Erratum: Quantum physics of kinks of dislocations [Phys. Rev. B 82, 014519 (2010)]

S. T. Chui

(Received 23 August 2010; published 2 September 2010)

DOI: 10.1103/PhysRevB.82.099902 PACS number(s): 67.80.−s, 99.10.Cd

In our paper, we study the quantum physics of kinks of dislocations and its implication on recent torsional oscillator experiments in solid He4. While the conclusions of this paper remain correct, an important detail has not been pointed out. In our paper, we claim that the dissipationless motion of dislocations can reduce the rotational kinetic energy of the He4 atoms. We did not point out that this is true only when the axis of rotation is not parallel to the dislocations. In real experimental situations it is unlikely that all dislocations are parallel to the axis of rotation and our conclusion remains correct. We explain this in detail next.

The kinetic energy of the system of mass density ρ is given by $E=\int d\mathbf{r}\rho v^2(r)/2$. A dislocation creates a displacement of an atom at position $\mathbf{r}$ by an amount $\mathbf{u}(\mathbf{r})$. As the dislocation is moved by an amount $\delta\mathbf{c}$, there is a change in the displacement given by $\delta\mathbf{u}=\delta\mathbf{c}\cdot\nabla\mathbf{u}$. For a particle at position $\mathbf{r}$ with velocity $\mathbf{v}(\mathbf{r})$, because of the motion of the dislocation, its velocity will be changed by an amount $\delta\mathbf{v}$ that is proportional to $\delta\mathbf{u}$. The change in kinetic energy of the particles is thus proportional to $\Delta E=\int d\mathbf{r}\rho\mathbf{v}(\mathbf{r})\cdot\delta\mathbf{u}$. For a rotational motion with angular velocity ω about an axis located at $\mathbf{r}_0$, the velocity of the particle is $\mathbf{v}=\boldsymbol{\omega}\times(\mathbf{r}-\mathbf{r}_0)$. Thus $\Delta E=\rho\int d\mathbf{r}\boldsymbol{\omega}\cdot\mathbf{w}$ where the local "circulation" $\mathbf{w}$ is defined by

$$\mathbf{w}(\mathbf{r})=\mathbf{r}\times\delta\mathbf{u}(\mathbf{r}). \tag{1}$$

The contribution $\int d\mathbf{r}\boldsymbol{\omega}\cdot\mathbf{r_0}\times\delta\mathbf{u}=0$ as δu is odd under a parity transformation. The displacement $\mathbf{u}(\mathbf{r}_\perp)$ of a dislocation lies in the plane perpendicular to the dislocation axis and is only a function of the perpendicular position from the axis. When the dislocation axis is not parallel to the rotation axis it is straightforward to obtain the component of the circulation along the rotation axis. To illustrate, consider the case so that the rotation axis lies in the same plane spanned by the Burger's vector and the dislocation, as is illustrated in Fig. 1. We call the angle between the dislocation and the rotation axis t. We assume that the dislocation moves in a direction parallel to the Burger's vector which we call the x axis. At height z, the dislocation core is at $\mathbf{r}'_\perp=(z\cos t,0)$. We write $\mathbf{r}_\perp-\mathbf{r}'_\perp$ in cylindrical coordinates with an azimuthal angle ϕ. For a single dislocation, we obtain[1]

$$w/\delta c=-0.5\cos(t)[(-1+2\sigma)(\cos^4(\phi)\cos^2(t)-\sin^4(\phi))+(3\cos^2(t)-3-2\cos^2(t)\sigma+2\sigma)\cos^2(\phi)\sin^2(\phi)]/[(\cos^2(\phi)\cos^2(t)+\sin^2(\phi))^2(-1+\sigma)2\pi] \tag{2}$$

where σ is the Poisson ratio. This local circulation is shown in Fig. 2 as a function of ϕ for t equal to 0, 30 and 60 degrees. The total circulation, obtained from integrating w with respect to ϕ, is shown in Fig. 3 as a function of t.

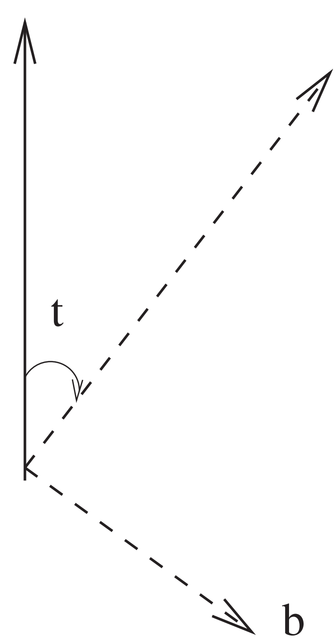

FIG. 1. A figure illustrating the rotation axis (solid line), the dislocation axis (dashed line) and the direction of the Burger's vector labeled by b.

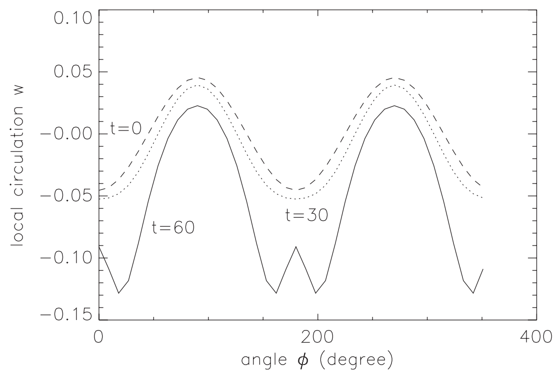

FIG. 2. The local circulation as a function of the azimuthal angle for different angles between the dislocation and the rotation axis.

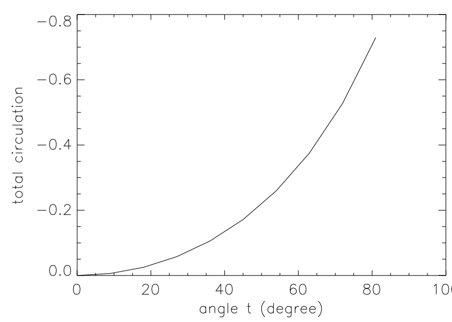

FIG. 3. The total circulation as a function of the angle t between the dislocation and the rotation axis.

As can be seen, the total circulation is zero **only** when $t=0$ and the dislocation is parallel to the rotation axis.

When the Burger's vector no longer lies in the same plane N formed by the dislocation line and the axis of rotation, a more complicated formula can be derived for w. The orientation of the Burger's vector can be specified by the Euler angle p, the amount of rotation about the dislocation axis required to move the Burger's vector to the plane N. We found

$$(1-\sigma)4\pi(x^2\cos^2 t+y^2)^2 w/\delta c = 2yx\sin 2p[(\cos^2 t-\sigma\cos^2 t-\sigma)x^2\cos^2 t-2(\sigma-1+\sigma\cos^2 t)y^2]-\cos 2p \times[(1-2\sigma)\cos t(x^4\cos^2 t-y^4)+(2\sigma-3)(\cos^2 t-1)y^2x^2\cos^2 t]$$

We thank the participants of the Paris supersolidity workshop for asking penetrating questions and to Norbert Mulders for helpful discussions.

[1] The last formula (w for $t=0$) in section VI is incorrect.