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Magnetocrystalline anisotropy of the GaAs-type semiconductors in a strong magnetic field

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Magnetocrystalline anisotropy of a GaAs-type semiconductor is theoretically investigated for the case of a strong, ultraquantum, magnetic field, zero temperature, and a high level of donor or acceptor doping. At these conditions, the crystal magnetization is expected to be mainly determined by conduction electrons or holes. So the magnetization value and the value of its anisotropy are determined by the (anisotropic) energy of the ground carrier Landau level. Analytical formulas for the magnetization revealing the crystal symmetry are obtained.

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The magnetic anisotropy effect is the deviation and dependance of the sample magnetization from/on the direction of the magnetic field $\mathcal{H}$. There are several mechanisms of this effect: the crystal lattice, the sample shape, and the stress mechanisms.[1] The magnetic anisotropy due to the lattice structure is called the magnetocrystalline anisotropy and will be considered in this study. The magnetocrystalline anisotropy can be bond with the anisotropic shape of molecules of the crystal (for dielectrics)[2] or with the spin-orbit interaction of the microscopic magnetic moments with the lattice potential (for metals).[3] The effect has been experimentally investigated for many solids: nonmetallic diamagnets and paramagnets,[2] Van Vleck paramagnets,[4] ferromagnets and antiferromagnets,[1,3,5] normal metals,[6] ferromagnetic thin films,[3,7] and, recently, for magnetic semiconductor alloys $\text{Ga}_{1-x}\text{Mn}_x\text{As}$.[8,9]

In this paper we will theoretically investigate the magnetocrystalline anisotropy of a donor or acceptor doped GaAs-type semiconductor. The enough high level of doping when carriers may be considered as an ideal Fermi gas is supposed.[10] For example, for n-InSb the typical doping level is $10^{14}-10^{17}$ cm^{-3} for this case to be held.[11] For simplicity, we will consider the ultraquantum limit and low temperatures, when only the bottom of the lowest spin sublevel of the ground Landau level is occupied. In this case the sample magnetization is expected to be mainly determined by conduction electrons or holes[12] and, consequently, the magnetocrystalline anisotropy is ruled by the band anisotropy (not only by spin-orbit terms). Analytical formulas for anisotropic corrections to the magnetization are derived.

First, let us consider the donor doped semiconductor. In the case of ultraquantum magnetic field and enough low temperature an inequality $k_{\mathcal{H}} l_\text{m} \ll 1$ is held for the equilibrium electrons, where $k_{\mathcal{H}}$ is the wave vector in the $\mathcal{H}$ direction and $l_\text{m} = \sqrt{c\hbar/e\mathcal{H}}$ is the magnetic length.[13] The electron Landau levels in a GaAs-type semiconductor do not have any anisotropy in the first, K^2, (linear by $\mathcal{H}$) approximation

$$E^{\text{K2}}_{n,\pm} = \hbar\omega_\text{c}\left(n+\frac{1}{2}\right) \pm \frac{g\mu_\text{B}\mathcal{H}}{2}, \qquad (1)$$

where $\omega_\text{c} = e\mathcal{H}/m^* c$ is the cyclotron frequency. Due to $k_{\mathcal{H}} l_\text{m} \ll 1$, the term $\hbar^2 k_{\mathcal{H}}^2/2m^*$ being the kinetic energy of the motion along the magnetic field direction is omitted in $E^{\text{K2}}_{n,\pm}$. The inequality $k_{\mathcal{H}} l_\text{m} \ll 1$ leads to a relation $\hat{K} \sim 1/l_\text{m}$. Thus, the perturbation theory first-order corrections from K^4 terms in the electron effective mass Hamiltonian to Landau levels' energies are quadratic by $\mathcal{H}$.

