Scaling the anomalous Hall effect: A connection between transport and magnetism

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An unconventional form of scaling—one based on the anomalous Hall effect—in $Fe_{0.8}Co_{0.2}Si$ demonstrates the close correlation between its magnetic and transport properties. In particular, this approach enables the universality class of its magnetic phase transition—in fields above 1 kOe—to be reliably established from the anomalous Hall conductivity, yielding $T_C=36.0\pm0.5$ K, $\delta=4.78\pm0.01$, $\beta=0.37\pm0.01$, and $\gamma=1.38\pm0.02$. These latter estimates are in excellent agreement with those deduced from conventional magnetic measurements over a comparable field range and lead to the inference that in modest fields the dominant interaction in the critical regime becomes near-neighbor isotropic exchange.

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I. INTRODUCTION

Transport properties related to the scattering of electronic *spin* rather than charge, specifically the anomalous (AHE) and spin Hall effects, have been the subject of considerable recent interest.[1] Here we report a study of the AHE in $Fe_{0.8}Co_{0.2}Si$ from which the critical exponents—and hence the universality class—of the transition to the ordered phase are deduced. Specifically, it is demonstrated that the latter can be reliably deduced from these transport data for this system.

Studies of critical behavior near continuous magnetic transitions are of fundamental importance, since establishing the associated universality class yields information on the range of the underlying interactions,[2,3] from which inferences regarding the likely interaction mechanism(s) can be made. Establishing the universality class characterizing magnetic critical behavior near a continuous paramagnetic to ferromagnetic (PM-FM) transition most often involves application of the scaling-law equation of state[2–4]

$$M(h,t) = |t|^{\beta} F_{\pm}\left(\frac{h}{|t|^{\gamma+\beta}}\right) \qquad (1)$$

to the analysis of magnetization, and to a lesser extent, susceptibility data. In Eq. (1), $t=|(T-T_C)|/T_C$ (T_C being the Curie temperature) is the reduced temperature and $h\sim H_i/T_C$ the reduced field (the internal field $H_i=H_a-N_DM$, in the usual notation); $F_{\pm}$ is the scaling function below (−)/above (+) T_C. Specifically, Eq. (1) lead to the following well-known *asymptotic* power-law predictions for the magnetization: (i) along the critical isotherm $M(H_i,T=T_C)\propto H_i^{1/\delta}$ [assuming the validity of the Widom equality, $\gamma+\beta=\beta\delta$ (Ref. 4)]; (ii) the spontaneous magnetization $M_S(H=0,T<T_C)\propto|t|^{\beta}$; and (iii) the inverse initial susceptibility $1/\chi_i(T>T_C)=(\partial H/\partial M)_{H=0}\propto|t|^{\gamma}$. These exponents can also be deduced *independently* from the ac susceptibility $\chi(h,t)=(\partial m/\partial h)$ (see, for example, Ref. 5).

While studies of critical behavior based on these latter techniques are well established and widely utilized, the advantages of a comparable approach based on transport measurements become clear when dealing with small sample sizes as, for example, in low-dimensional spintronic devices[6] (where signal-to-noise ratios are adversely affected by background and substrate contributions, thus compromising studies employing conventional methods. Here, it should be pointed out that assumptions regarding the dependence of the Hall constant on the longitudinal resistivity must be introduced as part of the analysis). Indeed, once proof-of-principle has been established for transport-based assessments of critical behavior (the principle focus of the present study), one immediate corollary would be its application to study the influence of size effects near phase transitions.

II. BACKGROUND

Thus the question of whether magnetic critical behavior can be assessed unambiguously from magneto*transport* data is an interesting—possibly fundamental—topic. Such a question is a natural consequence of the inherent connection between the magnetic and transport behavior of magnetic materials, aspects of which have been the subject of previous studies.[7] Specifically, following initial work by Craig *et al.*,[8] various attempts have been made to establish such links in a number of systems, including Ni (Ref. 8) and the dilute magnetic semiconductor (GaMn)As,[9] among others.[10] These studies, however, have often focused on the temperature-dependent, zero-field resistivity, from which estimates of a single exponent—that describing the behavior of the heat capacity—have been deduced.[8–10] In contrast, no *comprehensive* appraisal of the universality class of a magnetic phase transition, comparable to that provided by the techniques mentioned earlier, has been made to date based on magnetotransport data.

This report attempts to address this deficiency, it also appears timely, as recent debate centered on scaling between magnetization and resistivity/conductivity[7,11–13] confirms. Such debate frequently focuses on the occurrence of the Hall effect in magnets as it reflects contributions from the magnetization, as, for example, in discussions of the quantal phase in colossal magnetoresistance (MR) manganites.[7] The present study also confirms that the *anomalous* Hall conductivity is the parameter linked directly with the magnetization, and, subsequently, characterizes this link/proportionality quantitatively.[7,11–15] This issue plays a key role in any discussion of the fundamental correlation between magnetism and transport in magnetic materials and underlies the present investigation on a $Fe_{0.8}Co_{0.2}Si$ compound.

