Motion of electrons in liquid ^{4}He

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We present theoretical results on the stationary-state motion of electron bubbles in liquid ^{4}He both at zero and negative pressures. As the velocity increases, the moving electron bubble is squeezed along the direction of motion while it expands in the transverse directions. When the electron speed is large enough, as a consequence of this change in shape, a vortical fluid motion is induced around the bubble equator which eventually results in the formation and emission of a quantized vortex ring as a critical velocity is reached. This process occurs at zero pressure and at negative pressures down to ~ -1.2 bar. Below this value, the bubble becomes unstable and explodes as soon as the critical velocity is reached. Our results show that fast-moving electron bubbles explode in the pressure range where unidentified electron objects have been found to explode in recent cavitation experiments.

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I. INTRODUCTION

Electron bubbles (e bubbles) produced by excess electrons in bulk liquid ^{4}He have been investigated extensively, and many aspects of their behavior are well understood, see, e.g., Refs. 1 and 2, and references therein. In addition to these "normal" electron bubbles, there have been several observations of other negatively charged objects whose physical nature is still a mystery. These include the so-called fast ions and the "exotic" ions (whose mobilities are intermediate between those of the normal and the "fast" negative ions). Because they have a higher mobility than the normal electron bubble, these objects are believed to be small, probably with radius in the approximate range $R \sim 10-16$ Å. The exotic ions and the fast ions have been detected in several experiments by different groups.[3,4] Both fast and exotic ions under high electric fields have been studied in Ref. 5, where they found that the fast ions can reach the Landau critical velocity v_L while the exotic ions decay instead into charged vortex rings before being able to reach v_L.

No satisfactory explanation has been given so far for the existence of exotic ions. They cannot be electron bubbles in an excited state since they would be larger than normal electron bubbles. Since different species of such exotic ions were observed at the same time in ^{4}He, it is unlikely that these are negative ions of impurity atoms. In Ref. 5 their behavior has been tentatively attributed to a motion of an electron bubble undergoing a continuous sequence of escape-trapping-escape events of charged vortex rings.

A more radical proposal has been put forward,[6] where such objects are thought to be bubbles where only some fraction of the electron wave function is trapped. The trapping might occur, for instance, when a normal bubble is excited from the ground state to a higher quantum state and the bubble then undergoes a fissionlike process.[7,8] The existence of these electron fractions, however, has been criticized in Refs. 9–11.

Cavitation experiments performed with electrons trapped in liquid ^{4}He have yielded interesting information on the properties of e bubbles in their ground $1s$ and first-excited $1p$ states.[12] The critical *negative* pressure P_c at which the $1s$ bubble becomes unstable has been measured and theoretical results are found to be in good agreement with experiment.[13] Two new kind of e bubbles have been recently observed below a temperature (T) of 1 K, that explode above P_c.[14] The first class is made of e bubbles attached to quantized vortices in the superfluid. The second class is made of objects that are found to explode at even less negative pressures (P) than those attached to vortices. These so far unidentified electron objects (UEOs) have been tentatively associated with the presence of vortices in the liquid, such as an e bubble simultaneously attached to two vortices or to a doubly quantized vortex line.[14] However, detailed calculations seem to rule out the former possibility[15] whereas doubly quantized vortex lines are unstable against decaying into two singly quantized vortex lines, thus making unlikely also the latter possibility. In the present paper we put forward an alternative explanation for the UEO observed in cavitation experiments.

An e bubble moving through liquid ^{4}He at $T=0$ can dissipate kinetic energy for speeds in excess of the Landau critical velocity by creating rotons[16] that are most likely produced in pairs.[17] This process has been studied extensively, usually in pressurized ^{4}He, where the probability of roton pair creation is higher. A charged particle moving through superfluid ^{4}He can also spontaneously undergo a transition,[18] where a vortex ring becomes charged upon being captured by the e bubble. The nature of such process, however, remains unclear, despite the vast experimental and theoretical amount of data collected so far. It is assumed that, according to a Landau-generalized argument, there exists a critical velocity the bare ion must exceed for the transition to occur. Such velocity has been measured in pressurized ^{4}He.[19] Similar mobility measurements have been performed for negative ions moving in ^{4}He at low pressures,[20] and the mechanism limiting the velocity of the ion has been attributed to the continuous production of microscopic quantized vortex rings to which the electrons do not bind, rather that to the production of charged vortices. A similar mechanism was proposed long ago by Huang and Olinto.[21] The limiting drift velocity of negative ions measured in Ref. 20 can be extrapolated down to $P=0$, yielding a critical velocity slightly below 50 m/s.