In an uniform magnetic field the effective mass Hamiltonian in the fourth order by the kinematic moment $\hat{\mathbf{K}} = -i(\partial/\partial\mathbf{r}) + (e/\hbar c)\mathbf{A}$ has the form[14,15]

$$\hat{H} = \frac{\hbar^2\hat{K}^2}{2m^*} + \frac{1}{2} g\mu_\text{B}(\mathcal{H}\cdot\hat{\boldsymbol{\sigma}}) + \hat{H}_{4,\text{is}} + \hat{H}_\beta + \hat{H}_\tau + \hat{H}_\text{SO}. \qquad (2)$$

Here $\hbar^2\hat{K}^2/2m^*$ is the main (K^2) term, $g\ \mu_\text{B}(\mathcal{H}\cdot\boldsymbol{\sigma})/2$ is the Zeeman term, and $\hat{H}_{4,\text{is}}$ are the nonparabolic K^4, but isotropic terms. The other terms in Eq. (2) are anisotropic. The term $\hat{H}_\text{SO}$ is the Dresselhaus spin-orbit K^3 term which in presence of an external magnetic field looks as follows $\hat{H}_\text{SO} = \gamma[\hat{\boldsymbol{\kappa}}(\hat{\mathbf{K}})\cdot\hat{\boldsymbol{\sigma}}]$, where γ is the Dresselhaus constant, $\hat{\kappa}_i(\hat{\mathbf{K}}) = \hat{K}_{i+1}\hat{K}_i\hat{K}_{i+1} - \hat{K}_{i+2}\hat{K}_i\hat{K}_{i+2}$ (see Ref. 16 on the problem of choosing the order of the noncommuting operators $\hat{K}_j$ in $\hat{\kappa}_i$). The coordinate axes are assumed to be parallel to the $\langle 100\rangle$ crystallographic axes. The term $\hat{H}_\tau$ is a K^4 anisotropic spin-dependent term: $\hat{H}_\tau = \eta[\hat{\boldsymbol{\tau}}(\hat{\mathbf{K}})\cdot\hat{\boldsymbol{\sigma}}]$, where $\hat{\tau}_i(\hat{\mathbf{K}}) = \mathcal{H}_i\hat{K}_i^2$. It leads together with the $\hat{H}_\text{SO}$ term to an electron g-factor anisotropy in zinc-blende lattice semiconductors. The term $\hat{H}_\beta$ is a K^4 anisotropic but spinless term: $\hat{H}_\beta = \beta(\hat{K}_x^4 + \hat{K}_y^4 + \hat{K}_z^4)$. It can be called the effective mass anisotropy term.

The perturbation theory first-order corrections from $\hat{H}_\text{SO}$ are linear by $k_{\mathcal{H}}$. In the ultraquantum fields they are far smaller than the perturbation theory second-order corrections from $\hat{H}_\text{SO}$ which do not vanish at $k_{\mathcal{H}} \to 0$.[13] Thus, for the typical energy of the $\hat{H}_\text{SO}$ term corrections we have: $\nu(\mathcal{H}) = (\gamma/l_\text{m}^3)^2/\hbar\omega_\text{c} = e^2 m^* \gamma^2 \mathcal{H}^2/c^2\hbar^4$. This value is quadratic by $\mathcal{H}$. We conclude that the $\hat{H}_\beta$ and $\hat{H}_\tau$ terms in the perturbation theory first order and the $\hat{H}_\text{SO}$ term in the perturbation theory second order should be taken into account to describe the anisotropy of the electron spectrum.[17]

In the case when the Zeeman splitting is far greater than the typical values of the $\hat{H}_\tau$ and $\hat{H}_\text{SO}$ terms, the spin-orbit corrections of the wave function will be small. So the electron spin states can be classified on the base of the property of the spin in the state under discussion to be approximately codirected or approximately contradirected with the $\mathcal{H}$ direction ("+" and "−" states, correspondingly). Keeping in mind the examples of GaAs, GaSb, and InSb we will consider $g<0$. So the electron spin is codirected with the $\mathcal{H}$ direction at the ground state, i. e. the ground state is the 0^{+} state.

A calculation of the contribution from the $\hat{H}_{\eta}$ term in the Landau levels' energies was performed in Ref. 14

$$(\delta E_{\eta})_{n,\pm} = \pm\left(n+\frac{1}{2}\right)\frac{e\eta}{c\hbar}\mathcal{H}^{2}[1-P_{4}(\mathbf{h})], \tag{3}$$

where $\mathbf{h}=\mathcal{H}/\mathcal{H}$, $P_4(\mathbf{h})=h_z^4+h_y^4+h_z^4$ is the O_h (and T_d) group anisotropic invariant of the fourth order by $\mathbf{h}$. The $\hat{H}_{\beta}$ energy corrections are equal for both spin sublevels. They were calculated in Ref. 18. But the answer only for the difference $[(\delta E_{\beta})_1-(\delta E_{\beta})_0]$ was presented there. Here we have calculated the correction $(\delta E_{\beta})_n$ for $n=0$

$$(\delta E_{\beta})_{0} = \frac{3e^{2}\beta}{4c^{2}\hbar^{2}}\mathcal{H}^{2}[1+P_{4}(\mathbf{h})]. \tag{4}$$