The $Fe_xCo_{1-x}Si$ system is unusual in several respects. First, while its end members ($x=0$) and ($x=1$) are nonmagnetic, at intermediate compositions it displays magnetic order. Second, this ordered structure—at least in low applied fields—is helical, a reflection of the influence of a Dzyaloshinsky-Moriya-type interaction in this noncentrosymmetric cubic B-20 structure.[16,17] The latter has led to comparisons with MnSi, an interesting and well-studied system in itself, which displays a quantum phase transition under the application of hydrostatic pressure,[18] P (i.e., with $T_C \to 0$ as $P \to P_C \sim 15$ kbar); related effects accompany Co doping in $Fe_xCo_{1-x}Si$, attributable here, however, to carrier-density modifications. $Fe_xCo_{1-x}Si$ also exhibits unconventional—possibly unique—magnetotransport behavior, in particular, a positive magnetoresistance[11] and a large anomalous Hall effect.[12,13] The principal focus of the present study is on the latter in the vicinity of the magnetic order-disorder transition.

The reasons underlying this choice of material are several, being led principally by its large Hall resistivity,[12,13] mentioned above, and a transition temperature ($T_C \approx 36$ K) which enables the temperature regime both above and below T_C to be accessed with relative ease. Furthermore, this system is topical due to its displaying an extraordinary positive magnetoresistance $\{\Delta\rho = [\rho(H) - \rho(0)]/\rho(0)\}$ below 130 K,[11,12] the origin of which is a subject of ongoing debate.[11,12] The latter is, however, not the focus of the present study; rather, the convincing demonstration of a technique which enables the *anomalous* Hall coefficient to be extracted reliably from data in a situation where conventional extrapolation methods to estimate the *ordinary* Hall coefficient are clearly inadequate.[19,20]

An issue of particular importance to the present study is the often stated concern in evaluations of the critical response based on conventional magnetization measurements (and less widely used field-dependent ac susceptibilities) regarding the *asymptotic* nature of the predictions (i)–(iii). Specifically, data from the asymptotically low-field regime ($H_i \sim 0$) are generally excluded from scaling analyses. The reason for this is well established; the measured response contains both regular and critical contributions [the latter being the basis of Eq. (1)], thus comparisons with the above predictions are only appropriate when fields of sufficient magnitude to saturate the regular contributions (arising from domain-wall motion and/or coherent rotation, for example) have been applied. Issues surrounding demagnetization correction uncertainties are also paramount in this low-field regime. These constraints are of direct relevance to the $Fe_xCo_{1-x}Si$ system where, near the critical regime, quite modest applied fields (typically <1 kOe) are known to convert the helical order to essentially uniform ferromagnetism.[16,17] This is expected as the (uniform) applied field is the conjugate field for the latter. The response measured in fields *exceeding* a kilo-oersted is thus expected to display characteristics reminiscent of a more conventional PM-FM transition, as the following analysis (and associated ac susceptibility data[21]) confirms.

III. EXPERIMENTAL DETAILS

A polycrystalline specimen with nominal composition $x=0.8$ was grown as suggested by Manyala *et al.*[11] Powder x-ray diffraction revealed a single-phase structure. Magnetic and transport data were acquired on a sample with dimensions $(4 \times 1 \times 0.13)$ mm^3 using a commercial physical properties measurement system with fields applied along the largest sample dimension to reduce demagnetization corrections. Transport experiments were performed using a five-terminal technique on the same sample with an excitation current of 5 mA at 499 Hz. A linear current-voltage response at 2 K and above confirmed the absence of Joule heating at this current level. Possible Hall bar mismatch effects were cancelled by scanning the field from positive (+90 kOe) to negative (−90 kOe) values.

IV. RESULTS AND DISCUSSION

Figure 1 reproduces a selection of temperature- and field-dependent magnetic and transport data. Figure 1(a) displays the zero-field ac susceptibility, measured on warming following zero-field cooling; the peak evident near 36 K in is close agreement with previous reports at this composition,[11,12,22] around which measurements focusing on the transition region were implemented. Figure 1(b) reproduces the temperature-dependent transport data, which again agree overall with previously data reported on both polycrystal and single-crystal specimens.[11,12,22] In particular, the positive MR evident in the inset attests to the high quality of the present specimen.[12] In Fig. 1(c), the magnetization as a function of field at various fixed temperatures close to T_C is plotted in the form suggested by the modified Arrott-Noakes equation of state,[4] viz., $(H_i/M)^{1/1.387}$ versus $M^{1/0.365}$. The ensuing series of parallel straight lines indicate the applicability of essentially Heisenberg model exponents ($\gamma=1.387$, $\beta=0.365$, and $\delta=4.783$),[2–5] *in fields exceeding* 1 kOe. (i.e., fields sufficient to suppress the helical phase in favor of its uniform counterpart[17]), an unanticipated result given the itinerant characteristics displayed by this system (an issue returned to below). The individual exponent determinations are presented in Figs. 1(d)–1(f); the first of these, using data along the critical isotherm, yields an estimate for the exponent $\delta = 4.78 \pm 0.01$; the second and third yielding exponent values of $\gamma=1.38 \pm 0.02$ from the inverse initial susceptibility, and $\beta=0.37 \pm 0.01$ from the spontaneous magnetization.

