon coupling the one-dimensional system to two single-channel leads, the open system is described in terms of a time-dependent 4×4 unitary scattering matrix. Provided the system exceeds the attenuation length associated with the bulk energy gap, the transmission vanishes, and the scattering matrix is block diagonal. Each block, $\hat{r}_\alpha$, is a unitary 2×2 reflection matrix in spin space, where the index $\alpha=L,R$ refers to the left and the right lead, respectively. The average spin injected into lead α during a cycle can be expressed in terms of the spin current, $\vec{s}_\alpha=\text{Im tr}([dr_\alpha/dt]\vec{\sigma}r_\alpha^\dagger)$,[10] see Eq. (7) below. It can be readily verified that $\vec{s}_\alpha$ is invariant under gauge transformations $\hat{r}_\alpha\rightarrow e^{i\varphi_\alpha(t)}\hat{r}_\alpha$. This allows us to restrict our analysis to the $U(2)/U(1)\simeq \text{SU}(2)$ particle-hole symmetric part of $\hat{r}_\alpha$ which we denote by $\tilde{r}_\alpha$.

The pumping cycle defined in Eq. (1) provides a mapping from the periodic time, corresponding to a one-dimensional loop, S^1, to the SU(2) space of particle-hole symmetric reflection matrices, $\tilde{r}_\alpha$. Since the SU(2) group is topological equivalent to the three sphere, S^3, one may conclude that a general pumping cycle does not distinguish between different topological classes, since all closed contours on S^3 can be contracted to a single point. [The first homotopy class of S^3 is zero, $\Pi_1(S^3)=0$]. However, Eq. (1) restricts the mapping by associating $S(T/2+t)=\sigma_y S^T(T/2-t)\sigma_y$. In particular, at the two TRIM, the SU(2) part of the reflection matrix is given by $\tilde{r}_\alpha(t_i)=\pm\mathbb{1}$. This restricted mapping has two topological distinct classes, which may be categorized by

$$\tilde{r}_\alpha(0)\tilde{r}_\alpha(T/2)=(-1)^\nu\mathbb{1}. \tag{2}$$

Any loop $\tilde{r}_\alpha(t)$ on the three sphere characterized by $\nu=0$ can be contracted onto a single point while paths with $\nu=1$ are fixed by two distinct points at TRIM and cannot be contracted. Following Eq. (1), any additional points at which $\tilde{r}_\alpha=-\mathbb{1}$ away from the TRIM occur in pairs. We may, therefore, equivalently define ν as the parity of $\tilde{r}_\alpha=-\mathbb{1}$ moments traversed in a cycle. Away from the TRIM, however, these points are not protected by symmetry and can be removed by a perturbation larger than the level broadening introduced by coupling to leads, see Eq. (5). Hence, in the weak-coupling limit, we can disregard any such accidental points.

III. TOPOLOGICAL SPIN PUMP

The $\mathbb{Z}_2$ classification of the scattering matrix has a direct effect on the spin pumped during a cycle. We show below that the family of one-dimensional pumps belonging to the topological nontrivial class can, in contrast to its topologically trivial counterpart, operate as optimal pumps. To this end consider the effective Hamiltonian, $H=H_L+H_R$, for the left and right edge states of the bulk insulator,

$$H_\alpha(t)=\mu_\alpha(t)+\vec{h}_\alpha(t)\cdot\vec{\sigma}, \quad \alpha=L,R. \tag{3}$$

Here we have employed the fact that left and right edge states decouple and the Hamiltonian is a sum of two independent 2×2 Hermitian matrices in spin space, which can be parametrized by a unit matrix and the three Pauli matrices, $\vec{\sigma}$. For transparency of the arguments we first consider $\mu_\alpha=0$, where the spectrum of Eq. (3) is particle-hole symmetric, with eigenvalues $\pm|\vec{h}_\alpha(t)|\equiv\pm h_\alpha$ that vary during the course of a pumping cycle. Using the general expression for the scattering matrix $S=1+2i\pi W^\dagger(H-i\pi WW^\dagger)^{-1}W$, where W is a matrix that describes the coupling between the leads and the insulator,[11] and assuming that the leads couple equally well to up and down spins, so that $W_\alpha=w_\alpha\mathbb{1}$, the reflection matrix of each block is given by

$$\hat{r}(t)=\tilde{r}(t)=e^{i\phi(t)\vec{e}_\phi(t)\cdot\vec{\sigma}}, \tag{4}$$