A technique used for this calculation is analogous to the technique of the $\hat{H}_{\mathrm{SO}}$ contribution calculation described in details in Ref. 13. According to the results of Ref. 17, the $\hat{H}_{\mathrm{SO}}$ energy corrections must have the form

$$(\delta E_{\gamma})_{n,\pm} = \nu(\mathcal{H})[r_{n,\pm}+s_{n,\pm}P_{4}(\mathbf{h})+t_{n,\pm}P_{6}(\mathbf{h})], \tag{5}$$

where $P_6(\mathbf{h})=h_x^2h_y^2h_z^2$ is the O_h (and T_d) group anisotropic invariant of the sixth order by $\mathbf{h}$. Calculations analogous to one made in Ref. 17 for $[(\delta E_{\gamma})_{n,+}-(\delta E_{\gamma})_{n,-}]$ give: $r_{0,+}=-3/16$, $s_{0,+}=-13/16$, and $t_{0,+}=-23(5/8)$. Summarizing Eqs. (3)–(5), we obtain for the ground-state energy

$$E_{0,+}^{\mathrm{K4}} = E_{0,+}^{\mathrm{K2}} + \delta E_{0,+}^{\mathrm{is}} + \delta E_{0,+}^{\mathrm{anis}},$$

where $E_{0,+}^{\mathrm{K2}}=\lambda\mathcal{H}$ is the main K^2 term, $\lambda=(e\hbar/2m^{*}c)+(ge\hbar/4m_0c)$ is the conduction electron magnetic moment absolute value, $\delta E_{0,+}^{\mathrm{is}}=\lambda'\mathcal{H}^2$ is the isotropic $K^{3,4}$ correction to $E_{0,+}^{\mathrm{K2}}$, and the anisotropic $K^{3,4}$ correction $\delta E_{0,+}^{\mathrm{anis}}$ has the form

$$\delta E_{0,+}^{\mathrm{anis}} = A\frac{\mathcal{H}_x^4+\mathcal{H}_x^4+\mathcal{H}_z^4}{\mathcal{H}^2} + B\frac{\mathcal{H}_x^2\mathcal{H}_y^2\mathcal{H}_z^2}{\mathcal{H}^4},$$

$$A = \frac{1}{2}\frac{e\eta}{c\hbar}+\frac{3}{4}\frac{\beta e^{2}}{c^{2}\hbar^{2}}-\frac{13}{16}\frac{e^{2}m^{*}\gamma^{2}}{c^{2}\hbar^{4}}, \quad B = -23\frac{5}{8}\frac{e^{2}m^{*}\gamma^{2}}{c^{2}\hbar^{4}}. \tag{6}$$

$\delta E_{0,+}^{\mathrm{is}}$ contains isotropic contributions both from the $H_{4,\mathrm{is}}$ term in Eq. (2) and from the $\hat{H}_{\eta}$, $\hat{H}_{\beta}$, and $\hat{H}_{\mathrm{SO}}$ terms within the basis $\{1,P_4(\mathbf{h}),P_6(\mathbf{h})\}$. As the typical values of nonparabolicity terms in Eq. (2): $\nu(\mathcal{H})$, β/l_{m}^4, and $\eta\mathcal{H}/l_{\mathrm{m}}^2$, are far smaller than the character values of the quadratic terms: $\hbar^2/m^{*}l_{\mathrm{m}}^2$ and $|g|\mu_{\mathrm{B}}\mathcal{H}$ (this is the criterion of using the effective mass approximation for the ultraquantum case when $\hat{K}\sim 1/l_{\mathrm{m}}$), the inequalities $\delta E_{0,+}^{\mathrm{is,anis}}\ll E_{0,+}^{\mathrm{K2}}$ are carried out.