With these exponent values unambiguously determined (in fields above 1 kOe) by conventional means, the principal focus of the present study is presented, viz., an independent determination of the universality class of the transition via scaling the anomalous Hall conductivity. The occurrence of the so-called AHE in magnetic systems[23] originates from additional contributions associated with the presence of magnetic moments/magnetization, indeed the two are directly—but empirically—linked via[19,20,24,25]

$$\rho_{xy} = R_0 B + 4\pi R_S M. \tag{2}$$

In this equation, $B=[H+4\pi(1-N_D)M]$, and with the applied field H oriented perpendicular to the current flow, the demag-

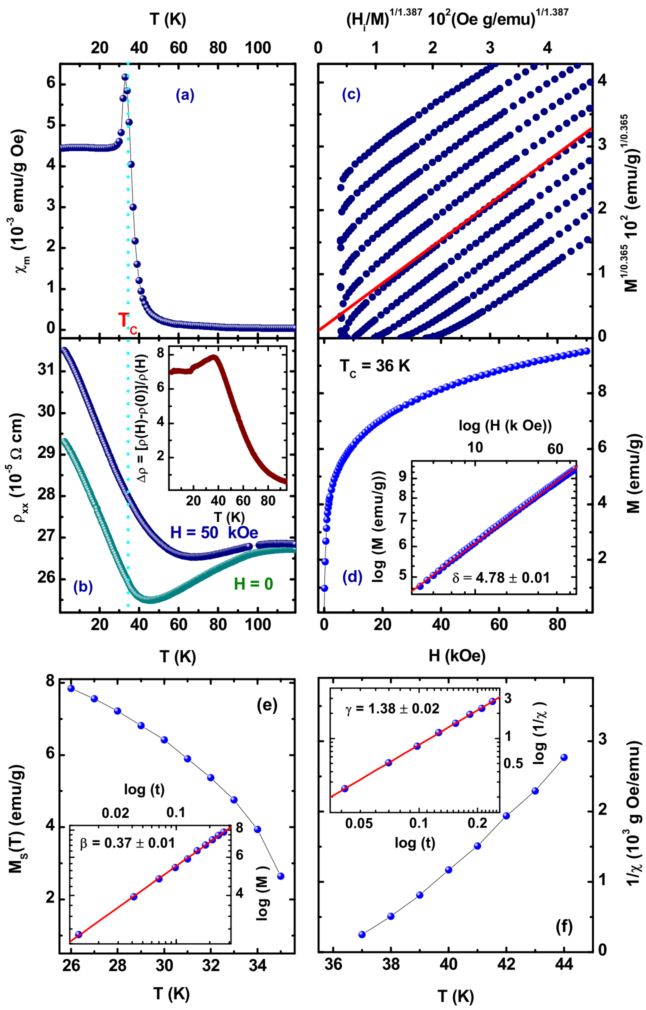

FIG. 1. (Color online) Summary of magnetic and transport measurements on $Fe_{0.8}Co_{0.2}Si$. (a) The zero-field ac susceptibility (measured on warming following zero-field cooling) with the peak near 36 K being in good agreement with previous reports. (b) The temperature-dependent transport data with a positive MR (expressed in percentages) reproduced in the insert. Magnetization isotherm data around T_C [26 K (top) to 44 K in 2 K steps, and 50 K (bottom)], replotted in the from suggested by a modified Arrott-Noakes equation of state [$(H_i/M)^{1/1.387}$ vs $M^{1/0.365}$], using Heisenberg model exponent values of γ=1.387, β=0.365, and δ=4.783. The critical isotherm passes through the origin. Data along the latter are fitted in figure (d) and yield $\delta=4.78\pm0.01$. (e) The spontaneous magnetization $M_S(T)$ plotted against temperature T: inset; M_S vs $t=(T_C-T)/T_C$ on a log-log scale, yielding $\beta=0.37\pm0.01$. (f) The inverse initial susceptibility, $1/\chi_i$, plotted against T: inset; $1/\chi_i$ vs t_m on a log-log scale, yielding $\gamma=1.38\pm0.02$.

netization factor $N_D\approx1$ for the present sample geometry, then $B\approx H$. $R_0=1/nec$ (e is electronic charge) is the ordinary Hall coefficient due to carrier's deflection—the influence of the Lorentz force—and yields the carrier type and concentration (n). R_S is the anomalous Hall coefficient, which can display temperature-/field-dependent features reflecting various types of magnetic order.[19,20,24,25] As the parameter central to the current analysis, its' accurate evaluation is imperative; this, in turn, implies a careful subtraction of the contribution from R_0. In systems such as $Fe_xCo_{1-x}Si$ where the magnetization remains *unsaturated* in available fields, this represents a significant challenge, the solution to which is central to the current discussion, and is detailed below.

