STM differential conductance of a disordered extreme type-II superconductor at high magnetic fields

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We calculate the tunneling conductance between a scanning tunneling microscope tip and the surface of an extreme type-II superconductor in a high magnetic field and at zero temperature. In a clean system, and with electrons tunneling only along vortex lines, we find that the spatially averaged differential conductance $\sigma(V)$ has a V-shape dependence on the bias voltage V, reflecting the presence of gapless points and near gapless regions in the quasiparticle excitation spectrum of a superconductor in high magnetic fields. Within a T-matrix approximation for a homogeneous superconductor, we investigate the influence of nonmagnetic impurities on the differential conductance. We find that in the presence of disorder the differential conductance becomes finite at zero bias and develops a nonlinear dependence on the bias voltage. We apply our theory to calculate the differential conductance $\sigma(V)$ of the superconductor $LuNi_2B_2C$ in the mixed state.

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In the last two decades most of the superconducting systems that hold the greatest promise for practical application are of the extreme type-II variety. They can be loosely defined as systems in which the semiclassical critical field at zero temperature $H_{c2}(0)$ in units of Tesla becomes comparable to, or even larger than T_c (measured in zero field) in units of Kelvin. High-temperature superconductors (HTSs), nonmagnetic nickel borocarbide superconductors,[1] A-15 superconductors[2] and the superconductor MgB_2 (Ref. 3) are true members of the extreme type-II species, as they satisfy this definition. Recent reports on upper-critical field measurements[4] in iron-based superconductors suggest that oxypnictides represent a new class of high-field superconductors with H_{c2} values surpassing 100 T. All indications so far suggest that these newly discovered iron-based superconductors are of the extreme type-II variety.

In extreme type-II systems, where superconductivity coexists with high magnetic field, it is necessary to use the high magnetic field limit as a theoretical starting point for understanding the mixed-state behavior of these systems. The high-field limit is often neglected when mixed-state properties are calculated, and most commonly, the low-field semiclassical approach of Abrikosov-Gor'kov theory[5] (and its iterations) is implemented. The criterion for distinguishing the high-field limit from its low-field limit counterpart is straightforward—If Landau level (LL) quantization of electronic energies in the magnetic field *within* the superconducting state is well defined and apparent in the spectrum, one is dealing with the high-field limit. In this limit the discreteness of the electronic energies has to be included in the description of the superconducting instability, both in the collective modes (i.e., Cooper pairs), as well as in quasiparticle excitations.[6] The high-field regime in which the Landau level structure is well resolved can be identified in the H-T phase diagram by a simple criterion, the cyclotron energy $\hbar\omega_c$ (where $\omega_c = eH/m*$, $m*$ is the effective mass of the electrons) must be larger than the BCS gap $\Delta(T,H)$, the thermal energy $k_B T$ and the inverse scattering rate due to disorder Γ. This high-field regime represents a large portion of the H-T phase diagram of an intrinsically extreme type-II superconductor and can be extended down to magnetic fields as low as $H \sim (0.2-0.5)H_{c2}(0)$ and temperatures as high as $T \sim 0.3T_c$. The size of this quantum region can be estimated from de Hass-van Alphen (dHvA) experiments in single crystal A-15 superconductors,[2] nonmagnetic borocarbide superconductors,[7,8] and, most recently, in the superconductor MgB_2.[9]

The subject matter of this work starts from the principal theme that at high magnetic fields and low temperatures, vortices are close to each other and their separation (given by the magnetic length $l = \sqrt{\hbar/eH}$) becomes much smaller than the electronic mean-free path. Therefore, in a fairly clean sample at low temperature, quasiparticle excitations can travel coherently over many unit cells of the vortex lattice. When, under these conditions, the LL quantization is exactly accounted for in the superconducting pairing, the solution of the BCS problem points to a qualitatively new *gapless* nature of the quasiparticle spectrum at high fields.[10] As a consequence, an s-wave, conventional superconductor in a high magnetic field becomes somewhat similar to a d-wave, unconventional superconductor with nodes in the gap. In the low-temperature and high-field regime, however, the nodes in the gap reflect the *center-off-mass* motion of the Cooper pairs in the magnetic field, in contrast to d-wave HTS cuprates where such nodes are due to the *relative orbital* motion in zero fields. Extensive numerical calculations of quasiparticle excitation spectra in the mixed state for both s-wave and d-wave superconductors[11] have found that for fields $H \gtrsim 0.5H_{c2}$, no qualitative difference in behavior can be seen between s-wave and d-wave cases. This gapless behavior in three-dimensional systems persists to surprisingly low magnetic fields $H^* \sim (0.2-0.5)H_{c2}$.[10–13] Below this critical field H^* in an s-wave superconductor, gaps start opening in the quasiparticle spectrum, and the system eventually reaches the low-field regime of localized states in the cores of isolated, well-separated vortices.[14] On the other hand, a d-wave system still exhibits the extended nature of low-lying quasiparticle excitations.[15] The estimate for H^* depends on the value of the s-wave gap function and can be much smaller than $H^* \sim (0.2-0.5)H_{c2}$ if the value of minimum gap in a strongly anisotropic (or multigap) *s*-wave case is significantly different from the accepted BCS value. Furthermore, recent low-field semiclassical calculations suggest that even in the isotropic *s*-wave superconductors there is a certain degree of delocalization of quasiparticles bound to the vortex core,[16] therefore careful consideration is needed when one identifies a gap symmetry by measuring physical quantities that are sensitive to the quasiparticle excitations in the mixed state.[17] The high-field gapless character of the excitation spectrum in an extreme type-II superconductor is not destroyed by a moderate level of nonmagnetic impurities present in either a dirty homogeneous superconductor[18] or a dirty inhomogeneous superconducting system.[19]