where we have dropped the lead index α for brevity. Here,

$$\cos\phi(t)=\frac{h(t)^2-\Gamma^2}{h(t)^2+\Gamma^2}, \quad \sin\phi(t)=\frac{2\Gamma h(t)}{h(t)^2+\Gamma^2}, \tag{5}$$

where $\Gamma=\pi|w|^2$ is the level broadening due to coupling to the leads and we introduced the unit vector,

$$\vec{e}_\phi(t)=\vec{h}(t)/h(t), \tag{6}$$

which defines a time-dependent rotation axis. We note that while the angle ϕ depends on the level broadening, the vector $\vec{e}_\phi$ is exclusively determined by the effective Hamiltonian. Equation (5) shows that whenever the edge state crosses the Fermi level $h(t_i)=0$, the angle $\phi=\pi$ and the reflection matrix is resonant $\hat{r}=-\mathbb{1}$.

Following Ref. 10 we express the spin injected into lead α during a cycle in terms of the reflection matrix,

$$\vec{S}_\alpha=\frac{\hbar}{2\pi}\oint dt\vec{s}_\alpha, \quad \vec{s}_\alpha(t)=\mathrm{Im}\,\mathrm{tr}\left(\frac{d\hat{r}_\alpha}{dt}\vec{\sigma}\hat{r}_\alpha^\dagger\right). \tag{7}$$

In general, the rotation axis $\vec{e}_\phi$ varies during the cycling process. As a result, one cannot identify a time-independent axis along which spin pumped during a cycle is quantized. The situation, however, changes when the coupling to the leads is weak. We consider the limit $1/T\ll\Gamma\ll E_\Delta$, where the level broadening Γ is small compared to the gap, yet large compared to the pumping rate, in order to allow for the pump to relax between cycles. In this limit the angle, Eq. (5), remains close to zero, $\phi(t)=\mathcal{O}(\Gamma/E_\Delta)$, and changes rapidly to π whenever a gapless edge state appears in the course of a cycle. The time duration of this transition can be estimated as $\delta t\sim\Gamma/[dh(t_i)/dt]\sim(\Gamma/E_\Delta)T\ll T$. In the weak-coupling limit, δt can be made arbitrarily short in comparison to the time scale for variations in $\vec{e}_\phi$, so that $\vec{e}_\phi(t)$ may be approximated by its value at the center of the resonance. The class of pumps with $\nu=1$ cross a single resonance at the TRIM. Hence, in the weak-coupling limit the reflection matrix of a topologically nontrivial pump describes a rotation around a fixed axis,

$$\hat{r}(t)=e^{i\phi(t)\vec{e}_\phi(t_i)\cdot\vec{\sigma}}\left[1+\mathcal{O}\left(\frac{\Gamma}{E_\Delta}\right)\right], \tag{8}$$

where t_i is the TRIM at which the resonance occurs. As a result, the spin injected into lead α by the class of topologically nontrivial pumps is quantized,

$$\vec{S}=\hbar\vec{e}_\phi(t_i)[1+\mathcal{O}(\Gamma/E_\Delta)], \tag{9}$$

where the quantization axis is determined by microscopic details of the system. Conversely, the class of pumps with $\nu=0$ either remains insulating during the entire cycle or traverses two resonances at the TRIM. In the weak-coupling limit, the former group can be approximated by a constant reflection matrix $r(t)\approx\mathbb{1}$ and thus does not pump spin. The second subgroup crosses two resonances during each pumping cycle. Each such resonance is associated with the value of the vector $\vec{e}_\phi(t_i)$. The two vectors $\vec{e}_\phi(0)$ and $\vec{e}_\phi(T/2)$, however, need not be aligned. As a result, one cannot identify a *time-independent* spin direction that would lead to a quantized spin pumped trough an insulator with $\nu=0$.

These observations have a direct implication on the spin noise. Following Refs. 12 and 13, the variance of the spin pumped during a cycle, in a direction $\vec{e}_q$, is determined by the time dependence of the vector $\vec{n}_{\vec{e}_q}\cdot\vec{\sigma}=r(t)^\dagger\sigma_{\vec{e}_q}r(t)$ and vanishes for a constant $\vec{n}_{\vec{e}_q}$.[13] In the weak-coupling limit, the reflection matrix of a topologically nontrivial pump, Eq. (8), describes a rotation around a fixed axis, $\vec{e}_\phi(t_i)$. As a result, the vector $\vec{n}_{\vec{e}_q}(t)$ with $\vec{e}_q=\vec{e}_\phi(t_i)$ remains constant during the course of a pumping cycle. It follows that the topological nontrivial spin pump allows for the noiseless pumping of quantized spin. Conversely, a trivial insulator that crosses two resonances during a cycle cannot be parametrized by a time-independent vector. Consequently, the trivial pump inevitably operates with generation of finite noise.