Using the well-established expression for the Hall conductivity

$$\sigma_{xy}=\frac{\rho_{xy}}{\rho_{xy}^2+\rho_{xx}^2}\approx\frac{\rho_{xy}}{\rho_{xx}^2}=\frac{R_0H+4\pi R_SM}{\rho_{xx}^2} \qquad (3)$$

the last form being valid when $\rho_{xy}\ll\rho_{xx}$, an inequality well satisfied in the present system as ρ_{xx} is at least 60 times larger than ρ_{xy}. In ferromagnets, with σ_{xy} written as $\sigma_{xy}=\sigma_{xy}^O+\sigma_{xy}^A$ in the usual notation,[20,25] i.e., with σ_{xy}^O representing the ordinary Hall conductivity, then the following expression for anomalous Hall conductivity σ_{xy}^A ensues:

$$\sigma_{xy}^A=\sigma_{xy}-\sigma_{xy}^O=\frac{4\pi R_SM}{\rho_{xx}^2}, \quad \sigma_{xy}^O=R_0H/\rho_{xx}^2. \qquad (4)$$

To proceed further, the issue of the dependence (or lack thereof) of R_S on the longitudinal resistivity, ρ_{xx}, also needs to be addressed.[19,20,24–32] R_S has various been taken as constant[26] or to display either a linear[27] or a quadratic[28–32] dependence on ρ_{xx}, depending on the dominant mechanism involved. Indeed, the field dependence of ρ_{xx} plays a pivotal role in the corresponding dependence of R_S. The central premise of the present report is that both this dependence on ρ_{xx} and an accurate evaluation of σ_{xy}^O can be addressed simultaneously using an approach analogous to that used in scaling approaches for the magnetization, viz., by introducing a function $G(R_S,\rho_{xx})=4\pi R_S/\rho_{xx}^2$. This enables Eq. (4) to be rewritten as

$$\sigma_{xy}^A=M\cdot G(R_S,\rho_{xx}). \qquad (5)$$

It follows directly from this equation that a plot of σ_{xy}^A vs M yields the functional form of $G(R_S,\rho_{xx})$, a result that actually has general application to other magnetic materials, rather than being specific to just $Fe_xCo_{1-x}Si$. This same equation also provides a means of establishing the appropriate dependence (if any) of R_S on ρ_{xx} in magnetic systems: specifically, if R_S varies as ρ_{xx}^Q, a plot of $\log(\sigma_{xy}^A/M)$ vs $\log(\rho_{xx})$ will yield the unspecified index Q, and hence the required dependence of R_S. Application of this approach is now demonstrated.

Figures 2(a) and 2(b) reproduce the field dependence of the Hall resistivity, ρ_{xy}, and the longitudinal resistivity, ρ_{xx}. Before implementing the above form of scaling, the issue of accurate estimates for the ordinary Hall coefficient, R_0, needs to be addressed.[19,20] While this can be done with reasonable accuracy in systems where the magnetization can be saturated in available fields, such a situation is not realized here; in $Fe_xCo_{1-x}Si$ the magnetization does not reach saturation even at 2 K in fields of 90 kOe.[16,17,22] It thus needs to be reiterated that conventional approaches are *not* appropriate for this system [as is also the situation in MnSi (Ref. 20)], as the following illustrates succinctly. Dividing Eq. (2) by H yields

$$\rho_{xy}/H=R_0+4\pi R_S\chi, \quad \chi=M/H, \qquad (6)$$

where the "susceptibility" $\chi=M/H$; thus plots of ρ_{xy}/H vs χ extrapolated from *high* field (*low* susceptibility) onto the