The ion/vortex-ring transition in isotopically pure superfluid ^{4}He has been studied in Ref. 22, and models for the emission of rings have been proposed. In the "peeling model,"[23] the nascent ring grows continuously from a small initial loop of vortex line localized near the equator of the ion. The application of a generalized Landau criterion, on the other hand, leads instead to the conclusion[24] that the nascent ring should appear spontaneously around the ion, girdling it along the equator. Vortex nucleation involving direct creation of rings encircling the negative ions has been also proposed in Ref. 25. Calculations based on hydrodynamics[23] indicate that the vortex nucleation is likely an activated process, and that the energy barrier for nucleation of small vortex loops (peeling) is lower than that required to create a vortex ring girdling the equator. Experiments[26] while measuring an energy barrier for vortex nucleation compatible with the loop emission model,[23] do not preclude, however, the occurrence of axially symmetric nucleation processes where the vortex ring is formed instead around the bubble equator. Our findings, as shown in the following, seem to agree with the latter scenario.

An instability of totally different nature has been predicted on classical grounds,[27] where the fast motion alters the shape of the e bubble because of the Bernoulli pressure field acting on its surface. The stability of a gas bubble moving inside a liquid with a constant velocity v is determined by the Weber number $W_e=2\rho v^2 R/\sigma$, where σ is the liquid-vapor surface tension, ρ the liquid-mass density, and R the bubble radius. At a critical Weber number $W_e \sim 3.37$, a gas bubble becomes unstable and grows without limits. Inserting the ^{4}He parameters in W_e one finds that the critical velocity for bubble explosion in liquid ^{4} is $v_c \sim 49$ m/s.[27]

II. METHOD

The high-velocity motion of e bubbles in liquid ^{4}He under the effect of a constant electric field has been recently studied for $P \geq 0$ within a time-dependent density-functional (DF) approach using a simplified, local DF.[28] One of the limitations of such approach is the impossibility of reproducing roton or vortex excitations in the superfluid. To overcome this shortcoming, we resort here to a zero temperature, more realistic nonlocal DF for ^{4}He.[29] An appealing feature of this functional, which is essential to perform accurate velocity-dependent DF calculations,[29,30] is that it reproduces quantitatively not only a number of static properties but also the observed excitations of bulk ^{4}He. Its use has allowed to explicitly show[31] that superfluid ^{4}He flowing under confinement at a velocity greater than the Landau critical velocity $v_c \sim 58$ m/s undergoes a phase transition from a spatially homogeneous state to a layered state characterized by a periodic-density modulation with a wavelength $\lambda = 2\pi\hbar/p_c \sim 3.58$ Å, where p_c is the roton momentum associated with the minimum in the bulk excitation spectrum.

Since a real time evolution of the electron motion based on such functional is computationally prohibitive, we restricted ourselves to the search of the stationary state where the e bubble moves with a constant velocity through the liquid ^{4}He. Our approach follows Refs. 8, 15, and 32. We refer the interested reader to these references for a thorough discussion of the method.

