

Photon antibunching and nonlinear effects for a quantum dot coupled to a semiconductor cavityF. Bello^{1,2} and D. M. Whittaker¹¹*Department of Physics and Astronomy, University of Sheffield, Sheffield S3 7RH, United Kingdom*²*Departament de Física, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Spain*

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The models presented simulate pumping techniques that can be used on modern semiconductor devices which are capable of coupling a quantum dot and cavity mode in order to determine a more efficient method of producing a single-photon emitter while taking into consideration typical parameters which are achievable given today's standards of coupling strength. Cavity quantum electrodynamics are incorporated in the calculations as we compare various pumping schemes for the system that either use on-resonant laser excitation or nonresonant excitation due to a wetting layer. In particular, we look to study how antibunching effects change for each method as the cavity finesse is increased toward the strong coupling regime. Experimentally these studies are equivalent to nonlinear pump-probe measurements, where a strong pump, either resonant or non-resonant, is used to excite the coupled system, and the resulting state is characterized using a weak, resonant probe beam.

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I. INTRODUCTION

Currently, a single atom trapped within a cavity is the standard model for cavity quantum electrodynamics (CQED) with the semiconductor analog of this being a quantum dot (QD) trapped within a dielectric cavity. In this paper, we theoretically investigate the antibunching of photons that occurs as a direct consequence of the interaction between matter and cavity field¹ with the purpose of determining a method for populating the system which enhances this purely CQED effect and produces a more consistent single-photon emitter. Strong coupling between matter and cavity modes has been shown to create a quantum nonlinearity under coherent excitation that subsequently allows the system to absorb only one photon at a time, preventing the bunching of photons and creating a photon blockade.² Since, further theoretical work has demonstrated that the same effect is possible in Kerr-type nonlinear media using coupled cavity modes³ as well as through an atom⁴ or polariton⁵⁻⁷ blockade where exciton-exciton or atom-atom scattering produces the desired antibunching effect. In addition, previous experimental work has been conducted on isolated quantum dots⁸⁻¹⁰ where no nonlinearity exists on the quantum level (and hence no photon blockade) yet very efficient antibunching of light is produced but this is without the known advantages of being strongly coupled to a cavity such as collection efficiency, enhanced emission rates, as well as the promotion of recombination dynamics. All these processes are capable of single-photon emission which is useful in many branches of physics such as quantum cryptography and quantum computation; fields that both require the entanglement and emission of single particles within discrete energy levels.

Antibunching effects have been investigated in a variety of structures and devices such as photon turnstiles,^{11,12} micropillars,^{13,14} and photonic crystals¹⁵⁻²⁰ to name but a few. Strong coupling is not easily obtained with atoms tending to offer a ratio of coupling vs decoherence time greater than 8:1 (Ref. 21) compared to less than 2:1 for quantum dots.¹⁵⁻¹⁷ Coupled QD-cavity systems have shown promise

as single-photon emitters while drawing much interest with the added benefits of not only being stable components of the solid state structure, but restricting particle movement in all three dimensions, while, in principle, an increase in scalability of quantum-dot structures will allow them to emit over a suitable range of optical frequencies. For these reasons we aim to improve their efficacy for producing antibunched light.

Our main interest lies within QD-cavity systems where a single exciton interacts with the cavity mode, however the pumping techniques presented, either coherent (resonant) or incoherent (nonresonant), could be similarly applied to atom-cavity systems. We therefore make frequent comparisons between them both considering the current standards of cavity finesse and coupling strength (g_c). Quantitatively, antibunching is demonstrated by computing a second-order correlation function at zero time delay [$g^{(2)}(\tau=0)$] that is less than one,²² a purely quantum-mechanical effect, with the ideal value for perfectly antibunched light being zero, thus guaranteeing single-photon emission. For coupled systems, the most efficient antibunching experiments thus far have reported a $g^{(2)}(0)$ approaching values of 10^{-2} for a QD strongly coupled to a photonic crystal cavity^{16,18} along with values of approximately 10^{-1} within strongly coupled atom-cavity systems.²¹ The focus of this research is to present a scenario that can potentially improve upon these results by examining under which pumping scheme one could more readily obtain antibunching effects given typical experimental parameters. In Sec. II, we introduce the incoherent and coherent methods utilized to populate the system, namely, via a wetting layer or resonant excitation with a laser followed by Sec. III where we discuss and analyze the results for each. We conclude with Sec. IV in which we compare each of the pumping methods and present possibilities on how to improve antibunching.