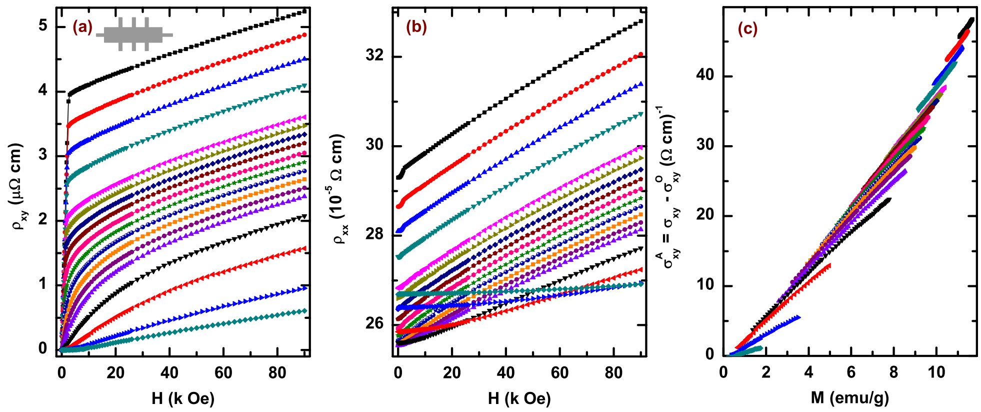

FIG. 2. (Color online) Compilation of magnetization and magnetotransport data. Field dependence (a) of the Hall resistivity, ρ_{xy}, and of the (b) longitudinal resistivity, ρ_{xx}, at 2 (top), 10, 15, 20, 26–44 (in 2 K steps), 50, 60, 80, and 100 K (bottom) in fields up to 90 kOe. Inset in (a) shows the schematic set up for transport measurements. Good data overlap from three sets of Hall Bar measurements confirms the homogeneity of the present system. (c) Plots of the anomalous Hall conductivity $\sigma^A_{xy} = \sigma_{xy} - \sigma^O_{xy}$ vs M, the linear dependence of which at each measuring temperature indicate $G(R_S, \rho_{xx}) = 4\pi R_S/\rho^2_{xx}$, enabling the proportionality factor/constant, $K(T)$, to be estimated.

ρ_{xy}/H axis should yield R_0.[19] Under the conditions prevalent in $Fe_xCo_{1-x}Si$, however, the values of R_0, so obtained, are overestimated (the carrier density n is underestimated), leading to significant uncertainties in R_S. A value for R_0 of the appropriate accuracy can be obtained in conjunction with the scaling approach [i.e., the establishment of the functional form of $G(R_S, \rho_{xx})$] by using the result that irrespective of this precise form, plots of σ^A_{xy} vs M *must* extrapolate to $\sigma^A_{xy} = 0$ (as inferred directly from the relationship $\sigma^A_{xy} = \frac{4\pi R_S M}{\rho^2_{xx}}$). Figure 2(c) demonstrates this technique; plots of $\sigma^A_{xy} = \sigma_{xy} - \sigma^O_{xy}$ vs M are reproduced in which the above constraint on the *intercept* has been implemented: it should be stressed that this intercept constraint *cannot* change any functional dependence of $\sigma^A_{xy} = \sigma_{xy} - \sigma^O_{xy}$ on M or ρ_{xx}. As such, this procedure has broad potential application to other systems which remain unsaturated in high field—$URu_{2-x}Re_xSi_2$, for example.[33]

The immediate outcome of this approach is improved values for the modified ordinary Hall coefficients, R_0, [Fig. 3(a)] and, correspondingly, the carrier density $n = 1/R_0ec \approx (3.75 \pm 0.04) \times 10^{22}$ cm^{-3} [Fig. 3(b)]. Note the expanded scale in these figures, the actual estimates vary by only some 2% below 100 K. An immediate corollary to the latter is that across the FM-PM transition, the ordinary Hall coefficient (R_0)—and hence carrier density (n)—does *not* exhibit any measurable critical characteristics, as similarly concluded in Ref. 13, at least in the system studied. Correspondingly, the technique utilized above can be used to address the question of whether R_0 and n exhibit critical behavior across FM-PM transitions (coincident here with a metal-insulator transition), an issue of importance in the understanding of carrier-mediated ferromagnetism, as, for example, in the dilute magnetic semiconductor (GaMn)As.[34]

The second outcome, one of fundamental importance and central to the present work, is that Fig. 2(c) establishes the functional dependence of σ^A_{xy} on M as being *linear* with considerable accuracy; the function $G(R_S, \rho_{xx})$ is thus a *constant* at any given temperature. An immediate consequence of the latter is that

$$R_S \propto \rho^2_{xx} \rightarrow R_S = K(T)\rho^2_{xx}, \qquad (7)$$

where $K(T)$ is, in general, a temperature-dependent coefficient. *Neither of these functional dependences established immediately prior to* Eq. (7) *can be modified by the intercept constraint discussed earlier*. The importance of the latter cannot be overemphasized, as the linear dependence of σ^A_{xy} on M ($\sigma^A_{xy} = K(T) \cdot M$) in $Fe_{0.8}Co_{0.2}Si$ established in this way enables scaling behavior based on the AHE to be implemented, and the universality class of the transition to be deduced.