Low-temperature gapless quasiparticle excitations strongly influence the phenomenology of superconductors at high fields, including thermodynamics,[20] thermal transport,[21] and acoustic attenuation.[22] Furthermore, the persistence of the dHvA signal deep within the mixed state of three-dimensional extreme type-II systems (A-15's,[2] nonmagnetic borocarbides,[7] and MgB_2 superconductors[9]) can be attributed to the presence of a small portion of the Fermi surface containing gapless and near gapless quasiparticle excitations, surrounded by regions where the gap is large.[23,24] The purpose of this work is to address yet another probe of low-temperature quasiparticle excitations—the tunneling current. We examine in detail the quasiparticle contribution to the tunneling differential conductance in the mixed state of a three-dimensional extreme type-II superconductor starting from the high-field limit of the Landau-level pairing scheme. We evaluate numerically the spatially averaged differential conductance $\sigma(V)$ (where V is the bias voltage) between an STM tip and the surface of an *s*-wave type-II superconductor separated by a vacuum barrier or a thin insulating layer in the mixed state under realistic assumptions for the microscopic properties of the materials studied (i.e., in the presence of impurities). Furthermore, we examine the field dependence of $\sigma(V)$ in the mixed-state $H_{c1} \ll H \lesssim H_{c2}$ of a real superconducting system. Within the Landau-level pairing scheme the magnetic field is measured by the number $n_c \sim 1/H$ of Landau levels occupied by the electrons participating in the superconducting pairing ($n_c = E_F/\hbar\omega_c$, where E_F is the Fermi energy) and is often quite large. In the borocarbide superconductor $LuNi_2B_2C$, the number of occupied Landau levels can be estimated as $n_c \sim 33$ at $H_{c2} = 7$ T and $n_c \sim 100$ at a field of $H = 2.5$ T. These numbers are sufficiently small that the computational part of the project is feasible and deems the superconductor $LuNi_2B_2C$ a suitable choice for our numerical calculations. Many reports in recent years suggest that there is an anisotropy in the *s*-wave gap function in this borocarbide superconductor,[25–29] which extends the range of validity of the theory presented in this paper to fields lower than the critical field H^*.[21]

In a planar junction model where electrons can tunnel only along the vortex lines, i.e., along the direction of the magnetic field, the tunneling matrix elements conserve the quasiparticle quasimomenta $\boldsymbol{q}$ perpendicular to the direction of the magnetic field.[10,30] For small tunneling voltages V such that $eV \leq \hbar\omega_c$, the tunneling into the superconductor will take place over a very narrow span of energies around the Fermi level.[31] Therefore, it is reasonable to assume that the tunneling will preserve the Landau level index n and that all the tunneling matrix elements will be the same constant evaluated at the Fermi momenta $k_z = \pm k_{Fn} = \sqrt{2m^*[\mu - \hbar\omega_c(n+1/2)]/\hbar^2}$. Moreover we assume that they are spin independent. While this model described well a planar tunneling junction, in this paper we are interested in an STM tunneling junction that can reveal information about the local density of states of a superconducting sample. In STM tunneling the local nature of the STM tip allows it to sample all quasimomenta perpendicular to the magnetic field. Then, it follows that the tunneling current due to an applied voltage V is related to the single-particle spectral function $A_N(\boldsymbol{r},\omega)$ of the normal metal in the STM tip and the single particle spectral function $A_S(\boldsymbol{r},\omega)$ of the superconductor at a position $\boldsymbol{r}$ on the surface as[31]

$$I(\boldsymbol{r},V,T) \propto \int d\varepsilon A_S(\boldsymbol{r},\varepsilon) A_N(\boldsymbol{r},\varepsilon + eV) \times [n_F(\varepsilon) - n_F(\varepsilon + eV)], \tag{1}$$

where $n_F(\omega)$ is the Fermi function. We assume that the spectral function of the normal metal is energy independent and evaluated at the Fermi surface. The superconducting spectral function is defined as

$$A_S(\boldsymbol{r},\omega) = -\frac{1}{\pi}\Im\mathrm{m}G(\boldsymbol{r},i\omega)|_{i\omega=\omega+i\delta}. \tag{2}$$