The total energy E of the electron-helium system under study is written as a functional of the electron wave function $\Phi(\mathbf{r})$ and the ^{4}He effective macroscopic wave function $\Psi(\mathbf{r})=\sqrt{\rho(\mathbf{r})}\exp[\imath S(\mathbf{r})]$, where $\rho(\mathbf{r})$ is the particle density and $\mathbf{v}(\mathbf{r})=\hbar\nabla S(\mathbf{r})/m_{\rm He}$ is the velocity field of the superfluid

$$E[\Psi,\Phi] = \frac{\hbar^2}{2m_{\rm He}}\int d\mathbf{r}|\nabla\Psi(\mathbf{r})|^2 + \int d\mathbf{r}\mathcal{E}(\rho,\mathbf{v}) + \frac{\hbar^2}{2m_e}\int d\mathbf{r}|\nabla\Phi(\mathbf{r})|^2 + \int d\mathbf{r}|\Phi|^2 V_{e\text{-He}}(\rho). \quad (1)$$

Inclusion of the electron-helium interaction $V_{e\text{-He}}$ is done as in Ref. 33. In this equation, $\mathcal{E}(\rho,\mathbf{v})$ represents the zero-temperature potential-energy DF described in Ref. 29, which has an explicit dependence on the local current-density field $j(\mathbf{r})=\rho(\mathbf{r})\mathbf{v}(\mathbf{r})$ through a term which accounts not only for the usual hydrodynamic current density but also for nonlocal "backflow" effects.

For a given P, we have solved the Euler-Lagrange equations which result from the variation with respect to Ψ^* and Φ^* of the zero temperature constrained grand-potential density $\tilde{\omega}(\Psi,\Phi)=\omega(\Psi,\Phi)-\varepsilon|\Phi|^2$, with

$$\omega(\Psi,\Phi) = \frac{\hbar^2}{2m_{\rm He}}|\nabla\Psi(\mathbf{r})|^2 + \mathcal{E}(\rho,\mathbf{v}) + \frac{\hbar^2}{2m_e}|\nabla\Phi|^2 + |\Phi|^2 V_{e\text{-He}}(\rho) - \mu\rho, \quad (2)$$

where μ is the helium chemical potential. The variation in the above functional yields two coupled equations

$$-\frac{\hbar^2}{2m_{\rm He}}\Delta\Psi + \left\{\frac{\delta\mathcal{E}}{\delta\rho} + |\Phi|^2\frac{\partial V_{e\text{-He}}(\rho)}{\partial\rho}\right\}\Psi \equiv \mathcal{H}\Psi = \mu\Psi, \quad (3)$$

$$-\frac{\hbar^2}{2m_e}\Delta\Phi + V_{e\text{-He}}(\rho)\Phi = \varepsilon\Phi, \quad (4)$$

where ε is the eigenvalue of the Schrödinger equation obeyed by the electron. We have self-consistently solved them using an imaginary-time method.

Since we are interested in stationary-state motions of the e bubble in ^{4}He, we minimize the above functional in the frame of reference moving with the e bubble, which we assume to move with a given constant velocity v along the x axis: the Hamiltonian $\mathcal{H}$ for the ^{4}He system [see Eq. (3)] thus acquires an additional term becoming

$$\mathcal{H}' = \mathcal{H} - v\hat{P}_x, \quad (5)$$

P_x being the ^{4}He total momentum component along the direction of motion.[34]

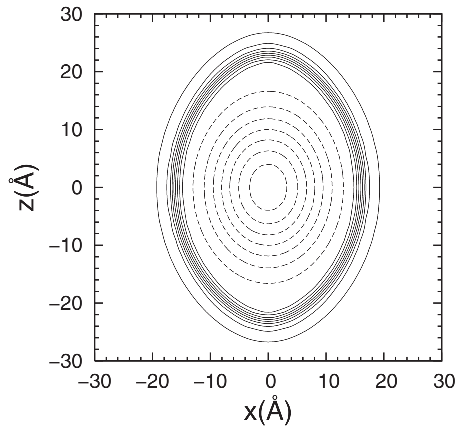

FIG. 1. Contour plot showing the e bubble stationary state at $P=0$ corresponding to $v=50.5$ m/s, i.e., just before the critical value $v_c=50.7$ m/s where a ring vortex is emitted. The equidensity lines for the ^{4}He density (solid lines) are plotted for values between $0.1\rho_b$ and $0.9\rho_b$, in steps of $0.1\rho_b$, $\rho_b=0.0218$ Å^{-3} being the ^{4}He bulk liquid density. The equidensity lines for the electron density (dashed lines) are plotted using nine lines between zero and the maximum value of $|\Phi|^2$.