In the ultraquantum limit, $E_{\mathrm{F}}\ll\hbar\omega_{\mathrm{c}}$, and at low temperatures, $k_{\mathrm{B}}T\ll|g|\mu_{\mathrm{B}}\mathcal{H}$, $\hbar\omega_{\mathrm{c}}$, only the lowest spin sublevel of the $n=0$ Landau level is occupied and the free energy of the electron gas has the form [Eq. (19.5) in Ref. 12]

$$F = n_{\mathrm{e}}E_{0,+}(\mathcal{H}) + \delta F(\mathcal{H},T,V,N_{\mathrm{e}}). \tag{7}$$

Here $n_{\mathrm{e}}=N_{\mathrm{e}}/V$ is the conduction electrons' concentration. The first term in Eq. (7) is the sum of the ground energies of all electrons. The δF term is a correction to the first term due to the electron one-dimensional (1D) dispersion, corresponding to the electron motion in the $\mathcal{H}$ direction with the wave vector $k_{\mathcal{H}}$. The value of δF substantially depends on the temperature. Using Eq. (19.5) in Ref. 12, we have obtained that in a K^2 approximation

$$\delta F = n_{\mathrm{e}}C(T)E_{\mathcal{H}}$$

if $k_{\mathrm{B}}T\lesssim E_{\mathcal{H}}$, where $E_{\mathcal{H}}=E_{\mathrm{F}}(E_{\mathrm{F}}/\hbar\omega_{\mathrm{c}})^2$, $C(T)$ is a dimensionless constant about unity, and

$$\delta F \approx -\frac{n_{\mathrm{e}}k_{\mathrm{B}}T}{2}\ln\left(\frac{k_{\mathrm{B}}T}{E_{\mathcal{H}}}\right)$$

if $k_{\mathrm{B}}T\gg E_{\mathcal{H}}$. Due to the inequalities $E_{0,+}\gg E_{\mathcal{H}}$ and $k_BT\ll\hbar\omega_{\mathrm{c}}$, the term δF is small relative to the first term in Eq. (7). Note that this is also true for the $n_{\mathrm{e}}E_{0,+}$ and δF terms' derivatives by $\mathcal{H}$.

The magnetization should be calculated as follows: $\mathbf{M}=-\partial F/\partial\mathcal{H}$. The first term $n_{\mathrm{e}}E_{0,+}$ in Eq. (7) is mainly linear by $\mathcal{H}$ when the main K^2 terms $K^2/2m^{*}$ and $\mu_{\mathrm{B}}(\hat{\sigma}\cdot\mathcal{H})/2$ prevails in the electron Hamiltonian. Therefore the saturation of the sample magnetization versus $\mathcal{H}$ must be approximately achieved: $\mathbf{M}(\mathcal{H})\to\mathbf{M}_0=-n_{\mathrm{e}}\lambda\mathbf{h}$. For example, this effect was observed in Ref. 19 for metallic compound LaRhIn$_5$ with small Fermi pockets.

The anisotropic corrections to δF are caused by the anisotropy of the 1D density of states function [Eq. (12.3) in Ref. 12] and, thus, by the anisotropy of the $n=0$ Landau level 1D dispersion $E_{0,+}(k_{\mathcal{H}},\mathcal{H})$. This dispersion is calculated on the base of Hamiltonian (2) [as the ground state energy $E_{0,+}^{\mathrm{K4}}(\mathcal{H})$]. Owing to the time inversion symmetry, it does not contain the odd degree terms by $k_{\mathcal{H}}$ and has the form

$$E_{0,+}(k_{\mathcal{H}},\mathcal{H}) = E_{0,+}^{\mathrm{K4}}(\mathcal{H}) + + \frac{\hbar^{2}k_{\mathcal{H}}^{2}}{2m^{*}} + a(\mathcal{H})k_{\mathcal{H}}^{2} + b(\mathcal{H})k_{\mathcal{H}}^{4}, \tag{8}$$

where $a(\mathcal{H})$ and $b(\mathcal{H})$ are due to the K^4 terms in Eq. (2). It is seen that $a(\mathcal{H})$ is proportional to l_{m}^{-2} multiplied by a K^4 constants, for example, β. Thus, comparing Eqs. (1), (6), and (8), we conclude that the anisotropic part of the dispersion $E_{0,+}(k_{\mathcal{H}},\mathcal{H})$ and, consequently, the anisotropic part of δF are of the same relative order as the anisotropic corrections to $E_{0,+}(\mathcal{H})$. Using this fact and the inequality $\delta F\ll n_{\mathrm{e}}E_{0,+}$, it was proved that the anisotropies of F and $\mathbf{M}$ are mainly caused by the anisotropy of $E_{0,+}$.