In a mean-field approximation, the superconducting Green's function $G(\boldsymbol{r},i\omega)$ for a clean superconductor is expanded in terms of a complete set of eigenfunctions in the magnetic sublattice representation (MSR).[10] In the Landau gauge $\boldsymbol{A} = H(-y,0,0)$, the eigenfunctions $\phi_{k_z,\boldsymbol{q},n}(\boldsymbol{r})$ belonging to the nth Landau level can be written as

$$\begin{aligned}\phi_{k_z,\boldsymbol{q},n}(\boldsymbol{r}) = & \sqrt{\frac{b_y}{2^n n! \sqrt{\pi} l L_x L_y L_z}} \exp(ik_z z) \\ & \times \sum_k \exp\left(i\frac{\pi b_x}{2a}k^2 - ikq_y b_y\right) \\ & \times \exp\left[i\left(q_x + \frac{\pi k}{a}\right)x - 1/2\left(y/l + q_x l + \frac{\pi k}{a} l\right)^2\right] \\ & \times H_n\left[\frac{y}{l} + \left(q_x + \frac{\pi k}{a}\right)l\right],\end{aligned} \tag{3}$$

where $\boldsymbol{a} = (a,0)$ and $\boldsymbol{b} = (b_x, b_y)$ are the unit vectors of the triangular or square vortex lattice, and $L_x L_y L_z$ is the volume of the system. $H_n(x)$ is the Hermite polynomial of the order n. Quasimomentum $\boldsymbol{q}$ is restricted to the first magnetic Brillouin zone (MBZ) that is spanned by vectors $\boldsymbol{Q_1} = (b_y/l^2, -b_x/l^2)$ and $\boldsymbol{Q_2} = (0, 2a/l^2)$. In this representation the normal and anomalous Green's functions can be constructed as[18]

$$G(\boldsymbol{r},i\omega) = \sum_{n,k_z,\boldsymbol{q}} \phi_{n,k_z,\boldsymbol{q}}(\boldsymbol{r}) \phi^*_{n,k_z,\boldsymbol{q}}(\boldsymbol{r}) \times \frac{i\omega + \varepsilon_n(k_z)}{(i\omega)^2 - E_{n,p}(k_z,\boldsymbol{q})^2},$$

$$F^{\dagger}(\boldsymbol{r},i\omega) = \sum_{n,k_z,\boldsymbol{q}} \phi^{*}_{n,-k_z,-\boldsymbol{q}}(\boldsymbol{r})\phi^{*}_{n,k_z,\boldsymbol{q}}(\boldsymbol{r}) \times \frac{-\Delta^{*}_{nn}(\boldsymbol{q})}{(i\omega)^2 - E_{n,p}(k_z,\boldsymbol{q})^2}, \tag{4}$$

where

$$E_{n,p}(k_z,\boldsymbol{q}) = p\hbar\omega_c \pm \sqrt{\varepsilon_n^2(k_z) + |\Delta_{n+p,n-p}(\boldsymbol{q})|^2},$$

$$\varepsilon_n(k_z) = \frac{\hbar^2 k_z^2}{2m^*} + \hbar\omega_c\left(n + \frac{1}{2}\right) - \mu \tag{5}$$

is the quasiparticle excitation spectrum in the mixed state of the superconductor in high magnetic fields near the points k_{Fn}. This spectrum is calculated within the diagonal approximation, where only the electrons belonging to the same Landau levels are involved in the superconducting pairing.[10,11,14] Contributions to the pairing from Landau levels separated by $\hbar\omega_c$ or more can be included in the renormalization of the BCS coupling constant $[g \rightarrow \tilde{g}(H,T)]$.[12] The gap $\Delta_{nm}(\boldsymbol{q})$ in the MSR representation can be written as

$$\Delta_{nm}(\boldsymbol{q}) = \frac{\Delta(T,H)}{\sqrt{2}} \frac{(-1)^m}{2^{n+m}\sqrt{n!m!}} \sum_k \exp\left(i\pi\frac{b_x}{a}k^2\right) \times \exp\left[2ikq_y b_y - \left(q_x + \frac{\pi k}{a}\right)^2 l^2\right] \times H_{n+m}\left[\sqrt{2}\left(q_x + \frac{\pi k}{a}\right)l\right], \tag{6}$$

where $\Delta(T,H)$ is the BCS gap. The gap $\Delta_{nm}(\boldsymbol{q})$ goes to zero on the Fermi surface at the set of points in the MBZ with a linear dispersion in q and also has many near gapless regions for large LL index n. The quasiparticle excitations from $p \neq 0$ bands in Eq. (5) are gapped by at least $\hbar\omega_c$ and their contribution to the tunneling current is insignificant for small bias voltages such that $k_B T \leq eV < \Delta_{BCS}(T,H) \leq \hbar\omega_c$. Once the off-diagonal contribution is included in the electronic pairing, a closed expression for the quasiparticle excitation spectrum and Green's functions is not possible. However, when the effects of the off-diagonal terms are analyzed analytically with perturbation theory, the qualitative behavior of the quasiparticle excitations as characterized by nodes in the MBZ remains the same.[12] This will be the case as long as the perturbation expansion itself is well defined, i.e., as long as the magnetic field is larger than the critical value $H^*(T)$. At $T \sim 0$, the critical field can be estimated from dHvA experiments to be $H^* \sim (0.2-0.5)H_{c2}$ for borocarbide superconductors[7,8] and can be even lower if a strong anisotropy of the s-wave gap[25–29] is taken into account.

