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Frequency-dependent ultrasound attenuation in superfluid ^{3}He in aerogel

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Ultrasound attenuation measurements in the B-like phase of superfluid ^{3}He embedded in 98% porosity aerogel have been performed at four frequencies between 3.6 and 11.3 MHz. At all of the pressures studied (14, 25, and 33 bar), the attenuation exhibits nontrivial frequency dependences that progressively deviate from the hydrodynamic ω^2 behavior as the temperature decreases. This behavior is related to the structure of the impurity states inside the gap and could offer a tool to profile the gap structure in this system.

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The observation of superfluid transitions in liquid ^{3}He embedded in high porosity silica aerogel[1,2] opened a way to introduce static disorder/impurities in this system and triggered immediate theoretical and experimental activities.[3] This keen interest arises from the intimate connection between superfluid ^{3}He and unconventional superconductors. For the case of ^{3}He, the nanometer scale aerogel structure presents a dilute concentration of elastic scattering centers in the system and the effects of impurities are unmistakably demonstrated in the low-temperature phase diagram of ^{3}He in 98% aerogel[3] as manifested by the substantial suppression of superfluid transition from the pure liquid,[1,2] the appearance of the disorder-driven quantum phase transition,[4] and the markedly different A-B transition.[5]

In addition to the Fermi wavelength, $\lambda_f \approx 0.3$ nm, and the bulk coherence length, $\xi_o = \hbar v_f / 2\pi T_c \simeq 20-80$ nm in the pressure range of 34–0 bar ($v_f \equiv$ Fermi velocity and $T_c \equiv$ bulk superfluid transition temperature), aerogel introduces a few other important length scales in the system. More specifically, the size of the aerogel strand, $\delta_a \simeq 3-5$ nm, the porosity-dependent structural correlation length, ξ_a ($\simeq 40$ nm for 98% porosity), and the geometric mean free path, ℓ_a ($\simeq 130$ nm for 98% porosity) are important parameters. To a first approximation, the aerogel can be considered as dilute random pointlike impurities, and theoretical models and/or ideas based on this argument have provided reasonable explanations for most of the phenomena observed in this system. Therefore, the effects of impurity scattering are reflected in the total scattering mean-free path in this system, ℓ, which should incorporate both the elastic scattering from the aerogel as well as the inelastic quasiparticle scattering, following Matthiessen's rule, $1/\ell = 1/\ell_e + 1/\ell_i$, where the elastic mean free path $\ell_e = \ell_a$ and ℓ_i is the inelastic quasiparticle-quasiparticle scattering mean free path. At low temperatures, the temperature-independent elastic scattering dominates the scattering process, preventing the divergent growth of the mean free path in a Fermi liquid. Thermal conductivity[6] and spin diffusion[7] measurements demonstrated the signatures of the temperature-independent ℓ_a at low temperatures. Furthermore, the first to zero sound crossover in the normal fluid of ^{3}He was found to be effectively inhibited in aerogel, keeping the system from entering into the collisionless limit ($\omega\tau \gg 1$) on cooling, where ω is the angular sound frequency and $\tau = \ell / v_f$ is the relaxation time.[8]

Sound propagation in the superfluid phases is also expected to remain in the hydrodynamic limit[9,10] but is further complicated by the presence of impurity states induced by pair-breaking scattering.[10,11] A recent ultrasound attenuation measurement performed in the B-like phase exposed many interesting features such as the absence of the order parameter collective modes and the presence of finite attenuation as $T \to 0$, evincing the existence of impurity states and gapless superfluidity.[11] These observations are consistent with the theoretical predictions based on the homogeneous isotropic scattering model (HISM) (Ref. 12) and a hydrodynamic two-fluid model.[10] However, it is important to recognize the unique aspects of aerogel originating from its structure. The importance of the correlation has already been raised in the context of inhomogeneity and anisotropy.[13,14] In addition, the nature of the scattering is expected to lie in an interesting territory between impurity and surface scattering, considering the size of aerogel, $\delta_a \approx 10\lambda_f$. The details of impurity states depend on the type of pairing mechanism and scattering.[13,15,16] Therefore, the full understanding of sound propagation in this system is complicated and should be explored by extensive theoretical and experimental studies.