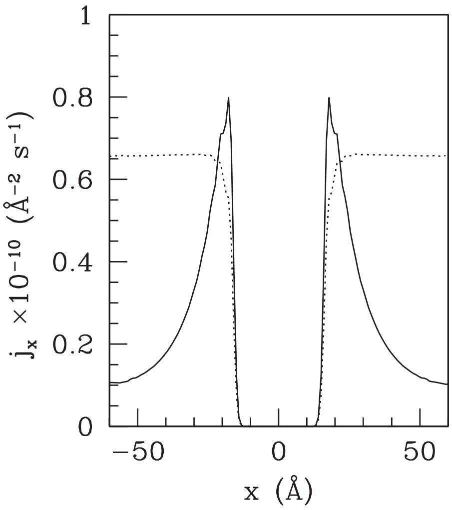

FIG. 3. Current density $j_x(\mathbf{r})=\rho(\mathbf{r})v_x(\mathbf{r})$ shown along the direction of motion. The dotted line shows (in arbitrary units) the ^{4}He density profile, whose asymptotic value is 0.0218 Å^{-3}.

III. RESULTS AND DISCUSSION

We have studied the stationary states representing an electron moving with a constant velocity through the liquid, both at $P=0$ and at negative pressures, down to -2 bar. One such state is shown in Fig. 1 by means of constant density contours. The e bubble appears squeezed along the direction of motion. As a consequence of this shape variation, the fluid current around it develops some vorticity around the bubble equator. This is shown in Fig. 2, where the calculated current-density field is shown for the same configuration.

We also show in Fig. 3 the current-density component j_x along a line parallel to the direction of motion, passing through the center of the bubble. The dotted line shows the actual ^{4}He density profile: it appears that the current reaches its maximum right in front of the bubble boundary while it decays to zero far from the bubble region. The velocity corresponding to the maximum of the current density shown in Fig. 3 coincides almost exactly with the value v imposed during the minimization of the energy functional.

We were able to compute such stationary states only up to a maximum velocity. In fact, as soon as the velocity exceeds a critical value $v_c=50.7$ m/s, during the minimization we observe the emission of vortex rings. The nascent ring appears spontaneously, girdling the bubble cavity along the equatorial line perpendicular to the direction of motion and then expands during the minimization procedure. One such ring, immediately after emission, is shown in Fig. 4. This process appears to be recurrent, i.e., once the emitted ring is removed, another one is emitted soon, and so on. We have checked that these ring are singly quantized, as expected.[30] The value found in our calculations for the critical velocity for vortex-ring emission is in agreement with the critical drift velocity of negative ions as measured at low pressures in Ref. 20, where a velocity slightly below 50 m/s can be extrapolated to $P=0$.

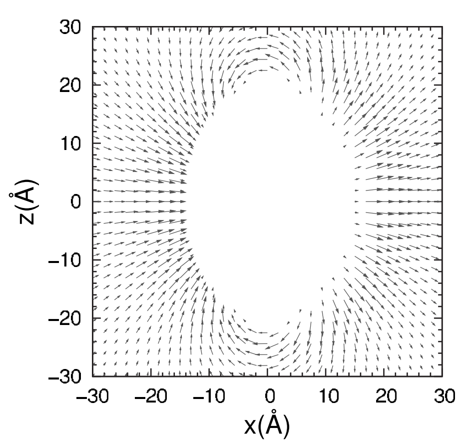

FIG. 2. Current-density $\mathbf{j}(\mathbf{r})=\rho(\mathbf{r})\mathbf{v}(\mathbf{r})$ map for the same configuration shown in Fig. 1. The length of the arrows is proportional to the value of $|\mathbf{j}|$. The maximum value shown in the figure corresponds to the peak in the following Fig. 3.

FIG. 4. (Color online) Surface isodensity plot showing a quantized ring emitted at $P=0$, just above v_c.