II. THEORETICAL MODELING

The separation between weak and strong coupling regimes is found by making a comparison of the ratio between

cavity and dot decay rates (γ_c, γ_d) and the size of the energy splitting for which one wants

$$\sqrt{ng_c} > \frac{|\gamma_c - \gamma_d|}{4} \quad (1)$$

with the number of photons, n , in the system increasing the energy separation between successive pairs of dressed state levels.^{23–25} At the moment decay rates for a typical cavity are usually between one to two orders of magnitude larger than that of a QD which are presented in the formalism for completeness and could potentially play a significant role on coherence. For the purpose of calculating antibunching effects we want to explicitly write the correlation function in terms of the definitions for the first ($i=1$) and second ($i=2$) order Green's functions given as $G^{(i)}(0) = \text{tr}[\rho(t)b_1^\dagger b_2^\dagger, \dots, b_1^\dagger b_1, \dots, b_2 b_1]$. Here we represent the cavity field using second quantization operators, b^\dagger and b , creating or annihilating a cavity photon, respectively, with $\rho(t)$ being the steady state solution to the density matrix. The normalized second order correlation function is then defined as

$$g^{(2)}(0) = \frac{G^{(2)}(0)}{|G^{(1)}(0)|^2} = \frac{\sum_{n,j} n(n-1)P_{n,j}}{|\sum_{n,j} nP_{n,j}|^2} \quad (2)$$

with the trace being over a product basis of QD and number states from which the summation includes the number of photons (n) as well as dot states ($j \in 1, 2$). $P_{n,j}$ is the probability of finding the system with n photons in the cavity with the denominator equal to the square of the average number of photons in the system defined as $\bar{n} = \sum nP_{n,j}$. Physically this calculation is analogous to a simultaneous readout taken by two detectors within the Hanbury Brown-Twiss experiment and computing the average over the fluctuations in output intensity. The ideal scenario, corresponding to having $g^{(2)}(0)=0$, occurs when the detectors cannot simultaneously count a photon hence insuring that one has a consistent single photon emitter with $P_{n>1}=0$.

A. Incoherent excitation

Special attention is placed on the type of pumping, either coherent or incoherent, that one would use in order to perform pump-probe-type experiments. First, we look at incoherent excitation of the dot via a wetting layer modeled using a three-level system coupled to a cavity mode as proposed by Swain *et al.*^{26,27} and depicted in Fig. 1. We consider the ground state as the source for excitations within the system, from where the wetting layer is coherently excited with Rabi frequency Γ . Pumping of the dot is subsequently modeled via the incoherent decay rate, γ_w , from the wetting layer to the excited state of the dot where an electron can pair with a hole in the valence band. This method of incoherent pumping differs from that thoroughly examined by Laussy *et al.* and del Valle *et al.*, in which the pumping of the dot is considered to be directly from a reservoir. Such incoherent pumping would adjust the strong coupling condition in Eq. (1).^{24,28} The Hamiltonian for the coupled three-level system is given in the rotating frame as

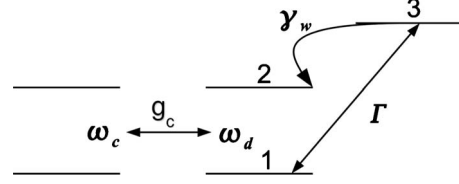


FIG. 1. Schematic representing a three-level system consisting of a wetting layer (3) as well as ground (1) and excited (2) states of the dot. The QD mode (ω_d) is coupled to a cavity mode (ω_c) via the coupling strength (g_c). The wetting layer is coherently driven by an on-resonant laser with Rabi frequency (Γ) while the excited state of the dot is being incoherently pumped directly from the wetting layer via the incoherent decay rate (γ_w).

$$H = \hbar \delta_d \sigma_2 + \hbar \delta_w \sigma_3 + \hbar \delta_c b_j^\dagger b_j + \hbar g_c (\sigma_{12}^+ b_j + b_j^\dagger \sigma_{12}^-) + \hbar \Gamma (\sigma_{13}^+ + \sigma_{13}^-), \quad (3)$$

where we have used a product basis consisting of an expansion of the Pauli spin matrices into three dimensions in order to describe the dot and wetting layer with its ground-state energy set equal to zero. Fock states are used to describe the cavity mode making use of the ladder operators b and b^\dagger . As defined analogous to Ref. 29, $\sigma_{rs}^+ = \sigma_{rs}^{-*} = \frac{1}{2}(S_x^{rs} + iS_y^{rs})$ are fermionic operators that describe interactions between levels r and s , and act on either the dot's ground (1) and excited (2) states along with the wetting layer's (3). $\sigma_i = |i\rangle\langle i|$ is a level shift operator whose expectation value reveals the population of energy level “ i ” while $\delta_{d,c,w}$ are the detunings for the dot (d), cavity (c), and wetting layer (w), respectively, relative to the laser frequency taken to be an on resonant cw π pulse. We look to solve for the density matrix, given below, of this coupled system in order to determine its dynamics.