In this work, we present ultrasound attenuation in the B-like phase of superfluid ^{3}He in 98% porosity aerogel measured at four different frequencies between 3.6 and 11.3 MHz. Our results reveal nontrivial frequency dependences in attenuation, gradually departing from the ω^2 dependence predicted in this system.[9,10] These results are interpreted in terms of the presence of impurity states inside the gap and provide a tool that can be refined to elucidate the details of the structure of the gap.

Two matched $LiNbO_3$ transducers were used as a transmitter and a receiver to implement a pulse-echo method. The transducers have primary resonances at 1.1 MHz, but only the third (3.69 MHz), the fifth (6.22 MHz), and the ninth (11.3 MHz) harmonics were chosen for the measurements. The transducers are separated by a Macor spacer maintaining a 3.05 ± 0.02 mm gap where the aerogel sample was grown *in situ*. A 3 μs pulse was generated by the transmitter and

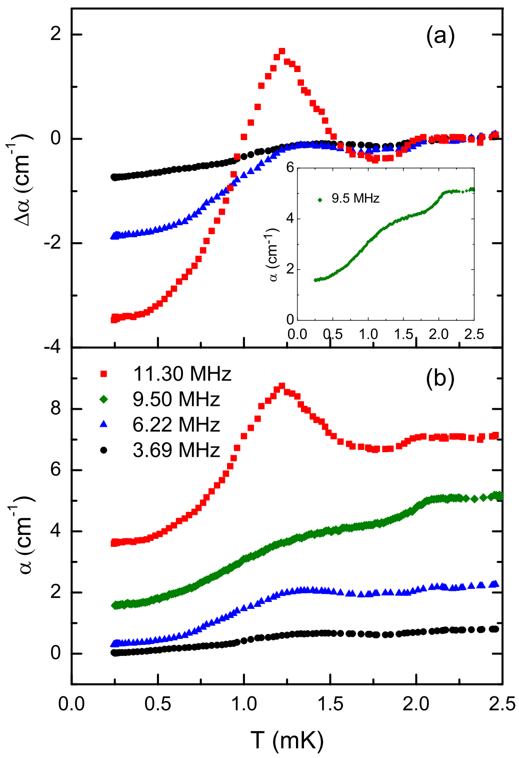

FIG. 1. (Color online) (a) Temperature dependence of relative attenuation at 3.69, 6.22, and 11.30 MHz taken at 33 bar. Inset: the temperature dependence of absolute attenuation at 9.5 MHz at the same sample pressure is reproduced from Ref. 11. (b) Temperature dependence of absolute attenuation for all frequencies at 33 bar. See text for the details.

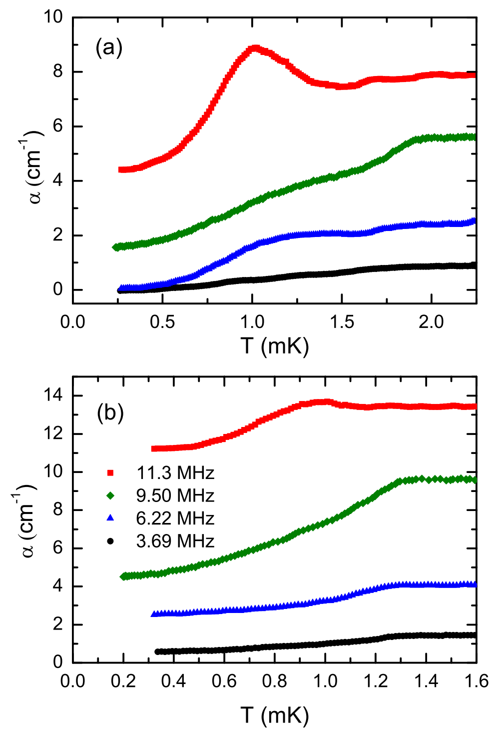

FIG. 2. (Color online) Temperature dependence of absolute attenuation for three frequencies along with the previous measurement at 9.5 MHz for (a) 25 bar and (b) 14 bar.