In STM experiments the tunneling current in Eq. (1) is typically characterized by a plot of the differential conductance $\sigma(\boldsymbol{r},V,T) = \partial I_{\parallel}/\partial V$ as a function of the bias voltage V at a particular position $\boldsymbol{r}$ of the microscope tip. In a perfectly clean type-II superconductor, where the tunneling is predominately along the vortex lines, the tunneling conductance can be derived from Eqs. (1), (2), and (4) as

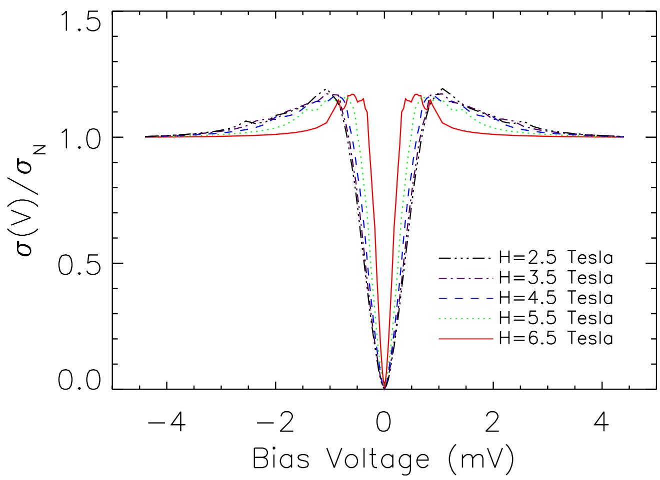

FIG. 1. (Color online) Spatially averaged differential conductance $\sigma(V)$ computed from Eq. (7) for a pure $LuNi_2B_2C$ superconductor as a function of magnetic field $0.35H_{c2} \leq H < H_{c2}$. We used experimentally determined values for $\Delta = 2.4$ meV and upper critical field $H_{c2} = 7$ T (Ref. 28), as well as the effective mass $m^* = 0.35m_e$ and Fermi velocity $v_F = 2.76 \times 10^7$ cm/s (Ref. 27). The differential conductance $\sigma(V)$ is rescaled by the normal state value σ_N in the corresponding field.

$$\frac{\sigma(\boldsymbol{r},V,T)}{\sigma_N} = -\frac{2\pi l^2}{N_{1F}} \sum_{n,k_z,\boldsymbol{q}} |\phi_{n,k_z,\boldsymbol{q}}(\boldsymbol{r})|^2 \times \{u^2_{n,k_z,\boldsymbol{q}} n'_F[\omega - E_{n,p=0}(k_z,\boldsymbol{q})] + v^2_{n,k_z,\boldsymbol{q}} n'_F[\omega + E_{n,p=0}(k_z,\boldsymbol{q})]\}, \tag{7}$$

where $n'_F = dn_F(E)/dE$ and the sum over n goes over all occupied Landau levels up to n_c. N_{1F} is the normal-state one-dimensional density of states at the Fermi surface of the electrons that are moving along the direction of the magnetic field. The differential conductance in Eq. (7) at some magnetic field strength H is rescaled by the normal state value σ_N evaluated at H. The Bogoliubov-deGennes amplitudes $u_{n,k_z,\boldsymbol{q}}$ and $v_{n,k_z,\boldsymbol{q}}$ are written in standard form as

$$u^2_{n,k_z,\boldsymbol{q}} = \frac{1}{2}\left[1 - \frac{\varepsilon_n(k_z)}{E_{n,p=0}(k_z,\boldsymbol{q})}\right],$$

$$v^2_{n,k_z,\boldsymbol{q}} = \frac{1}{2}\left[1 + \frac{\varepsilon_n(k_z)}{E_{n,p=0}(k_z,\boldsymbol{q})}\right]. \tag{8}$$