Substituting Eq. (6) into the definition of $\mathbf{M}$, we obtain for the magnetocrystalline anisotropy: $\mathbf{M}=\mathbf{M}_0+\delta\mathbf{M}$, where $\delta\mathbf{M}=-2n_{\mathrm{e}}\lambda'\mathcal{H}\mathbf{h}+\delta\mathbf{M}_{\mathrm{anis}}$, $\delta\mathbf{M}_{\mathrm{anis}}=\delta\mathbf{M}_{\beta}+\delta\mathbf{M}_{\eta}+\delta\mathbf{M}_{\gamma}$ is the anisotropic part of the magnetization, $\delta\mathbf{M}_p=-[\partial(\delta E_p)_{0,+}/\partial\mathcal{H}]_{\mathrm{anis}}$, $p=\beta,\eta,\gamma$

TABLE I. The conduction band parameters and the values of the different contributions to the magnetocrystalline anisotropy for n-doped GaAs, GaSb, and InSb ($\mathcal{H}=10$ T).

	GaAs	GaSb	InSb
m/m_0	0.067	0.041	0.014
g	−0.44	−8	−49.5
γ, eV Å^3	24	187	220
η, eV cm^2 Oe^{-1}	6.5×10^{-23}		7.23×10^{-21}
β, eV Å^4	670	2900	2.19×10^{4}
$\delta M_\eta/M_0$	7.75×10^{-4}		0.0274
$\delta M_\beta/M_0$	1.83×10^{-3}	5.74×10^{-3}	0.0190
$(\delta M_\gamma)_{h4}/M_0$	1.50×10^{-5}	4.05×10^{-4}	8.40×10^{-5}
$(\delta M_\gamma)_{h6}/M_0$	2.40×10^{-5}	6.45×10^{-4}	1.34×10^{-4}

$$\delta\mathbf{M}_{\mathrm{anis}} = 2n_{\mathrm{e}}A\mathcal{H}[-2\mathbf{v}_3(\mathbf{h}) + P_4(\mathbf{h})\mathbf{h}] + 2n_{\mathrm{e}}B\mathcal{H}[-h_xh_yh_z\mathbf{v}_2(\mathbf{h}) + 2P_6(\mathbf{h})\mathbf{h}], \tag{9}$$

$\mathbf{v}_3(\mathbf{h})=(h_x^3,h_y^3,h_z^3)$, and $\mathbf{v}_2(\mathbf{h})=(h_yh_z,h_xh_z,h_xh_y)$. It may be useful to extract the components $\delta\mathbf{M}_\parallel$ and $\delta\mathbf{M}_\perp$ from $\delta\mathbf{M}_{\mathrm{anis}}$ that are parallel and perpendicular to the $\mathcal{H}$ direction.

Estimations of the value of the effect proposed are presented in Table I. In the last four rows of Table I the maximum variations of the anisotropic part of each mechanism contribution to $\delta\mathbf{M}_\parallel$ relative to M_0 are presented.

Let us make some comments how the "exotic" band parameters used in Table I were taken from the experimental data. The value of β for GaAs and InSb was extracted from the cyclotron resonance measurements (the paper[20] for GaAs and the paper[21] for InSb). In these papers the data on the Landau levels anisotropic shifts, different for different levels, were expressed in terms of the effective mass anisotropy. We have reinterpreted this Landau level shifts on base of Eq. (4) and analogous equation for $n=1$ to find the β parameter. The value of η for GaAs was extracted from the experimental data of Refs. 18, 20, and 22 on the anisotropy of the cyclotron resonance line $0^\pm\rightarrow1^\pm$ spin splitting. A detailed analysis of extracting of η from the data of Refs. 18, 20, and 22 was done in Ref. 17. Briefly speaking, it was based on Eqs. (3) and (5). The value of η for InSb was taken from the spin-resonance measurements,[23] where the anisotropy of the ground Landau level spin splitting was measured and presented in the form of the g factor anisotropy. We have reinterpreted this $n=0$ level splitting anisotropy on base Eq. (3) to extract η. The values of γ for GaAs, GaSb, and InSb was taken from the spin relaxation data presented in the review.[24] The value of β for GaSb was taken from the Shubnikov-de Gaas effect measurements discussed in Ref. 25. In that work the experimental data on the anisotropy of the frequencies of the magnetoresistance oscillations were analyzed in terms of the effective mass anisotropy. We have reinterpreted the effective mass anisotropy obtained in Ref. 25 on the base of the formula for the electron dispersion: $E(\mathbf{k})=\hbar^2k^2/2m^*+\hat{H}_\beta(\mathbf{k})$, to find the β value.