detected by the receiver using a commercial spectrometer, LIBRA/NMRKIT II (Tecmag Inc., Houston, TX). Eight received signals were averaged in a phase alternating pulse sequence for each measurement. The temperature was determined by a melting curve thermometer for $T \geq 1$ mK and a Pt-NMR thermometer for $T \leq 1$ mK. Our experimental scheme is described in detail elsewhere.[5]

With the 9.5 MHz transducers used in our earlier experiment,[11] we were able to resolve the multiple echoes in the received signal from which the absolute attenuation was extracted. However, long ringing in the transducers used in this work prevented us from resolving the echoes. Therefore, we could only determine the relative attenuation using the area under a part of the received signal by $\Delta\alpha = \alpha(T) - \alpha(T_c) = -(1/d)\ln\{A(T)/A(T_c)\}$, where d is the path length and $A(T)$ is the acoustic amplitude at temperature T.[5] The relative attenuation measured on warming at 3.69, 6.22, and 11.30 MHz for the sample pressure of 33 bar is displayed in Fig. 1(a). In all of these traces, the superfluid transition in aerogel, T_{ca}, is well established by smooth change in attenuation around 2.1 mK and the characteristic temperature dependence is consistent with that of 9.5 MHz absolute attenuation reproduced in the inset including the conspicuous bump structure.[11]

A theoretical model for longitudinal sound propagation in liquid ^{3}He in aerogel was developed by a Hiroshima group.[17] This model, based on the Landau-Boltzmann transport theory, considers the impurity scattering off aerogel as well as the relative motion between aerogel and liquid ^{3}He. Specifically, a ^{3}He quasiparticle impinged on aerogel strand transfers momentum and causes a dragged motion of the aerogel, the collisional drag effect. When this process generates a relative motion between these two systems, it gives rise to an additional damping mechanism. Therefore, the damping of the fast mode[18] arises from the friction as well as the usual shear viscosity (η) and becomes

$$\alpha_f = \frac{\omega^2/2c_f}{1+\rho_a\rho_s/\rho_n\rho}\left(\frac{\rho_a^2\tau_f/\rho\rho_n}{1+\rho_a/\rho_n} + \frac{4\eta/3\rho c_1^2}{1+\rho_a\rho_s/\rho_n\rho}\right), \qquad (1)$$

where c_f is the fast mode velocity, c_1 is the first sound velocity in bulk, ρ_a is the aerogel density, $\rho_{n(s)}$ is the normal (superfluid) density ($\rho = \rho_n + \rho_s$), and τ_f is the frictional relaxation time. The full temperature dependence was provided from the microscopic calculation of ρ_s, τ_f, and η incorporating the midgap impurity states calculated using HISM. This calculation[17] captures the important features observed in our experiments and the direct comparison with the experimental results at 9.5 MHz can be found in Ref. 11. While τ_f initially increases right below T_{ca} (even above the normal fluid value for high pressure) due to rather rapid opening of superfluid gap, τ_f eventually approaches zero as $T \to 0$, displaying a broad peak in attenuation. This effect, reflected in the size of the peak in τ_f, is more pronounced at a higher pressure where the pair-breaking effect is less significant, and accordingly, the bump structure in the attenuation gradually fades as the sample pressure is lowered as shown in Figs. 1 and 2. Although there is no direct experimental confirmation for the

