

Theoretical study of the dynamics of magnetization on the topological surface

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We investigate theoretically the dynamics of magnetization coupled to the surface Dirac fermions of a three-dimensional topological insulator by deriving the Landau-Lifshitz-Gilbert (LLG) equation in the presence of charge current. Both the inverse spin-galvanic effect and the Gilbert damping coefficient α are related to the two-dimensional diagonal conductivity σ_{xx} of the Dirac fermion, while the Berry phase of the ferromagnetic moment to the Hall conductivity σ_{xy} . The spin-transfer torque and the so-called β terms are shown to be negligibly small. Anomalous behaviors in various phenomena including the ferromagnetic resonance are predicted in terms of this LLG equation.

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Topological insulator (TI) provides a new state of matter topologically distinct from the conventional band insulator.¹ In particular, the edge channels or the surface states are described by Dirac fermions and protected by the band gap in the bulk states, and backward scattering is forbidden by the time-reversal symmetry. From the viewpoint of the spintronics, it offers a unique opportunity to pursue novel functions since the relativistic spin-orbit interaction plays an essential role there. Actually, several proposals have been made such as the quantized magnetoelectric effect,² giant spin rotation,³ magnetic properties of the surface state,⁴ magnetotransport phenomena,⁵ and superconducting proximity effect including Majorana fermions.⁶⁻⁸

Also, a recent study focuses on the inverse spin-galvanic effect in a TI/ferromagnet interface, predicting the current-induced magnetization reversal due to the Hall current on the TI.⁹ In Ref. 9, the Fermi energy is assumed to be in the gap of the Dirac dispersion opened by the exchange coupling. In this case, the quantized Hall liquid is realized, and there occurs no dissipation coming from the surface Dirac fermions.

However, in realistic systems, it is rather difficult to tune the Fermi energy in the gap since the proximity-induced exchange field is expected to be around 5–50 meV. Therefore, it is important to consider the generic case where the Fermi energy is at the finite density of states of Dirac fermions, where the diagonal conductivity is much larger than the transverse one, and the damping of the magnetization becomes appreciable. Related systems are semiconductors and metals with Rashba spin-orbit interaction, where the spin-galvanic effect and current-induced magnetization reversal have been predicted¹⁰ and experimentally observed.^{11,12} Compared with these systems where the Rashba coupling constant is a key parameter, the spin and momentum in TI are tightly related to each other corresponding to the strong-coupling limit of spin-orbit interaction, and hence the gigantic spin-galvanic effect is expected.

In this Rapid Communication, we study the dynamics of the magnetization coupled to the surface Dirac fermion of TI. Landau-Lifshitz-Gilbert (LLG) equation in the presence of charge current is derived microscopically, and (i) inverse

spin-galvanic effect, (ii) Gilbert damping coefficient α , (iii) the so-called β terms, and (iv) the correction to the Berry phase, are derived in a unified fashion. It is found that these are expressed by relatively small number of parameters, i.e., the velocity v_F , Fermi wave number k_F , exchange coupling M , and the transport lifetime τ of the Dirac fermions. It is also clarified that the terms related to the spatial gradient are negligibly small when the surface state is a good metal. With this LLG equation, we propose a ferromagnetic resonance (FMR) experiment, where modifications of the resonance frequency and Gilbert damping are predicted. Combined with the transport measurement of the Hall conductivity, FMR provide several tests of our theory. Throughout this Rapid Communication, we take the unit of $\hbar=1$.

Derivation of LLG equation. By attaching a ferromagnet on the TI as shown in Fig. 1, we can consider a topological surface state where conducting electrons interact with localized spins, S , through the exchange field,

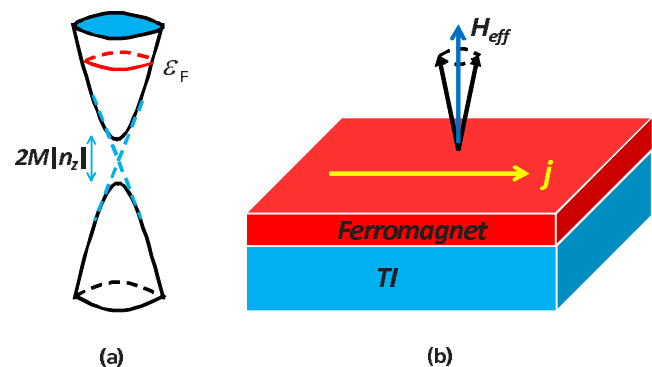


FIG. 1. (Color online) (a) Illustration of the Dirac dispersion on top of TI. The Fermi level ϵ_F is far above the surface gap opened by magnetization in the ferromagnetic layer. (b) Sketch of FMR experiment in the soft magnetic layer. The substrate in the figure is TI, which is capped by a layer of soft ferromagnet. The magnetization precesses around the external magnetic field H_{eff} .