We would like to emphasize that the signs of the parameters η and β in Table I are indeed correct. The positive sign of β follows from the cyclotron resonance data[20,21] or from the Shubnikov-de Gaas measurements data[25] treated in terms of the effective mass anisotropy and is the same as the effective mass sign. As follows from the data in Ref. 18, the sign of η in GaAs is the same as the sign of g_1 in the term $g_1\hat{K}^2(\mathcal{H}\cdot\hat{\boldsymbol{\sigma}})$. The positiveness of η and g_1 has been verified by us on the basis of the 14-band model formulas in Ref. 15 and is in agreement with the experiential data discussed in Ref. 17. As follows from the analysis of the spin-resonance experiment,[23] the sign of η in InSb is opposite to the g-factor sign. Thus, η is also positive in InSb. Note that the sign of γ is not important for the obtained answer in Eq. (9).

A magnetocrystalline anisotropy of an acceptor doped semiconductor can be studied in a similar manner. For holes, there are anisotropic terms in the K^2 effective mass Hamiltonian already. The K^2 hole Hamiltonian may be expressed in the form[26,27]

$$\hat{H} = \hat{H}_{\mathrm{is}} + \hat{H}_{\mathrm{anis}},$$

$$\hat{H}_{\mathrm{is}} = -[\tilde{\gamma}_1 + (5/2)\tilde{\gamma}_2]\hat{K}^2 + 2\tilde{\gamma}_3\sum_i \hat{K}_i^2\hat{J}_i^2 + 4\tilde{\gamma}_3\sum_{(i,j)}\{\hat{K}_i,\hat{K}_j\} \times\{\hat{J}_i,\hat{J}_j\} + 2\mu_{\mathrm{B}}\mathcal{K}(\hat{\boldsymbol{J}}\cdot\mathcal{H}),$$

$$\hat{H}_{\mathrm{anis}} = 2(\tilde{\gamma}_2-\tilde{\gamma}_3)\sum_i \hat{K}_i^2\hat{J}_i^2 + 2\mu_{\mathrm{B}}q\sum_i \mathcal{H}_i\hat{J}_i^3, \tag{10}$$

where $\{\hat{A},\hat{B}\}=(\hat{A}\hat{B}+\hat{B}\hat{A})/2$, $i=x,y,z$, $(i,j)=(x,y),(x,z),(y,z)$, $\hat{J}_i$ are the 3/2 angular momentum operator matrixes, and $\tilde{\gamma}_i$, $\mathcal{K}$, and q are the K^2 valence band parameters [$\tilde{\gamma}_i=(\hbar^2/2m_0)\gamma_i$, where γ_i are the dimensionless Luttinger parameters]. As it is true that $|\gamma_2-\gamma_3|\ll\gamma_3$ and $q\ll\mathcal{K}$ for many semiconductors, the anisotropic Hamiltonian part $\hat{H}_{\mathrm{anis}}$ can be treated as a perturbation to the isotropic one $\hat{H}_{\mathrm{is}}$. As for electrons, in the enough high magnetic field only the bottom of the ground Landau level subband is occupied. Thus, it can be proved that, as for electrons, we can put $k_{\mathcal{H}}=0$ for holes in that subband and $F=n_{\mathrm{h}}E_{\mathrm{g}}(\mathcal{H})$, where n_{h} is the hole concentration and E_{g} is the energy of the ground hole Landau level.