The above arguments generalize to finite chemical potential μ_α. Once particle-hole symmetry is broken, the energy levels cross the chemical potential at two different moments, $h(t_\pm)=\pm\mu$. From Eq. (1) it follows that these occur symmetrically around the TRIM, $t_\pm = t_i \pm \delta t$. By diagonalizing the Hamiltonian at the crossing point, one can show that the width of each transition is determined by Γ, and in the weak-coupling limit, is associated with a fixed vector $\vec{e}_\phi(t_\pm)$. We note that due to Eq. (1), $\vec{e}_\phi(t_+)$ and $\vec{e}_\phi(t_-)$ are colinear. Hence, $\hat{r}(t)$ describes a rotation around a fixed axis, and the spin pumped through a topologically nontrivial pump is noiseless and quantized in a direction which also depends on μ.

IV. TWO EXAMPLES

To illustrate these ideas we next consider two examples which demonstrate the difference between the two topological classes of pumps. To model a topologically nontrivial pump we consider the particle-hole symmetric Hamiltonian $H=H_0+V_b+V_{\rm st}+V_{\rm so}$ (Refs. 7 and 14) with

$$H_0 = \tau_0 \sum_{i,\alpha} (c^\dagger_{i,\alpha}c_{i+1,\alpha} + c^\dagger_{i+1,\alpha}c_{i,\alpha}),$$

$$V_b = b(t) \sum_{i,\alpha,\beta} (-1)^i \sigma^z_{\alpha,\beta} c^\dagger_{i,\alpha}c_{i,\beta},$$

$$V_{\rm st} = \tau_{\rm st}(t) \sum_{i,\alpha} (-1)^i (c^\dagger_{i,\alpha}c_{i+1,\alpha} + c^\dagger_{i+1,\alpha}c_{i,\alpha}),$$

$$V_{\rm so} = \sum_{i,\alpha,\beta} i\vec{e}_{\rm so}\vec{\sigma}_{\alpha,\beta}(c^\dagger_{i,\alpha}c_{i+1,\beta} - c^\dagger_{i+1,\alpha}c_{i,\beta}). \qquad (10)$$

Here sums run over N sites and spin indices, and $\tau_{\rm st}(t) = \tau^0 \cos(2\pi t/T)$, $b(t)=b^0 \sin(2\pi t/T)$. To simplify the illustration we choose $\vec{e}_{\rm so}=e_{\rm so}\hat{x}$, such that $\hat{r}(t)$ depends only on two Pauli matrices and can be represented as a point on the two sphere. Finally, first and last sites are coupled to the leads via spin-independent hopping elements $W_\alpha = w_\alpha \mathbb{1}$ while electrons inside the leads are described by a tight-binding model, H_0 in Eq. (10).

The transmission and reflection coefficients, $\hat{t}_{RL}$ and $\hat{r}_L$, are found from the transfer matrix $\mathcal{T}_{RL}$ by solving

$$\hat{t}_{RL}\begin{pmatrix} e^{ik_R a(N+2)} \\ e^{ik_R a(N+1)} \end{pmatrix} = \mathcal{T}_{RL}\begin{pmatrix} 1+\hat{r}_L \\ e^{-ik_L a} + e^{ik_L a}\hat{r}_L \end{pmatrix}, \qquad (11)$$

where $k_{R,L} \approx \pm\pi/(2a)$ are the Fermi wave vectors in right and left lead at half filling, and the transfer matrix relates the states $(\psi_{N+2},\psi_{N+1})^T = \mathcal{T}_{RL}(\psi_0,\psi_{-1})^T$, and is derived from Eq. (10). The resulting $\hat{t}_{RL}$ is exponentially suppressed over the length associated with the gap. To leading orders in $\delta\tau = \tau_{\rm st}/\tau_0$, $\delta b = b/\tau_0$, and $\delta e = e_{\rm so}/\tau_0$ the reflection matrix is given by Eq. (4) with