Figure 1 shows the differential conductance (averaged over the unit cell of the vortex lattice) as a function of magnetic field at $T \sim 0$ calculated from Eq. (7) for a pure $LuNi_2B_2C$ superconductor in the mixed state such that $H^* \leq H \leq H_{c2}$. In calculating the differential conductance we used the experimentally determined effective mass of $0.35m_e$ and Fermi velocity $v_F = 2.76 \times 10^7$ cm/s.[27] The values of other physical quantities needed in our calculation, the value of the BCS gap $\Delta = 2.4$ meV and upper critical field $H_{c2} = 7$ T in $\Delta(H) = \Delta\sqrt{1 - H/H_{c2}}$, are taken from experiments.[28] We assume that the vortex lattice has square symmetry as observed at high fields.[32] The differential conductance shows a V-shape structure with the minimum at zero bias. For small bias voltages it displays a linear voltage dependence and has a peak at the voltage approximately equal to the gap value $\Delta(H)$ at the corresponding magnetic field. We attribute this behavior to the creation of coherent, gapless and near gapless quasiparticle excitations at the Fermi surface in the mixed state of an extreme type-II superconductor at high fields such that $H^* \leq H \leq H_{c2}$. In high magnetic fields, vortices are very close to each other so that at low temperatures quasiparticles can propagate coherently over many unit cells resulting in a minimum of the differential conductance at zero bias voltage. The differential conductance in Fig. 1 has an increasing slope as the magnetic field increases, reflecting the larger abundance of gapless excitations in higher fields than in lower fields. Recent quasiclassical studies based on the microscopic Eilenberger theory in low fields, such that $H \leq H^*$, demonstrated that the V-shape density of states might be a property of the vortex state regardless of the underlying gap structure (isotropic, point node, or line node) or the strength of the magnetic field.[17]

Now we turn our interest to a *dirty* but *homogeneous* superconductor in the presence of nonmagnetic (scalar) impurities. In such systems the coherence length ξ is much longer than the effective distance ξ_{imp} over which the impurity potential changes, i.e., $\xi/\xi_{imp} \gg 1$, so that the superconducting order parameter is not affected by the impurities (apart from its overall magnitude) and still forms a perfect vortex lattice. In order to account for the disorder, the bare Green's functions for the clean superconductor in Eq. (4) are dressed via scattering through the normal $\Sigma^N(i\omega)$ and anomalous $\Sigma^A(\boldsymbol{q},i\omega)$ self-energies. Dressed Green's functions are obtained by replacing $i\omega$ with $i\tilde{\omega} \equiv i\omega - \Sigma^N(i\omega)$ and $\Delta_{nn}(\boldsymbol{q})$ with $\tilde{\Delta}_{nn}(\boldsymbol{q}) \equiv \Delta_{nn}(\boldsymbol{q}) + \Sigma^A(\boldsymbol{q},i\omega)$ in Eq. (4). We follow a T-matrix approximation that was originally developed for heavy fermion superconductors[33] and adapted by us in order to treat self-consistently impurity scattering in high magnetic fields.[18] We briefly outline this procedure in the current work as follows. Within the T-matrix approximation the normal and anomalous self-energies are related to the diagonal, and off-diagonal elements of the 2×2 scattering matrix $\hat{\mathrm{T}}(\boldsymbol{r},\boldsymbol{r}_{\prime},i\omega)$ that obeys the Lippmann-Schwinger equations

$$\hat{T}(\boldsymbol{r},\boldsymbol{r}_{\prime};i\omega) = U(\boldsymbol{r})\,\delta(\boldsymbol{r}-\boldsymbol{r}_{\prime})\hat{\sigma}_z + \int d\boldsymbol{r}_1 U(\boldsymbol{r}) \times \hat{\sigma}_z \hat{G}(\boldsymbol{r},\boldsymbol{r}_1;i\omega)\hat{T}(\boldsymbol{r}_1,\boldsymbol{r}_{\prime};i\omega), \tag{9}$$

where $\hat{G}$-matrix elements are given by Eq. (4) and $\hat{\sigma}_z$ is a Pauli matrix. $U(\boldsymbol{r}) = \Sigma_i U_0 \delta(\boldsymbol{r}-\boldsymbol{R}_i)$ is a short-range impurity potential where the scalar scattering amplitude U_0 is assumed to be isotropic. The locations of scattering centers $\boldsymbol{R}_i$ are randomly distributed everywhere in the system. In high magnetic fields we can assume that the scattering potential is weak compared to the separation between Landau levels (given by $\hbar\omega_c$) so that electrons scatter into states within the same Landau level. Therefore, in solving Eq. (9) we neglect inter-Landau-level scattering and define the superconducting self-energies as

$$\Sigma^N_{nn}(i\omega) = n_i \langle T^{11}_{nn}(k_z,\boldsymbol{q},\omega)\rangle_{\boldsymbol{R}_i},$$

$$\Sigma^A_{nn}(\boldsymbol{q};i\omega) = -n_i \langle T^{12}_{nn}(k_z,\boldsymbol{q},i\omega)\rangle_{\boldsymbol{R}_i}, \tag{10}$$

where $\langle \ldots \rangle_{\boldsymbol{R}_i}$ denotes the average over the impurity positions and n_i is the impurity concentration. $T^{ij}_{nn}(k_z,\boldsymbol{q},i\omega)$ are the coefficients in the T-matrix expansion over the complete set of MSR eigenstates in Eq. (3). Then the Lippmann-Schwinger Eqs. (9) are averaged over the impurity positions and the self-energies are expressed as

$$\Sigma^N(i\omega) = n_i \frac{\sum' G_m(k_z,\boldsymbol{k},i\omega)}{1/U_0^2 - \left[\sum' G_m(k_z,\boldsymbol{k},i\omega)\right]^2},$$