The ground Landau level within the Hamiltonian $\hat{H}_{\mathrm{is}}$ is[26]

$$E_{\mathrm{g}}^{\mathrm{is}} = \frac{-\tilde{\gamma}_1+\tilde{\gamma}_3}{l_{\mathrm{m}}^2} + \frac{\mu_{\mathrm{B}}\mathcal{K}\mathcal{H}}{2}$$

(if $\mathcal{K}>0$) and a wave function

$$\psi_{\mathrm{g}} = \pi^{-1/4}\exp(-\xi^2/2)(0,1,0,0)^{\mathrm{T}}$$

corresponds to it. Here ξ is the oscillator coordinate and the wave function is written in the standard basis $\psi_m^{3/2}$ (Ref. 27) in the "magnetic" coordinate system x',y',z' (in this coordinate system $\mathcal{H}\parallel\mathbf{e}_{z'}$, see Ref. 13). Following to Ref. 13, we have expressed the vector components $\hat{K}_i$, $\hat{J}_i$, and $\mathcal{H}_i$ in the crystallographic coordinate system through the vectors components $\hat{K}_{i'}$, $\hat{J}_{i'}$, and $\mathcal{H}_{i'}$ in the magnetic coordinate system

TABLE II. The valence band parameters and the values of the different contributions to the magnetocrystalline anisotropy for *p*-doped GaAs, GaSb, and Ge.

	GaAs	GaSb	Ge
γ_1	6.8	13.1	13.3
γ_2	2.3	4.5	4.2
γ_3	2.9	6	5.6
$\mathcal{K}$	1.2	3.5	3.3
q	0.04		0.06
$\delta M_\gamma / M_0$	0.22	0.42	0.32
$\delta M_q / M_0$	0.011		0.01

$$a_x = \cos\theta\cos\varphi a_{z'} + \sin\varphi a_{x'} + \sin\theta\cos\varphi a_{y'},$$

$$a_y = \cos\theta\sin\varphi a_{z'} - \cos\varphi a_{x'} + \sin\theta\sin\varphi a_{y'},$$

$$a_z = \sin\theta a_{z'} - \cos\theta a_{y'},$$

where $\mathbf{a}=\hat{\mathbf{K}}$, $\mathcal{H}$, or $\hat{\mathbf{J}}$. In the magnetic coordinate system, the matrixes $\hat{J}_i$ have the standard form and $\mathcal{H}_{x',y'}=0$, $\mathcal{H}_{z'}=\mathcal{H}$, $\hat{K}_{x'}=-\xi$, $\hat{K}_{y'}=-id/d\xi$, and $\hat{K}_{z'}=0$. All these formulas were substituted in $\hat{H}_{\text{anis}}$ and then the first-order perturbation theory correction $\delta E_{\text{g}}^{\text{anis}}=\langle\psi_{\text{g}}|\hat{H}_{\text{anis}}|\psi_{\text{g}}\rangle$ to the energy E_{g}^{is} was calculated. As the formula for $\hat{H}_{\text{anis}}$ contains fourth-order vector components expressions, the function $\delta E_{\text{g}}(\mathbf{h})$ is the fourth order polynom. An exact calculation gives

$$\delta E_{\text{g}}^{\text{anis}} = D\mathcal{H}P_4(\mathbf{h}),$$

where

$$D = \frac{e\hbar}{2m_0 c}\left[\frac{3}{2}(\gamma_2 - \gamma_3) - \frac{9}{8}q\right].$$

Thus, a corresponding anisotropic correction to the magnetization $\mathbf{M}=-n_{\text{h}}\partial E_{\text{g}}/\partial\mathcal{H}$ contains the fourth-order term by $\mathbf{h}$ only, does not depend on $\mathcal{H}$ and has the form

$$\delta\mathbf{M}_{\text{anis}} = -n_{\text{h}}D[4\mathbf{v}_3(\mathbf{h}) - 3P_4(\mathbf{h})\mathbf{h}]. \tag{11}$$

As for *n*-type semiconductor, it may be useful to extract the components $\delta\mathbf{M}_\parallel$ and $\delta\mathbf{M}_\perp$ from $\delta\mathbf{M}_{\text{anis}}$. For example, $\delta\mathbf{M}_\parallel=-n_{\text{h}}DP_4(\mathbf{h})\mathbf{h}$. The magnetization orbital (δM_γ) and *g*-factor (δM_q) anisotropic parts for the three semiconductors with small values of $(\gamma_2-\gamma_3)$ are shown in Table II. All band parameters are taken from the book.[27] In the last two rows of Table II the maximum variations of the anisotropic part of each mechanism contribution to $\delta\mathbf{M}_\parallel$ relative to $M_0=-n_{\text{h}}\partial E_{\text{g}}^{\text{is}}/\partial\mathcal{H}$ are presented. It is seen from Tables I and IIthat the free hole magnetization anisotropy is far greater than the free electron one.