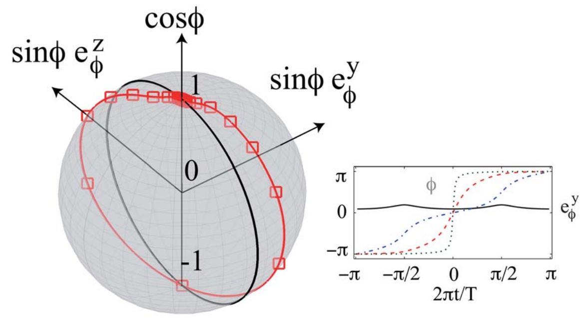

FIG. 1. (Color online) Time evolution of $\hat{r}(t)$ of model, Eq. (10), in the absence of spin orbit (black curve), and for $\delta e/\delta\tau=0.3$ and $\Gamma/\tau_0=0.1$ (red/light gray curve). Squares indicate equally spaced time intervals and illustrate the time duration of the transition to $\hat{r} = -\mathbb{1}$. The loops cross points $\pm\mathbb{1}$ at the TRIM 0 and $T/2$, respectively, corresponding to a nontrivial $\mathbb{Z}_2$ index $\nu=1$. Inset shows the time dependence of the y component of $\vec{e}_\phi(t)$ (black curve) and of $\phi(t)$ for $\delta e/\delta\tau=0.3$ and $\Gamma/\tau_0=1$, 0.1, and 0.01 (dashed-dotted blue, dashed red, and dotted green lines, respectively). At $\Gamma/\tau_0\to 0$, the transition becomes sharper and $\hat{r}(t)$ describes a rotation around a fixed axis, $e_\phi(\vec{T}/2)$, implying that the pump is optimal.

$$\vec{e}_\phi = \left(0, \frac{\beta_1}{\sqrt{\alpha_1^2+\beta_1^2}}, \frac{-\alpha_1}{\sqrt{\alpha_1^2+\beta_1^2}}\right)^T, \quad h = \frac{b}{\sqrt{\alpha_1^2+\beta_1^2}}. \qquad (12)$$

Here we introduced $\alpha_1 = ({\rm Re}\,\Delta_1 + 2\delta\tau)$ and $\beta_1 = ({\rm Im}\,\Delta_1 + 2\delta e)$ with $\Delta_1 = \sqrt{\delta b^2 + 4(\delta\tau + i\delta e)^2}$.

Figure 1 shows the time evolution of the reflection matrix, $\hat{r}(t)=\tilde{r}(t)$, of model, Eq. (10), with and without spin-orbit coupling. The inset shows the time dependence of $\phi(t)$ and the y component of the vector $\vec{e}_\phi(t)$, at finite spin-orbit coupling and for different coupling strengths. At the TRIM, $t=0$ and $T/2$, the phase takes the value $\phi=0$ and $\phi=\pi$, respectively. Consequently, $\hat{r}(t)$ belongs to the class of pumps with nontrivial $\mathbb{Z}_2$ index $\nu=1$, see Eq. (2). The loops that represent the time evolution of $\hat{r}(t)$ are, therefore, fixed by the values at the two TRIM, and cannot be contracted to a single point by a continuous deformation of the microscopic Hamiltonian that preserves condition (1). In the absence of spin-orbit coupling, $\hat{r}(t)$ follows a geodesic, corresponding to the rotation around a fixed axis (black curve). At finite spin-orbit coupling, the reflection matrices trace out curved loops, indicating the absence of a fixed rotation axis. At moderately weak coupling, $\Gamma/\tau_0=0.1$ the reflection matrix follows a geodesic during most of the cycle (red/light gray curve), whose tangent is determined by the spin-orbit vector $\vec{e}_{\rm so}$. In the limit $\Gamma/\tau_0\to 0$ the loops converge to a geodesic, implying that the pump works optimally.

To model a topologically trivial pump we consider Hamiltonian (10) with a double spatial periodicity of the time-dependent parameters,

$$V_b = b(t) \sum_{n,\alpha,\beta} (-1)^n \sigma^z_{\alpha,\beta}(c^\dagger_{2n,\alpha}c_{2n,\beta} + c^\dagger_{2n+1,\alpha}c_{2n+1,\beta}),$$

$$V_{\rm st} = \tau_{\rm st}(t) \sum_{n,\alpha} (-1)^n (c^\dagger_{2n+1}c_{2n+2} + c^\dagger_{2n+2}c_{2n+1}). \qquad (13)$$

By solving Eq. (11) to leading order in $\delta\tau$, δb, δe, we find that $\hat{r}$ is given by Eq. (4) with