In detail, the above results imply that the normalized anomalous Hall conductivity $\sigma^A_{xy}/K(T)$ is simply a linear function of the magnetization M, a relation confirmed in Fig. 3(d), which incorporates data between 2 and 100 K in field

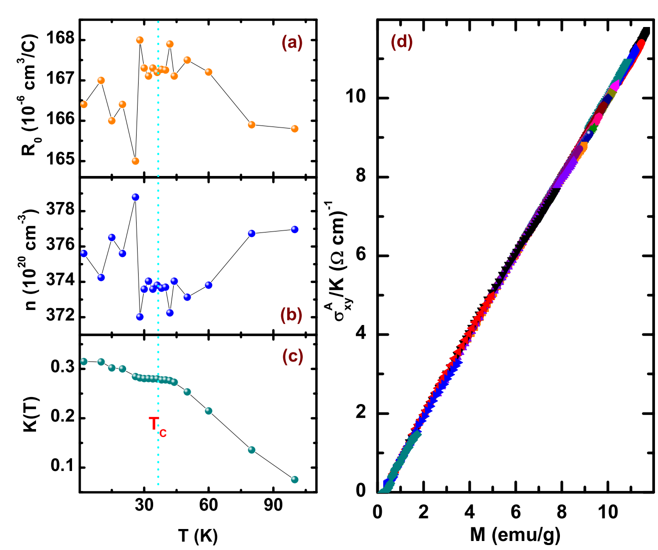

FIG. 3. (Color online) (a) The modified ordinary Hall coefficient, R_0, and (b) the corresponding carrier density, n. (c) The proportionality constant, $K(T)$, from Fig. 2(c). (d) Linear scaling plot of the normalized anomalous Hall conductivity $\sigma^A_{xy}/K(T)$ vs M, incorporating measured values of ρ_{xx}, ρ_{xy}, and M.

up to 90 kOe. Of particular importance in the present context is that the σ_{xy}^A vs M relationship in *the critical regime* assumes a particularly simple form; here, as Fig. 3(c) demonstrates, $K(T)$ is essentially *constant* in the vicinity of T_C for the present system [At this stage, it should be pointed out that since the AHE is proportional to $\lambda \boldsymbol{E} \times \boldsymbol{M}$, $\boldsymbol{E}$ being the electric field, and $\boldsymbol{M}$ the magnetization tensors, the parameter $K(T)$ coupling transport/AHE and magnetism should reflect the spin-orbit parameter λ (Refs. 25 and 35)]. The ensuing conclusion of the direct linearity of the σ_{xy}^A-M relationship throughout the critical regime can be further tested by constructing an a equation of state plot based on the *transport* parameter σ_{xy}^A—the analog of the conventional Arrott-Noakes equation of state[4]—by the simple replacement of the magnetization, M, by the anomalous Hall conductivity, σ_{xy}^A, viz.

$$\left(\frac{\sigma_{xy}^A}{\sigma_1}\right)^{1/\beta} = \frac{T_C - T}{T_C} + \left(\frac{H}{\sigma_{xy}^A}\right)^{1/\gamma} \tag{8}$$

σ_1 being a material specific constant. The consequences of this direct linearity prediction are tested in Fig. 4(a). Here the ensuing series of parallel straight lines resulting from the adoption of three-dimensional (3D) Heisenberg model exponents[2–5] together with an ordering temperature estimate of $T_C = 36.0 \pm 0.5$ K [found from the critical anomalous Hall conductivity isotherm, $\sigma_{xy}^A(T_C)$, passing through the origin], confirm unequivocally the applicability of Eq. (8), and, indirectly, the assumptions on which it is based. Note, in particular, that the data incorporated in this figure are from σ_{xy}^A *alone*, and the exponents values used agree with those found *independently* from magnetization (and ac susceptibility data[21]). Figure 4(a) thus demonstrates scaling based *solely* on AHE data in the critical region. The self-consistency of this approach, and the exponent values it yields, can be assessed using techniques analogous to those implemented with conventional magnetization studies,[4,5] as follows.

The intercepts of these linearized plots on the abscissa, $(\sigma_{xy}^A/\sigma_1)^{1/\beta}$ at $H=0$, yield the corresponding "spontaneous" anomalous Hall conductivity, designated $\sigma_{xy\text{-}Spon}^A$, which, this same model approach predicts, is governed by the power-law $\sigma_{xy\text{-}Spon}^A(t) \propto t^{\beta}(T<T_C)$ [$t=(T-T_C)/T_C$ being the reduced temperature]. Correspondingly, those on the ordinate axis $((H/\sigma_{xy}^A)^{1/\gamma})$ yield a quantity analogous to the (inverse) initial susceptibility $[1/\chi_i(T) = (\partial H/\partial M)_{H=0}]$, predicted by Eq. (8) to vary as $H/\sigma_{xy}^A \propto t^{\gamma}(T>T_C)$. Finally, along the critical isotherm—the σ_{xy}^A isotherm passing through the origin—corresponding to the critical temperature, T_C, at which $\sigma_{xy\text{-}Spon}^A$ first emerges, the field dependence of σ_{xy}^A is characterized by $\sigma_{xy}^A(H, T=T_C) \propto H^{1/\delta}$. Self-consistency is demonstrated by taking these intercepts and (re)testing them against the corresponding reduced temperature/field power-law relationships given above, making *small* adjustments until self-consistency is achieved.[4,6] The final choices are shown in Figs. 4(b)–4(d), and their inserts, yielding $\delta = 4.78 \pm 0.01$, $\beta = 0.37 \pm 0.01$, and $\gamma = 1.38 \pm 0.01$ with $T_C = 36.0 \pm 0.5$ K. These values not only demonstrate consistency with those found from magnetization (and ac susceptibility[21]) but are also very close to 3D Heisenberg model exponents,[2–5] which