$$\Sigma^A_{nn}(\boldsymbol{q};i\omega) = n_i \frac{\sqrt{2} f^*_{nn}(\boldsymbol{q}) \sum' f_{mm}(\boldsymbol{k}) F^\dagger_m(k_z,\boldsymbol{k},i\omega)}{1/U_0^2 - \left[\sum' G^{11}_m(k_z,\boldsymbol{k},i\omega)\right]^2}, \tag{11}$$

where $\Sigma' \equiv (b_y/L_x L_y L_z \sqrt{\pi} l)\Sigma_{k_z,\boldsymbol{k},m}$ and we define $f_{nn}(\boldsymbol{k}) = \Delta_{nn}(\boldsymbol{k})/\Delta$. Substituting $\tilde{\omega}$ for ω and $\tilde{\Delta}_{nn}(\boldsymbol{q})$ for $\Delta_{nn}(\boldsymbol{q})$ in Eq. (4), and performing the analytical continuation to real frequencies ($i\omega \rightarrow \omega + i\delta$), the self-energies in Eq. (11) can be combined in a single nonlinear integral equation for the complex function $u = \tilde{\omega}/\tilde{\Delta}$

$$u = \frac{\omega}{\Delta} + \zeta \frac{\sum_n [m^*/4\pi^3 k_{Fn} N(0)] \int d\boldsymbol{q} [1-\sqrt{2}|f_{nn}(\boldsymbol{q})|^2] u/\sqrt{u^2 - |f_{nn}(\boldsymbol{q})|^2}}{c^2 - \left\{\sum_n [m^*/4\pi^3 k_{Fn} N(0)] \int d\boldsymbol{q} \cdot u/\sqrt{u^2 - |f_{nn}(\boldsymbol{q})|^2}\right\}^2}, \tag{12}$$

where $\zeta = \Gamma/\Delta(H)$ and $N(0)$ is the normal state density of states at the Fermi level. In the T-matrix approximation disorder is characterized by the parameter $\Gamma = n_i/N(0)\pi$, which measures the concentration of impurities relative to the electron density, and the parameter $c = 1/[\pi N(0) U_0]$, which is a measure of scattering strength. The weak-scattering limit in a high magnetic field is approached when $c \gtrsim 1$ since then c^2 is much larger than the second term in the denominator of expression (12). On the other hand, in the strong scattering limit $c \to 0$. Equation (12) is an implicit equation for the complex function $u = f(\omega/\Delta)$ that has to be calculated numerically for different values of disorder parameters $g = \Gamma/\Delta$ and c. The values of all other quantities characterizing the microscopic properties of a superconducting system (the BCS gap Δ, upper critical field H_{c2}, and Fermi energy E_F) have to be taken from experiments. Once the complex function u is known as a function of ω/Δ, and replacement of ω with $\tilde{\omega}$ as well as analytical continuation have been performed in the spectral functions in Eq. (2), the differential conductance in the presence of the disorder can be calculated from Eqs. (1) and (2). In the limit when temperature $T \to 0$ the STM differential conductance $\sigma(\mathbf{r}, V)$ in the disordered superconductor in high magnetic field is given by

$$\frac{\sigma(\boldsymbol{r},V)}{\sigma_N} = \sum_{n=0}^{n_c} \frac{m^*}{4\pi^3 k_{Fn} N(0)} \int d\boldsymbol{q} |\phi_{n,\boldsymbol{q}}(\boldsymbol{r})|^2 \times \Im\mathrm{m} \frac{u}{\sqrt{u^2 - |f_{nn}(\boldsymbol{q})|^2}}, \quad (13)$$

where $u = f[V/\Delta(H)]$ and V is the bias voltage. In deriving Eqs. (12) and (13) we assume that the impurity scattering does not change the $\boldsymbol{q}$ dependence of the quasiparticle excitation spectrum, i.e., $\tilde{\Delta}_{nn}(\boldsymbol{q}) = \tilde{\Delta} f_{nn}(\boldsymbol{q})$ is given by Eq. (6) and disorder only renormalizes the BCS gap $\tilde{\Delta}$. This assumption is valid around the gapless points at the Fermi surface and is less accurate for the quasiparticle excitations gapped by large Δ.[18] However, in the low-temperature limit ($k_B T \ll \Delta$) the behavior of the tunneling current for small bias voltage will be governed by excitations around the gapless and near gapless regions on the Fermi surface while the contribution from the gapped regions will be exponentially small. In the limit of $k_B T \gtrsim \Delta$ a fully self-consistent $\tilde{\Delta}$ in the presence of disorder has to be determined. The most challenging part of our computational work is finding the numerical solution of the system of Eqs. (12) for real superconducting systems in a high magnetic field. For the typical range of magnetic fields used in various experiments in the mixed state, the number of Landau levels involved in the superconducting pairing $n_c = E_F/\hbar\omega_c$ can be quite large. In the borocarbide superconductor $LuNi_2B_2C$ this number can be estimated to be $n_c \sim 33$ at a field of $H_{c2} = 7$ T and $n_c \sim 100$ at a field of $H = 2.5$ T. The disorder parameters of our theory, g and c, can be connected to the normal state inverse scattering rate Γ_0 by letting $f_{nn}(\boldsymbol{q}) \to 0$ and $\omega \to 0$ in Eq. (12). This procedure yields $\Gamma_0 = \Gamma/(1+c^2)$, a result consistent with a study of the transport properties of the normal metal in a high magnetic field.[34]