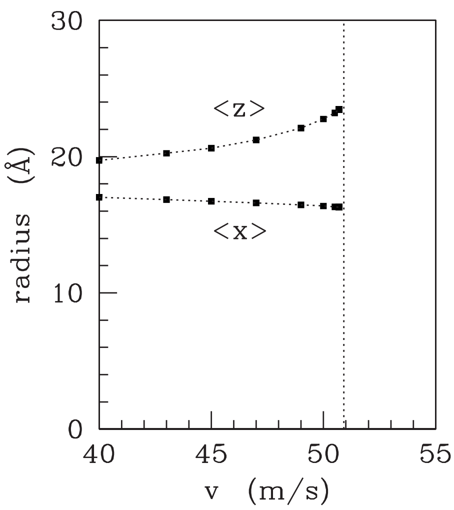

FIG. 5. Average longitudinal ($\langle x\rangle$) and transverse ($\langle z\rangle$) dimensions, with respect to the direction of motion, of the moving e bubble at $P=0$ for different values of the bubble velocity. The vertical dotted line shows the critical velocity for ring-vortex emission.

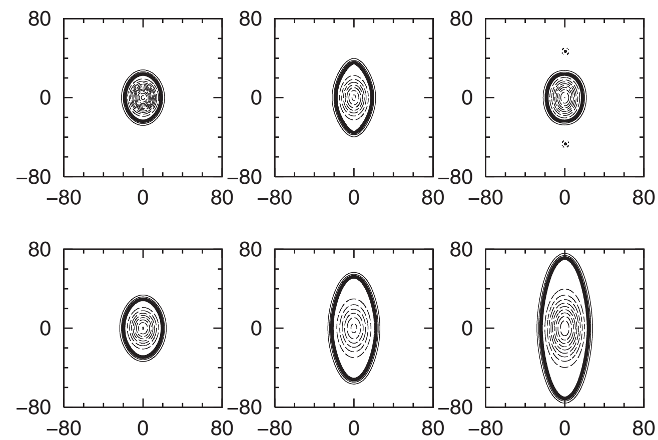

FIG. 6. Comparison between the two-decay mechanisms of a moving bubble at negative pressures, at velocities just above the critical ones. The upper panels are for $P=-1$ bar (ring emission) and the lower panels for $P=-1.3$ bar (bubble explosion). We show several helium equidensity lines (solid) spanning the surface region of the electron bubble, as well as several electron equiprobability density lines (dashed). A lens-shaped e bubble seems to be always associated with ring emission whereas a more rounded, ellipsoidal shape is kept instead during the explosion.

Figure 5 shows the calculated average dimensions along the direction of motion ($\langle x\rangle$) and perpendicular to it ($\langle z\rangle$) of the moving bubble at $P=0$, for different values of the bubble velocity.

The value of the critical velocity found for ring emission is rather robust: we have found essentially the same value using different unit cells/spatial meshes and different initial conditions at the beginning of the minimization process. It has been suggested that vortex nucleation by a moving impurity in liquid ^{4}He occurs when the velocity at the equator of the impurity reaches the local speed of sound.[35,36] This is not confirmed by our calculations. Indeed, the velocity field never exceeds the imposed value v anywhere around the bubble surface.

Our approach, being based on fully three-dimensional calculations, does not impose any *a priori* symmetry during the imaginary-time evolution. In spite of this, the ring emanates from the equatorial line rather that from any other place on the bubble surface, as assumed in the peeling model:[23] this is a consequence of the peculiar vorticity field arising around the equatorial line discussed above. We have indeed verified, by imprinting an off-centered ring vortex field on the e bubble in the initial configuration, that during the functional minimization at a given velocity, such ring shrinks to zero while eventually the bubble spontaneously produces a centered vortex ring, as discussed above.

The ring emission also occurs (albeit with different critical velocities) at negative pressures but only when P is not lower than ~ -1.2 bar. One such event is shown in the three upper panels of Fig. 6, where we plot a section of the ^{4}He +electron system during the imaginary-time evolution at $P=-1$ bar. The horizontal axis coincides with the direction of motion. As it tries to accommodate to the imposed value of the velocity, the e bubble becomes squeezed along the direction of motion, and suddenly it emits a vortex ring (the two tiny features above and below the bubble in the right upper panel of Fig. 6) while the bubble shrinks back to a more compact shape. At pressures below (roughly) -1.2 bar, however, the e bubble no longer emits rings but rather increases in size without limits during the imaginary-time evolution. One such explosion is shown for $P=-1.3$ bar in the three lower panels in Fig. 6. This process is reminiscent of the mechanical instability that should occur for gas bubbles, as discussed at the beginning, when the critical Weber number is reached.