Let us discuss the question of separating the lattice and the free carriers parts of the sample magnetization. The magnetic field is expected to be extremely high when the lattice part of the magnetization is saturated. This comes from the nonquantum criterion[5] $e^2/r_{\text{atom}}^2 \sim (e/c)v_{\text{atom}}\mathcal{H}$ saying that the atomic force acting on a core electron is of the same order as the Lorentz force. Thus, for the fields discussed we have $\mathbf{M}_{\text{core}}=\chi_{\text{core}}\mathcal{H}$. For example, for *n*-doped InSb with $n_{\text{e}}=2\times10^{17}$ cm^{-3} the saturated conduction electrons part of magnetization M_0 is equal to the linear by $\mathcal{H}$ extrapolated low field susceptibility magnetization (measured in Ref. 11) at $\mathcal{H}=10$ T (in the low fields limit the main part of $\mathbf{M}$ is the lattice contribution[11]). Thus, below this field, but when $\hbar\omega_{\text{c}} \gg E_{\text{F}}$ the conduction electrons part of the magnetization is greater than the core electrons part of magnetization. The origin of this situation, when the conduction electrons or holes contribution to the magnetization prevails on the core electrons contribution in strong magnetic fields, is the small value of the carrier effective mass, $m^* \ll m_0$, and, thus, the inequality $\lambda \gg \mu_{\text{B}}$ for the magnetic moments of free carriers and core electrons.[12] Finally, as it was done in Ref. 11, measurements on a large set of samples with different levels of doping may let extract the free carriers part of the magnetization and its anisotropy after analyzing the dependencies $\mathbf{M}(n_{\text{e}})$.

It should be noted that core electrons of doping element atoms can give a contribution to the magnetization as well. In particular, the spin-orbit coupling of doping atom core electrons with the crystal field can give a contribution to the magnetization anisotropy. This effect can be studied on the base of calculation of the crystal field effect on the fine structure of the doping atom terms, analogously to the theory Ref. 28 of the magnetocrystalline anisotropy in 3*d*-4*f* intermetallics. Here we only mention that for a light doping element, with atoms not heavier than the lattice atoms (thus, with $v_{\text{atom}}^{\text{dop}} \leq v_{\text{atom}}$), the doping atom core electrons contribution to the magnetization per one doping or lattice atom is of the same order as the lattice atom core electrons contribution discussed above.

Let us briefly discuss the experimental techniques to measure the proposed magnetocrystalline anisotropy. Usually, the precision of a technique increases with the increase of the sample volume as in each technique the value of the signal is proportional to the magnetization absolute value. In Ref. 11 the Guye method based on the measuring of the force acting on a long sample in a nonuniform magnetic field was used. Its precision was about 2% what approaches to the necessary level for *n*-doped semiconductors and is enough for *p*-doped semiconductors (see Tables I and II). There are some force methods to measure the difference between $\mathbf{M}_\parallel(\mathbf{h}')$ and $\mathbf{M}_\parallel(\mathbf{h}'')$ directly.[2] It exists modifications of the Gyue method with better precision based on vibrating and rotating samples.[4] If a sample has a good facet, it is possible to use the magnetic Kerr effect technique.[7] The superconducting quantum interference device (SQUID) magnetometer, which was used to measure the magnetization and its anisotropy of the dilute magnetic semiconductor alloys[9,29,30] recently, has a very high precision. For example, the precision of the Quantum Design MPMS 5XL SQUID magnetometer used in Ref. 31 is in the range of 10^{-9} emu for $\mathcal{H}=0$ to ±5 T.

In conclusion, we would like to mention that the proposed effect of the anisotropy of $\mathbf{M}$ can be employed as a new method to determine the semiconductor band parameters γ, η, β, $\gamma_{2,3}$, and q [see Eqs. (9) and (11)].

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