$$\begin{aligned} \dot{\rho} = & -\frac{i}{\hbar} [H_s, \rho] - \frac{\gamma_c}{2} [b^\dagger b \rho(t) + \rho(t) b^\dagger b - 2b \rho(t) b^\dagger] \\ & - \frac{\gamma_d}{2} [\sigma_{12}^+ \sigma_{12}^- \rho(t) + \rho(t) \sigma_{12}^+ \sigma_{12}^- - 2\sigma_{12}^- \rho(t) \sigma_{12}^+] \\ & - \frac{\gamma_w}{2} [\sigma_{23}^+ \sigma_{23}^- \rho(t) + \rho(t) \sigma_{23}^+ \sigma_{23}^- - 2\sigma_{23}^- \rho(t) \sigma_{23}^+]. \quad (4) \end{aligned}$$

Explicitly shown are decay terms ($\gamma_{c(d)}$) taking into consideration the interaction of the cavity and QD with a surrounding thermal reservoir for which we have set $\gamma_d = \frac{\gamma_c}{10}$.³⁰ These terms are well known and derived using the Markovian approximation.^{24,31}

B. Coherent excitation

We now introduce the pumping scheme for on-resonant excitation of a cavity mode with a laser. For the case of a coherently pumped cavity we have the following Hamiltonian given in the rotating frame as

$$H = \hbar \delta_d \sigma_z + \hbar \delta_c b_j^\dagger b_j + \hbar g_c (\sigma^+ b_j + b_j^\dagger \sigma^-) + \hbar \Gamma (b^\dagger + b). \quad (5)$$

The Pauli spin matrices are used to describe the two-level dot with the subscripts 1,2 on $\sigma^{+(-)}$ dropped. Second quanti-

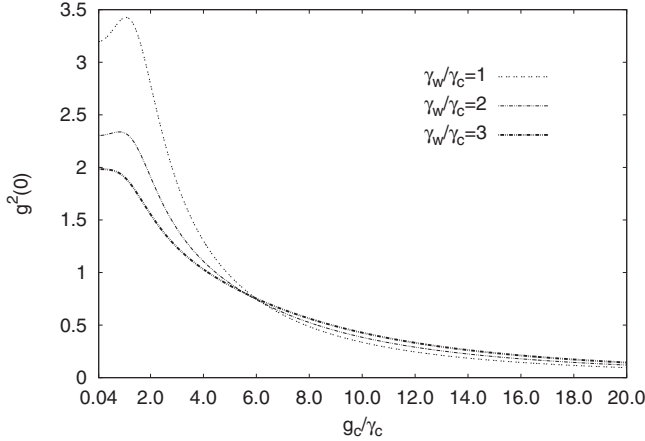


FIG. 2. $g^{(2)}(0)$ shown as a function of coupling over decay for incoherent pumping of the dot via a wetting layer with γ_c kept constant. Using a classical cw pulse to populate the wetting layer, Γ is taken to be π . Pumping from the wetting layer increases in order from lowest to highest going from a dotted to a more progressively solid line. The coupling g_c is varied for different pumping rates from the wetting layer to the excited state of the dot. A crossover appears around the weak to strong coupling transition at which point the affect of pump strength on antibunched light is inverted.

zation is used once again for the cavity mode which is excited by a classical driving field with the master equation given by the following expression:

$$\dot{\rho} = -\frac{i}{\hbar}[H_s, \rho] - \frac{\gamma_c}{2}[b^\dagger b \rho(t) + \rho(t) b^\dagger b - 2b \rho(t) b^\dagger] - \frac{\gamma_d}{2}[\sigma^+ \sigma^- \rho(t) + \rho(t) \sigma^+ \sigma^- - 2\sigma^- \rho(t) \sigma^+]. \quad (6)$$