In Fig. 2 we plot the differential conductance $\sigma(V)$ computed from Eq. (13) and averaged over the unit cell of the vortex lattice in the weak scattering limit ($c = 1$) as a function of bias voltage V for the superconductor $LuNi_2B_2C$ placed in a magnetic field H such that $0.35H_{c2} \lesssim H \lesssim H_{c2}$. In calculating the differential conductance we used the experimentally determined effective mass of $0.35m_e$ and Fermi velocity $v_F = 2.76 \times 10^7$ cm/s,[27] as well as BCS gap $\Delta = 2.4$ meV and

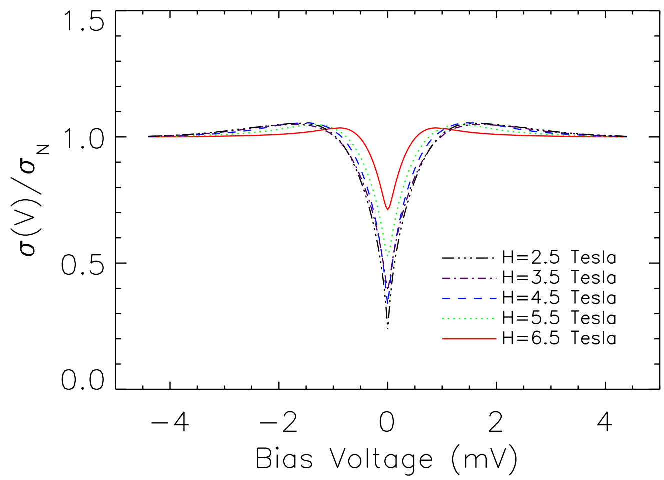

FIG. 2. (Color online) Spatially averaged differential conductance $\sigma(V)$ computed from Eq. (13) in the weak scattering limit ($c = 1$) for the superconductor $LuNi_2B_2C$ as a function of magnetic field $0.35H_{c2} \leq H < H_{c2}$. Impurity concentration parameter $g = \Gamma/\Delta$ of 0.2 yields the normal state inverse scattering rate $\Gamma_0 = 0.24$ meV observed in experiments (Refs. 26 and 35).

upper critical field $H_{c2} = 7$ T.[28] The impurity concentration parameter $g = \Gamma/\Delta$ is chosen to be 0.2, yielding the normal state inverse scattering rate of $\Gamma_0 = 0.24$ meV observed in a recent dHvA experiment[35] and a point-contact Andreev reflection spectroscopy experiment.[26] This particular choice of disorder parameters c and g corresponds to a fairly clean system typically used in a tunneling experiment. Close inspection of Fig. 2 leads us to the conclusion that the effect of scalar disorder in the mixed state is to introduce a finite conductance at zero bias. This feature in $\sigma(V)$ means that a finite density of states in the vortex state is created at the Fermi level in the presence of nonmagnetic impurities in a high magnetic field at low temperature. This behavior contrasts with a perfectly pure superconductor where the density of states exhibits a linear dependence at low energies as seen in Fig. 1. The finite differential conductance at zero bias signals the broadening of gapless points in the excitation spectrum in Eq. (5) into gapless regions in the MBZ. Furthermore, it can been seen in Fig. 2 that the differential conductance $\sigma(V)$ diminishes as the magnetic field is lowered from $H \lesssim H_{c2}$ to $H \sim 0.35H_{c2}$ and displays increasingly nonlinear behavior for small bias voltages. This is a consequence of the depletion of gapless and near gapless quasiparticle excitations at the Fermi surface at lower magnetic fields. On the other hand, the conductance peak is suppressed and broadened by the disorder in comparison to the conductance peak in the clean superconductor shown in Fig. 1.

In Fig. 3 we plot the differential conductance $\sigma(V)$ computed from Eq. (13) and averaged over a unit cell in the weak scattering limit ($c = 1$) for the superconductor $LuNi_2B_2C$ in a magnetic field $H = 5.5$ T as a function of impurity concentration parameter $g = \Gamma/\Delta$. As seen in Fig. 3, an increasing concentration of impurities (a) creates increasing conductance at zero bias voltage and (b) washes away the conductance peak that is pronounced in Fig. 1. At the same time, the dependence of the conductance $\sigma(V)$ on bias voltage V changes from linear to polynomial as the impurity concentration increases.