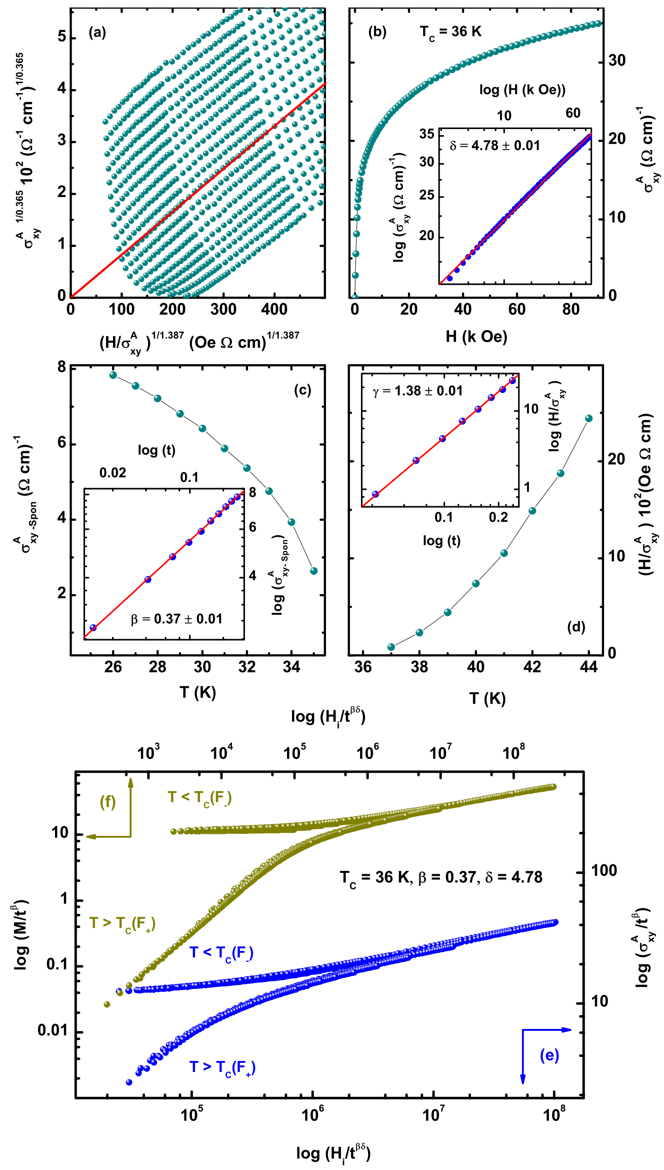

FIG. 4. (Color online) Critical analysis based on the anomalous Hall conductivity σ_{xy}^A incorporating data between 26 and 44 K (in 1 K step). (a) σ_{xy}^A-field (H) data at the above temperatures reproduced in the form $\sigma_{xy}^{A\ 1/0.365}$ vs $(H/\sigma_{xy}^A)^{1/1.387}$; the resulting series of parallel straight lines confirm the exponent assignments with the line passing through the origin yielding $T_C = 36 \pm 0.5$ K. (b) The critical anomalous Hall conductivity $\sigma_{xy}^A(T_C)$ plotted against H; the inset shows the same data replotted on a double logarithmic scale, the slope of which yields $\delta = 4.78 \pm 0.01$ for $2 < H < 90$ kOe. (c) The spontaneous anomalous Hall conductivity $\sigma_{xy\text{-}Spon}^A$ plotted against T; inset, $\sigma_{xy\text{-}Spon}^A$ vs t on a log-log scale, the slope of the straight line drawn yielding $\beta = 0.37 \pm 0.01$. (d) The quantity H/σ_{xy}^A, plotted against T; inset, double logarithmic plot of H/σ_{xy}^A against t, yielding $\gamma = 1.38 \pm 0.01$. (e) A comprehensive scaling plot for σ_{xy}^A, using the critical exponents and T_C value listed above. The data collapse onto the $\log_{10}(\sigma_{xy}^A/t^{\beta})$ vs $\log_{10}(H/t^{\beta\delta})$ plot demonstrates convincingly the self-consistent determination of T_C and the critical exponents γ, β, and δ. (f) Conventional scaling plot of the corresponding magnetization M using the same exponent and T_C values. Upper/lower branches in (e) and (f) correspond to data below (F_-)/above (F_+) T_C, respectively.

thus appear to describe the transition to the ordered phase *in fields above* 1 kOe. The emergence of localized model exponents in a system displaying itinerant features is discussed below.