This is essentially the conventional Jaynes-Cummings model using a classical driving field, for which expect similar results for an atom coupled to a cavity mode. If we change the operators in the Γ term from $b^\dagger(b)$ to $\sigma^+(\sigma^-)$, we are able to consider the laser field coherently exciting the QD. By shining the laser at an angle of incidence that avoids scattering with the cavity radiation, it could be possible to directly excite a QD mode rather than a cavity mode. Recently within photonic crystal cavities, it has been shown to be able to isolate single quantum dots (SQD) (Ref. 15) as well as to efficiently locate QDs and shift the cavity off-resonance making it possible to excite solely the QD.³²

III. RESULTS

A. Incoherent excitation

Antibunching effects are calculated by solving for the second-order Green's function presented in Eq. (2). Shown in Fig. 2, as coupling is increased bunching begins to decrease and eventually we start to see sub-Poissonian statistics where a crossing occurs. Increasing the coupling strength beyond this point, the lowest pumping (incoherent decay) rate from the wetting layer yields the best antibunching results. As coupling increases, it begins to dominate the inco-

herent pumping rate and dictates the number of particles within the cavity making the role of γ_w become less effectual in populating the system bringing the curves closer together. This contrasts the results before the crossing where although a lower coupling strength expectedly yields an increase in the second-order correlation function, the increase in pumping causes the QD to self-quench due to the incoherent pumping rate overcoming the coherent dynamics of the system, thus preventing the cavity from being populated.²⁴ Hence, higher pumping is yielding a lower result for $g^{(2)}(0)$. Using the incoherent pumping scheme proposed, the distribution of light is a complicated function depending on the intricate balance between the ratios of γ_w , $\gamma_{c(d)}$, and g_c that is neither necessarily thermal (exponential) or lasing (Poissonian) in form.³³ Initial varying of the Rabi frequency, Γ , has shown that the laser can be utilized to adjust the dot, and hence cavity, population given a sufficient incoherent pump rate and coupling strength. This is useful in regards to controlling antibunching, for which we expect an increase in photon bunching corresponding to an increase in the number of particles within the cavity.^{6,36} Considering the parameters given, the average number of photons in the cavity varies within the range on the order of 1 to 10^{-1} for both the incoherent and coherent pumping schemes presented. This is similar to that of Birnbaum *et al.*²¹ although antibunching has been measured in systems where weak pumping has been used to ensure that only the energy levels within the vacuum Rabi splitting are being excited with $\bar{n} \approx 10^{-4}$.³² We will further discuss and demonstrate the effect of pump strength on photon bunching in Sec. IV when we make comparisons between all pumping methods.

B. Coherent excitation

For the case of coherent excitation of the cavity mode, Fig. 3 shows that when the light field is approximately on-resonant with the energy splitting (g_c) we witness the strongest antibunching of photons by creating a photon blockade effect. As the cavity linewidth becomes more comparable to the energy splitting the strongest antibunching effect tends to move away from the exact value for g_c where we see an increase in bunching.²⁰ For instance, the coupling to cavity decay ratio of $\frac{g_c}{\gamma_c} = 5$ shows a minimum value of 0.873 for $g^{(2)}(0)$ occurring at approximately $1.3 \times g_c$. As detuning increases we return to the expected value of one for a coherent driving field. As we increase coupling strength, one sees a general increase in antibunching creating a more reliable single photon emitter with the best results for the strongest coupling. At zero detuning a strong bunching of photons will occur on the detectors. Here, although the absorption of an initial photon is difficult, the subsequent absorption of additional photons at higher energy levels on the Jaynes-Cummings ladder contributes to the overall bunching in the system. Additional troughs in Fig. 3 are due to the occupation of higher energy levels in the dressed state model. In particular, for a ratio of $\frac{g_c}{\gamma_c} = 40$, we witness antibunching at the two-particle eigenstate energy splitting of $\pm \sqrt{2}g_c$, corresponding to $\delta_c = \frac{g_c}{\sqrt{2}}$ on the graph, where a dip in the correlation function yields a value of 0.859.^{20,34} We can also see this