The values of the critical velocities found as a function of the studied pressures are summarized in Fig. 7. The horizontal dotted line divides the phase diagram into an upper part, where ring emission occurs as the critical velocity is ex-

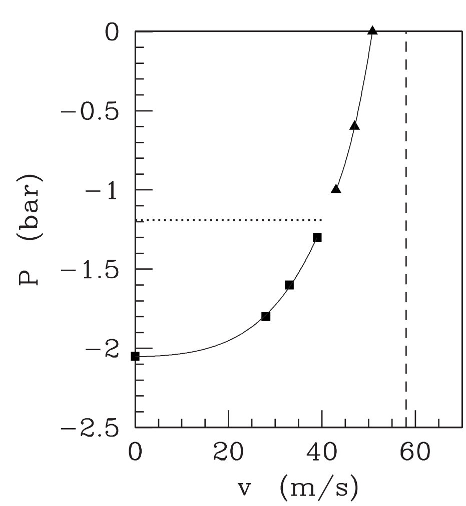

FIG. 7. Values of the critical velocities for ring emission (triangles) and bubble explosion (squares), as a function of pressure. The solid line has been drawn as a guide to the eyes. The vertical dashed line is the Landau critical velocity at $P=0$ and $T=0$. The horizontal dotted line separates these two regimes.

ceeded, and a lower part, where the bubble explodes as soon as the critical velocity is reached.

We use now these results to discuss the experimental findings of Ref. 14. At low temperatures, the cavitation events found by Ghosh and Maris are collected into three distinct classes. The first kind of events, occurring around $P=-2$ bar, are identified as low velocity, free e bubbles, whose theoretical description using DF theory is found to be in good agreement with the experiments.[13] Another type of e bubbles are found to explode at less negative pressures of about -1.8 bar. These events are qualitatively described by models in which the e bubble is attached to a vortex line.[15] The third type of events are found in the -1.5 to -1.7 bar pressure range, and occur only when a sufficiently high dc voltage, $V\sim 200$ V, is applied at very low temperatures ($T<1$ K), i.e., they are likely associated with the explosion of fast moving, high-mobility electron bubbles. Note that the above voltage, corresponding to a field of about 100 V/cm at the acoustic focus[37] in the experimental setup of Ref. 14, would result in a drift velocity of the observed e bubbles in the 20–30 m/s range, i.e., where, according to our calculations, the e bubble should explode when P is in the experimental range -1.5 to -1.7 bar.

It is thus tempting to associate the UEO of Ghosh and Maris with fast-moving free-electron bubbles exploding because of the mechanical instability described above.

This possibility was already considered in Ref. 14 but promptly discarded because on the one hand, these authors lacked of the detailed calculations presented here, and on the other hand, the observed cavitation probability was not sensibly increasing with the duration of the sound pulses used to locally generate negative pressures. Additional analysis[37] has shown that this experimental observation would not really exclude fast-moving bubbles as candidates for the UEO but only at the cost of assuming that the escape rate of electrons trapped on vortices is sufficiently high. In this case the number of UEO would be comparable to the total number of the observed events whereas experimentally this is not the case, as the number of UEO is about ten times smaller than the number of exploding normal e bubbles. At present, we have no convincing answer to this conundrum. However, we remark that a comparison of our theory with the experiments of Ref. 14 is complicated by the presence of ^{3}He impurities in the experimental ^{4}He sample, which are known to alter in a significant way the vortex-ring parameters of isotopically pure ^{4}He, thus making difficult to estimate the negative pressure at which a crossover from vortex nucleation to explosion occurs.

To conclude, we have shown that fast-moving electron bubbles in liquid ^{4}He explode in the pressure range where unidentified electron objects have been found to explode in cavitation experiments, thus constituting an alternative explanation for the observed UEO.

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