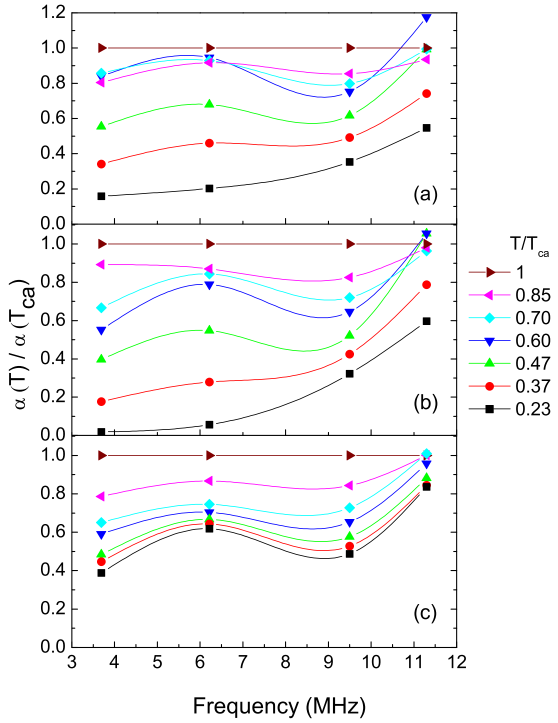

FIG. 3. (Color online) Sound attenuation as a function of frequency for select reduced temperatures at (a) 33 bar, (b) 25 bar, and (c) 14 bar. The sound attenuation is normalized by the attenuation at T_{ca}. The lines going through the data points are guides for eyes.

ω^2 dependence in attenuation as in Eq. (1), this claim is supported by the observation of strong frequency dependence in attenuation.[8,11] Additionally, the experimentally determined ℓ_a in 98% aerogel by thermal conductivity (90 nm),[6] spin diffusion (130 nm),[7] and sound attenuation (120 nm) (Ref. 11) makes $\omega\tau < 1$ for $\omega/2\pi < 20$ MHz for the entire pressure range and at all temperatures.

With this notion, we deduced the absolute attenuation from the relative attenuation measured in this experiment using the normal fluid absolute attenuation at 9.5 MHz as a fixed point through

$$\alpha(T) = \frac{\alpha_o}{\omega_o^2}\omega^2 + \Delta\alpha, \tag{2}$$

where α_o is the attenuation at 9.5 MHz at T_{ca} from Ref. 11 and $\omega_o/2\pi = 9.5$ MHz. The attenuation calculated following this recipe is displayed in Fig. 1(b) for 33 bar and in Fig. 2 for 25 and 14 bar. The sound attenuation at 9.5 MHz for the corresponding pressure is also reproduced in the same panel. It is noteworthy that they all share similar qualitative features as already described.

To illuminate the frequency dependence, the sound attenuation is potted as a function of frequency at a constant temperature in Fig. 3. The sound attenuation is normalized by the value in the normal liquid near T_{ca}, effectively by ω^2, and the data for the same set of the reduced temperatures are chosen for all pressures in this plot. One can clearly see the evolution of the frequency dependence deviating from the ω^2 dependence (flat line in the plot) as temperature lowers. For

TABLE I. Important parameters estimated for three pressures used in this work, assuming $\ell_a = 130$ nm and taking $\omega/2\pi = 11.3$ MHz.

P (bar)	$\omega\tau$	$\frac{\Delta_0(0)}{2\pi\hbar}$ (MHz)	$\frac{\Delta_a(0)}{2\pi\hbar}$ (MHz)	$\frac{\hbar\omega}{\Delta_a(T)}$ $0.23T_{ca}$	$0.60T_{ca}$	$0.85T_{ca}$
33	0.30	96.17	62	0.18	0.20	0.26
25	0.27	91.13	57	0.20	0.22	0.29
14	0.23	77.37	31	0.36	0.39	0.53

33 and 25 bar, the attenuation establishes a significant frequency dependence beyond the quadratic behavior in the zero-temperature limit after going through a nonmonotonic frequency dependence on cooling. For 14 bar, however, the attenuation shows a dramatically different behavior than the one observed at higher pressures. The hump structure around 6 MHz starts to develop from the lower value of T/T_{ca} at 14 bar compared to the other higher pressures but continues to grow to the lowest temperature. At 25 and 33 bar, this feature seems to be associated with the broad bump structure in the attenuation. However, the similar frequency dependence persists down to the lowest temperature at 14 bar while the anomalous bump structure almost vanishes.