Finally these anomalous Hall conductivity data, σ_{xy}^A, have been scaled using the well-established scaling equation of state modified to reflect the linear σ_{xy}^A-M relationship, i.e.,

$$\sigma_{xy}^A(H,t) = |t|^{\beta} F_{\pm}\left(\frac{H}{|t|^{\gamma+\beta}}\right). \qquad (9)$$

Figure 4(e) demonstrates convincingly that data from Fig. 4(a) can be scaled onto two "branches" expected to result from measurements taken below, F_-, and above, F_+, T_C. This figure confirms unequivocally the reliability of the estimates of critical exponents and the ordering temperature in this system using anomalous Hall conductivity data alone; it also shows that this system falls into the universality class of the isotropic, near-neighbor 3D Heisenberg model (again, in fields in excess of 1 kOe). The more conventional scaling of the magnetization data using the analogous equation of state, Fig. 4(f), provides a final verification (as does the scaled ac susceptibility,[21] the latter yielding $\delta = 4.78 \pm 0.01$, $\beta = 0.38 \pm 0.01$, $\gamma = 1.37 \pm 0.01$, and $T_C = 35.9 \pm 0.2$ K, again in fields above 1 kOe).

Such a result has interesting consequences since, as stated in Sec. I, the universality class of a transition is intimately linked with the range of the dominant interactions in the system. In particular, there exists a general consensus that whereas the critical behavior for localized spins coupled via short-ranged (Heisenberg) interactions in insulators is determined by the space/lattice dimensionality, d, and the order parameter/spin dimensionality, k, the corresponding behavior in metallic systems is markedly different.[3,6,36] A preconception that might exist, referred to earlier, based on the semimetallic nature of $Fe_{1-x}Co_xSi$ and its significant Rhodes-Wohlfarth parameter,[37] is that the present system is more likely to resemble the latter rather than the former.

For the latter, renormalization-group calculations[36] predict that for long-ranged attractive spin-spin interactions decaying with distance r as $J(r) \approx r^{-(d+\sigma)}$, mean-field behavior occurs for $\sigma \le d/2 = 3/2$ here [i.e., if $J(r)$ decreases with r slower than $r^{-4.5}$]. In contrast, for $\sigma > 2$ short-ranged critical behavior ensues, while in the intermediate regime, $d/2 \le \sigma \le 2$, the critical exponents depend on the value for σ. The present data indicate, possibly unexpectedly, though unequivocally that in $Fe_{0.8}Co_{0.2}Si$, short-range interactions—specifically near-neighbor isotropic exchange—emerge as the dominant interaction in the critical regime [at least in quite modest applied fields (<1 kOe) (Ref. 17)]. In the same counterintuitive vein, near-neighbor 3D Heisenberg model exponents also appear to describe the universality class of the transition to ferromagnetism in metallic alloys such as PdMn (Ref. 38) and PdFe (Ref. 39) which display much more marked itinerant characteristics than the present system.

Another aspect of the present study is the observation of $\sigma_{xy}^A = K(T) \cdot M$, which supports the exclusion of an extrinsic origin for the AHE (Refs. 11 and 12) in this system, at least from asymmetric skew scattering,[27] for which $R_S \propto \rho_{xx}$; side-jump scattering[28] has been ruled out by studies over a wider composition range.[12] The above observation is also consistent with a recent theoretical calculation based on the sum of Berry-phase curvature, a key prediction of which is that the intrinsic anomalous Hall conductivity $\sigma_{IAH} \propto M$.[32] Indeed, the approach summarized above enables the normalized anomalous Hall conductivity $\sigma_{xy}^A / K(T)$ to be directly correlated with the magnetization, M, over wide temperature (2–100 K) and field (90 kOe) ranges.

V. SUMMARY AND CONCLUSIONS

The present results demonstrate unequivocally that the universality class of magnetic phase transitions can be accurately deduced from measurements of the AHE, specifically, the *anomalous* Hall conductivity rather than the "general" Hall resistivity/conductivity. Such a result could prove pivotal in improving our current understanding of the correlation between magnetism and transport in magnetic metals/semiconductors, specifically those that exhibit a substantial AHE. The wider investigation of the applicability of the present approach is planned for the immediate future, especially to device-scale spintronic materials with extreme weak magnetic signals, raising the prospects for a study of size effects of the AHE in such devices in general, and the role played by dimensionality in critical behavior in particular. As a final note, the scaling implemented here incorporates measurements of ρ_{xx}, M, and ρ_{xy} and is thus more comprehensive than those advocated previously which involve just ρ_{xx} and M.[10,11]

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