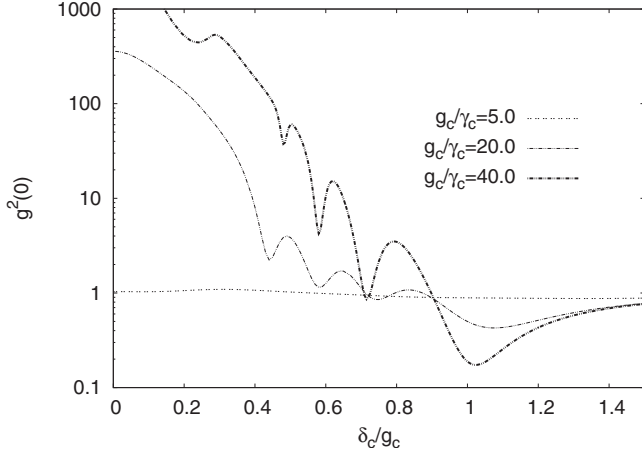


FIG. 3. $g^{(2)}(0)$ shown as a function of detuning ($\delta_c = \delta_d$) over coupling (g_c) for various coupling parameters with $\Gamma = \pi$. Here, the driving field has directly excited the cavity mode. Antibunching effects [$g^{(2)}(0) < 1$] occur when the driving field is roughly on-resonance with the two-photon energy splitting of $\sqrt{2}g_c$ as well as the vacuum Rabi splitting, g_c , at which point $g^{(2)}(0)$ tends toward zero creating a single photon emitter. Highly off-resonance we return to the expected value of one for a coherent driving field. As we move closer to the weakly coupled regime, the correlation function remains approximately the classical barrier of one for a coherent laser field.

feature occur for energy level splittings of $\sqrt{3}g_c$, $\sqrt{4}g_c$, and $\sqrt{5}g_c$.

Figure 4 considers the direct excitation of the dot via use of a coherent driving field. Antibunching effects are shown to be more easily obtainable within this method with the value of the correlation function going well below 0.1 compared to values for similar parameters in the case of a coherently pumped cavity. When the laser is on-resonance with the dot,

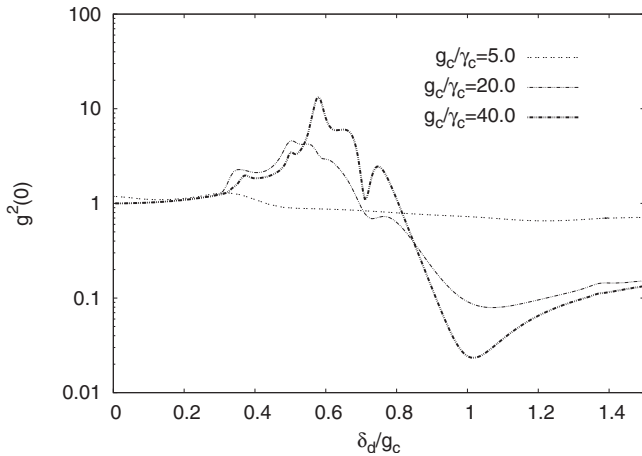


FIG. 4. $g^{(2)}(0)$ shown as a function of detuning ($\delta_c = \delta_d$) over coupling (g_c) with $\Gamma = \pi$ for a coherent driving field that directly excites the dot which in turn populates the cavity. Antibunching effects occur when the driving field is roughly on-resonance with the two-photon energy splitting of $\sqrt{2}g_c$ as well as the vacuum Rabi splitting, g_c , at which point $g^{(2)}(0)$ tends toward zero and is approximately an order of magnitude smaller than the case of a coherently driven cavity mode.

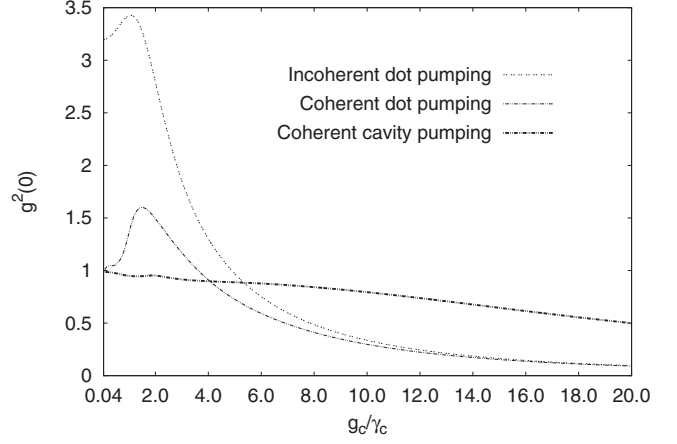


FIG. 5. Comparison of $g^{(2)}(0)$ for incoherent dot pumping as well as coherent dot or cavity excitation while varying g_c . For the coherently driven cases data presented is for when detuning is equal to the energy level splitting, $\hbar g_c$. Initially we only see antibunching effects in the system for a coherently driven cavity mode, further into the strong coupling regime dot pumping shows greater antibunching effects at the present day standards for atom-cavity coupling with $\frac{g_c}{\gamma_c} = 8$. At this point a coherently driven cavity yields a value of $g^{(2)}(0) = 0.842$ while for incoherent or coherent dot pumping we see values of approximately half that at 0.485 and 0.412, respectively.