In the presence of aerogel, an unusual circumstance arises where $\omega \sim \Delta/\hbar$ and $\omega < 1/\tau$, and this situation cannot occur in bulk ^{3}He except for the region very close to T_c. More quantitatively, Table I lists several important quantities pertinent to our discussion, where $\ell_a = 130$ nm is used. The average gap in aerogel at zero temperature used in this table is from Halperin *et al.*[3] The superfluid gaps at finite temperatures are obtained assuming the same temperature dependence as in the bulk and using the specific heat jump measurements by Choi *et al.*[19] The superfluid gap is significantly suppressed at all pressures and the degree of the suppression is more severe at lower pressure, 14 bar. Although the sound is in the hydrodynamic limit, the sound frequency is still comparable to the size of the superfluid gap even in the zero-temperature limit. Therefore, the sound attenuation cannot be described purely from the hydrodynamic mechanism. Nagai and Wölfle[16] provided the full expression for sound attenuation in a viscoeleastic model

$$\alpha = \frac{2\omega^2}{3\rho c_1^3}\mathrm{Re}\left[\frac{\eta}{1 - i\omega\tau_\eta}\right], \tag{3}$$

where τ_η is the viscous relaxation time. This expression is valid in the superfluid as well as in the normal liquid with the appropriate τ_η. The finite frequency correction in this model always gives rise to a negative contribution on the order of $(\omega\tau_\eta)^2$, which is diametrically different from our observation.

Hirschfeld *et al.*[20] calculated the electromagnetic absorption in isotropic as well as anisotropic *p -wave* superconducting states, considering various absorption mechanisms involving impurity bound states. According to their results, the resonant electromagnetic absorption occurs through a quasiparticle scattering from filled to empty impurity states, filled impurity states to gap edge, and filled gap edge to unoccupied impurity state, which leads to potentially a nontrivial frequency dependence (see Fig. 2 of Ref. 20). At $T=0$, the optical conductivity in an isotropic *p*-wave superconducting state, e.g., the Balian-Werthamer (BW) state, can be simplified to $\sigma(\omega) \propto \int_0^{\omega} d\tilde{\omega} \hat{N}(\tilde{\omega}) \hat{N}(\tilde{\omega}-\omega)$, where $\hat{N}(\omega)$ is the normalized density of states. Although this specific calculation was performed for electromagnetic absorption, the absorption processes should be similar in ultrasound. Considering the excitation frequency relative to the significantly suppressed superfluid gap, the above mentioned processes must be important contributions to the total sound attenuation in aerogel. Based on this argument, one can expect the strict hydrodynamic ω^2 dependence only with a flat distribution of midgap impurity states with no gap between the impurity states and gap edge. Therefore, knowledge of the detailed spectrum of the impurity states is crucial. However, there are no theoretical calculations of impurity states in superfluid ^{3}He specifically addressing the structure of aerogel such as its finite size. In an extreme limit, the full spectrum of the Andreev bound states (ABS) from surface scattering[15,16,21] has been investigated theoretically. The spectrum of ABS is rather different from those resulting from impurity scattering in the BW state. One interesting feature of the ABS is the appearance of a small gap (Δ^* in Ref. 21) between the edge of the ABS band and the gap edge only in diffusive surface scattering.[21] A series of recent experiments[22,23] have provided convincing evidence for ABS and confirmed many features predicted by the theory.

Thuneberg *et al.*[24] calculated the spectrum of the bound states resulted from scattering off a localized negative ion in ^{3}He-*B*. They found that multiple bound states appeared at different energies and with different weight depending on the angular momentum channel for a finite-size impurity. It is reasonable to expect a similar effect in aerogel, which has the similar size to a negative ion ($\simeq 1-2$ nm). The location and the number of bound states caused by a cylindrical rather than spherical object would be distinct and the distribution of these objects with correlation could produce nontrivial impurity bound states. We interpret the nontrivial frequency dependence of the attenuation to be directly related to the spectrum of impurity states induced inside the gap. The frequency dependence at different temperatures and pressures may be understood qualitatively based on the value of $\omega/\Delta_a(T,P)$. The most prominent nonmonotonic frequency dependence appears around $\omega/\Delta_a(T,P) \simeq 0.3$ for all pressures although one cannot ignore the effect of thermally excited quasiparticles.

In summary, we observed nontrivial frequency-dependent ultrasound attenuation in the *B*-like phase of superfluid ^{3}He in 98% aerogel. Our observation cannot be understood by a simple hydrodynamic mechanism. The sound attenuation should reflect the structure of the impurity states that might be characteristically different from those induced by conventional random impurities. Additional measurements with a finer resolution and a wider range in frequency might elucidate the structure of impurity states induced by (correlated) finite-size scatters.

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