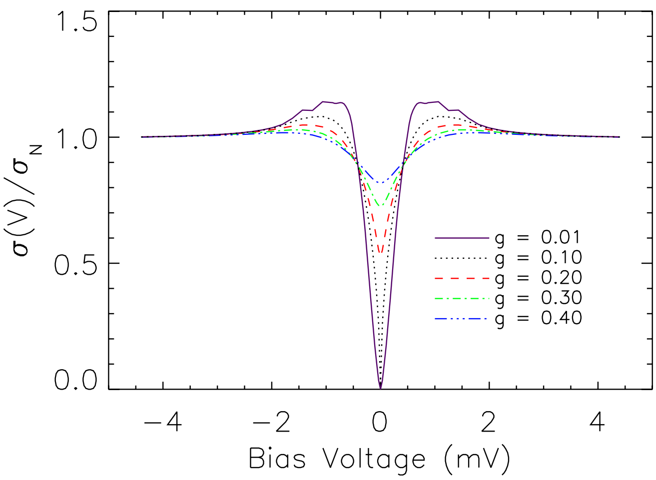

FIG. 3. (Color online) Spatially averaged differential conductance $\sigma(V)$ computed from Eq. (13) in a weak scattering limit ($c=1$) as a function of impurity concentration parameter $g=\Gamma/\Delta$ for the superconductor $LuNi_2B_2C$ in a magnetic fields of $H=5.5$ T.

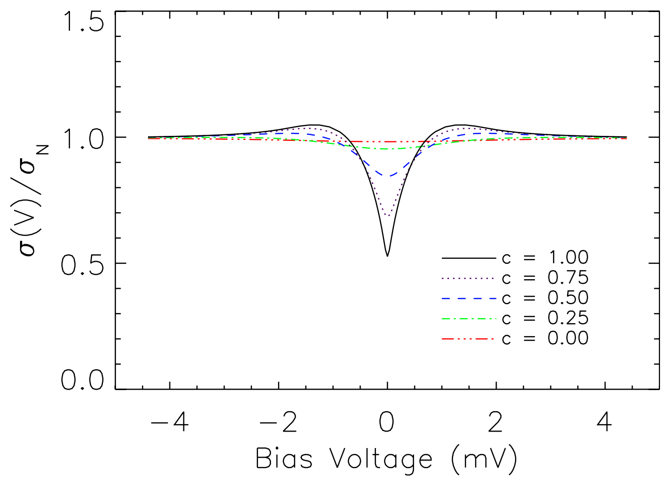

FIG. 4. (Color online) Spatially averaged differential conductance $\sigma(V)$ computed from Eq. (13) for the superconductor $LuNi_2B_2C$ in a magnetic fields of $H=5.5$ T vs the scattering strength parameter $c\sim 1/U_0$. The impurity concentration parameter is fixed at $g=\Gamma/\Delta=0.2$.

In Fig. 4 we plot the differential conductance $\sigma(V)$ computed from Eq. (13) and averaged over a unit cell for the superconductor $LuNi_2B_2C$ in a magnetic field $H=5.5$ Tesla for a fixed value of the impurity concentration parameter $g=0.2$ but as a function of the scattering strength parameter c. As one moves away from the weak scattering limit $c=1$ to the strong scattering limit as $c\rightarrow 0$, all the prominent tunneling features of the superconductor in high magnetic field seen in Figs. 1 (clean system), 2 and 3, mainly the V-shape density of states and conductance peak, are smeared. In the strong scattering limit $c=0$, the conductance approaches its normal state value. This is an indication that the gap has been suppressed everywhere on the Fermi surface. However, we have to point out that the approximation of neglecting inter-Landau-level electron scattering by impurities, made in deriving the superconducting self-energies in Eqs. (10) and (11), is increasingly unreliable as $c\rightarrow 0$ since the strength of the scattering potential U_0 becomes larger than the LL separation given by $\hbar\omega_c$. Therefore, in the strong scattering limit when $c\rightarrow 0$ our theory is stretched to its limit and quantitative accuracy diminishes.

In summary, we have developed expressions for the zero-temperature differential conductance $\sigma(V)$ between an STM tip and an extreme type-II superconductor placed in a high magnetic field both for a clean system as well as in the presence of nonmagnetic impurities. Using experimentally determined parameters, we calculate the spatially averaged $\sigma(V)$ as a function of magnetic fields and various disorder parameters for the borocarbide superconductor $LuNi_2B_2C$. In the range of magnetic fields where our theory is applicable, $H^* \leq H \lesssim H_{c2}$, the calculated $\sigma(V)$ has a characteristic V shape as a function of small bias voltage, a feature that becomes less pronounced as disorder increases. We attribute this behavior to the presence of gapless points and near gapless regions at the Fermi surface in the quasiparticle excitation spectrum of the extreme type-II superconductor in high magnetic field. Our calculations, performed in the high field limit of the Landau-level pairing scheme, are consistent with a recent quasiclassical study that starts from the low field regime[17] and finds a V-shape density of states in the vortex state regardless of the underlying gap structure (isotropic, point node, or line node) or the strength of the magnetic field. Further experimental tunneling studies of extreme type-II superconductors are needed to help disentangle their mixed-state behavior.

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