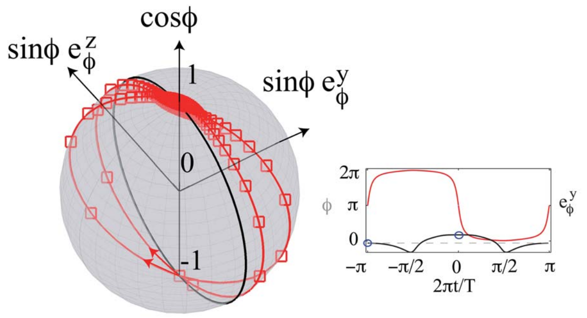

FIG. 2. (Color online) Time evolution of the $\hat{r}(t)$ of model, Eq. (13), with ($\delta e/\delta\tau=0.1$, red/light gray curve) and without (black curve) spin-orbit coupling, for $\Gamma/\tau_0=0.1$. The loops cross two resonances during a cycle and thus belong to the trivial class of pumps. At finite spin-orbit coupling, $\hat{r}(t)$ follows two different geodesics, with nonparallel tangents at the TRIM, indicated by the red arrows. The inset shows the time dependence of the y component of $\vec{e}_\phi(t)$ (black curve) and of $\phi(t)$ for $\delta e/\delta\tau=0.1$ and $\Gamma/\tau_0=0.1$ (red/light gray curve). The different values of $\vec{e}_\phi(t_i)$ at the resonances, $\phi=\pi$, are marked by the blue circles.

$$\vec{e}_\phi=\left(0,\frac{\delta\tau\beta_2+2\delta e\alpha_2}{\gamma_2\sqrt{\alpha_2^2+\beta_2^2}},\frac{2\delta e\beta_2-\delta\tau\alpha_2}{\gamma_2\sqrt{\alpha_2^2+\beta_2^2}}\right)^T,\quad h=\frac{2b\gamma_2}{\sqrt{\alpha_2^2+\beta_2^2}}. \tag{14}$$

Here $\alpha_2=\mathrm{Re}\,\Delta_2-(\delta b^2-\delta\tau^2)$, $\beta_2=\mathrm{Im}\,\Delta_2+4\delta e$, $\Delta_2=\sqrt{(\delta b^2-\delta\tau^2-4i\delta e)^2+(2\delta b\,\delta\tau-4i\delta e\,\delta b)}$, and $\gamma_2=\sqrt{\delta\tau^2+4\delta e^2}$.

Figure 2 shows the time evolution of the reflection matrix, $\hat{r}(t)$, for model, Eq. (13), with (red/light gray curve) and without (black curve) spin-orbit coupling. The loops cross two resonances at the TRIM, corresponding to the topologically trivial class of pumps, $\nu=0$, Eq. (2). In contrast to the nontrivial class of pumps, these loops are fixed by a single point. As a result, they can be contracted to this single point by a continuous deformation of the starting Hamiltonian without violating condition (1). This is manifested at finite spin-orbit coupling, where the loop avoids the north pole, $\hat{r}=\mathbb{1}$, while remaining fixed to the south pole, $\hat{r}=-\mathbb{1}$, as illustrated in the red/light gray curve. The inset shows the time dependence of the y component of the rotation axis $\vec{e}_\phi(t)$ and the angle $\phi(t)$ for finite spin-orbit coupling. The angle crosses two resonances, $\phi=\pi$, during a pumping cycle, which correspond to different values of the vector $\vec{e}_\phi(t_i)$ (see blue circles in the inset). The resulting reflection matrix traces two loops, which in the limit $\Gamma/\tau_0\to 0$ converge to two different geodesics with nonparallel tangents at the TRIM, indicated by the arrows, see red/light gray curve in main figure. Hence, in the presence of a finite spin-orbit coupling, the spin pumped during a cycle is not quantized and the pump operates with the generation of finite noise.

V. CONCLUSIONS

We have studied the class of one-dimensional pumps with a time-reversal restriction on the pumping cycle, Eq. (1). These systems can be viewed as a mapping of two-dimensional time-reversal invariant bulk insulators placed on a cylinder, where t corresponds to a magnetic flux threading the cylinder. We found that the scattering matrices of the pumps are dichotomized by the $\mathbb{Z}_2$ topological index. We have shown that the class of pumps characterized by a nontrivial topological index allows, in contrast with its topologically trivial counterpart, for the noiseless pumping of quantized spin, even if spin is not conserved. This observation sheds light on the topological classification of two-dimensional time-reversal invariant insulators.

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