the emitted light field resembles that of the coherent source yielding a $g^{(2)}(0) = 1$. The improvement in antibunching could be explained due to the use of a QD to populate the cavity instead of a coherent field. The two-level dot acts as a natural antibunching device for light permitting only one photon to enter the cavity at a time. With a sufficient coupling to cavity decay ratio, the photon will leave the cavity before the dot will reemit another into it and thus not allow particles to bunch in the system.

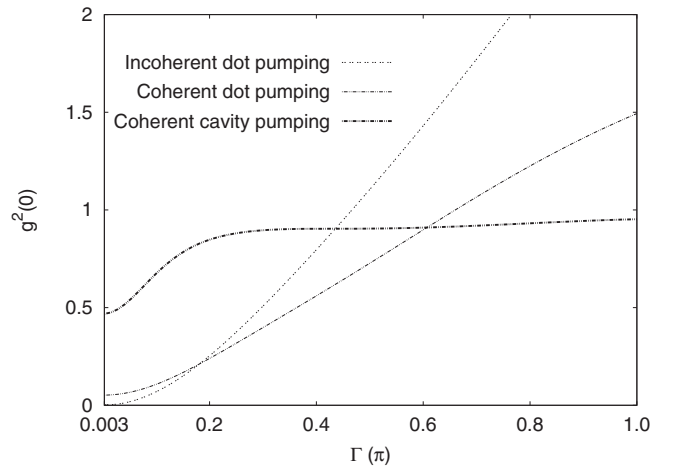


FIG. 6. Comparisons of pumping methods presented for $\frac{g_c}{\gamma_c} = 2$ while varying the strength of the driving field, Γ . As the driving field is lowered, we see an increase in antibunching due to the smaller average number of photons in the cavity. Values of $g^{(2)}(0) = 0.842$.

IV. DISCUSSION AND CONCLUSIONS

To conclude, we have presented two alternative methods for populating a QD-cavity system in lieu of a coherently driven cavity mode. Both of which show potential to enhance the effects of antibunching via incoherent decay from a wetting layer or coherent excitation of a QD mode. A comparison of $g^{(2)}(0)$ under all pumping schemes is presented in Fig. 5 with the incoherent case presented being for $\gamma_w = \gamma_c$. Considering our relatively strong pumping along with today's current standards for coupling strength to cavity linewidth in a QD-cavity system ($\frac{g_c}{\gamma_c} < 2$), antibunching effects would only be seen for the case of coherent excitation of the cavity and one would need state-of-the-art values of $\frac{g_c}{\gamma_c}$ in order to take advantage of the potential benefits of the coherent dot and incoherent pumping methods presented. Indeed, ultra strong coupling is not needed in order to witness effective photon antibunching if one uses a smaller pumping rate and focuses on the Rabi vacuum splitting only. With an increase in pumping, we expect this to cause an increase in bunching.^{24,35,36} In Fig. 6, we adjust the strength of the driving field (Γ) for each scheme considering $\frac{g_c}{\gamma_c} = 2$. Results

show that as pumping is decreased very effective antibunching occurs for incoherent pumping via a wetting layer and coherent dot pumping with results of $g^{(2)}(0) = 0.002$ for $\bar{n} = 8.4 \times 10^{-3}$ and a $g^{(2)}(0) = 0.05$ for $\bar{n} = 3.3 \times 10^{-4}$, respectively, for the case of $\frac{\Gamma}{\pi} = 0.003$. This is a vast improvement upon $g^{(2)}(0)$ if one was to utilize a coherently driven cavity which yielded a value of 0.47 for similar parameters.

Results such as these are highly noteworthy in regards to today's capabilities for manufacturing and manipulating strongly coupled QD-cavity systems. The alternative pumping methods in lieu of coherent cavity excitation utilized in this paper coupled with current technological advances such as SQDs, offer increased possibilities for single-photon sources laxing the current restraints on producing microcavities, QDs, and large Q factors.

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