$$H_{\text{ex}} = -M \int d\mathbf{r} n(\mathbf{r}) \cdot \hat{\boldsymbol{\sigma}}(\mathbf{r}). \quad (1)$$

Here, we set $\mathbf{S} = S\mathbf{n}$ with a unit vector \mathbf{n} pointing in the direction of spin, $\hat{\boldsymbol{\sigma}}(\mathbf{r}) = c^\dagger(\mathbf{r})\boldsymbol{\sigma}c(\mathbf{r})$ represents (twice) the electron-spin density, with $c^\dagger = (c_\uparrow^\dagger, c_\downarrow^\dagger)$ being electron creation operators, $\boldsymbol{\sigma}$ the Pauli spin-matrix vector, and M being the exchange-coupling energy. The total Hamiltonian of the system is given by $H_{\text{tot}} = H_S + H_{\text{el}} + H_{\text{ex}}$, where H_S and H_{el} are those for localized spins and conducting electrons, respectively.

The dynamics of magnetization can be described by the LLG equation,

$$\dot{\mathbf{n}} = \gamma_0 \mathbf{H}_{\text{eff}} \times \mathbf{n} + \alpha_0 \dot{\mathbf{n}} \times \mathbf{n} + \mathbf{t}'_{\text{el}}, \quad (2)$$

where $\gamma_0 \mathbf{H}_{\text{eff}}$ and α_0 are an effective field and a Gilbert damping constant, respectively, both coming from H_S . Effects of conducting electrons are contained in the spin torque,

$$\mathbf{t}'_{\text{el}}(\mathbf{r}) \equiv s_0 \mathbf{t}'_{\text{el}}(\mathbf{r}) = M \mathbf{n}(\mathbf{r}) \times \langle \hat{\boldsymbol{\sigma}}(\mathbf{r}) \rangle_{\text{ne}}, \quad (3)$$

which arises from H_{ex} . Here, $s_0 \equiv S/a^2$ is the localized spin per area a^2 . In the following, we thus calculate spin polarization of conducting electrons perpendicular to \mathbf{n} , $\langle \hat{\boldsymbol{\sigma}}_{\perp}(\mathbf{r}) \rangle_{\text{ne}}$, in such nonequilibrium states with current flow and spatially varying magnetization to derive the β term, or with time-dependent magnetization for Gilbert damping. Here and hereafter, $\langle \cdots \rangle_{\text{ne}}$ represents statistical average in such nonequilibrium states.

Following Refs. 13–15 we consider a small transverse fluctuation, $\mathbf{u} = (u^x, u^y, 0)$, $|\mathbf{u}| \ll 1$, around a uniformly magnetized state, $\mathbf{n} = \hat{z}$, such that $\mathbf{n} = \hat{z} + \mathbf{u}$. In the “unperturbed” state, $\mathbf{n} = \hat{z}$, the electrons are described by the Hamiltonian,

$$\mathcal{H}_0 = \sum_{\mathbf{k}, \sigma} c_{\mathbf{k}\sigma}^\dagger [v_F(k_y \sigma^x - k_x \sigma^y) - M \sigma^z - \varepsilon_F] c_{\mathbf{k}\sigma} + V_{\text{imp}}, \quad (4)$$

where V_{imp} is the impurity potential given by $V_{\text{imp}} = u \sum_i c^\dagger(\mathbf{R}_i) c(\mathbf{R}_i)$. We take a quenched average for the impurity positions \mathbf{R}_i . The electron damping rate is then given by $\gamma = 1/(2\tau) = \pi n_i u^2 v_F$ in the first Born approximation. Here, n_i is the concentration of impurities and $v_F = \varepsilon_F / (2\pi v_F^2)$ is the density of states at ε_F . We assume that $\gamma \ll v_F k_F = \sqrt{\varepsilon_F^2 - M^2}$, M , and calculate spin-transfer torque in the lowest nontrivial order.

In the presence of $\mathbf{u}(\mathbf{r}, t) = \mathbf{u}(\mathbf{q}, \omega) e^{i(\mathbf{q}\cdot\mathbf{r} - \omega t)}$, the conducting electrons feel a perturbation (note that $H_{\text{el}} + H_{\text{ex}} = \mathcal{H}_0 + \mathcal{H}_1$)

$$\mathcal{H}_1 = -M \sum_{\mathbf{k}\sigma} c_{\mathbf{k}+\mathbf{q}\sigma}^\dagger \boldsymbol{\sigma} c_{\mathbf{k}\sigma} \cdot \mathbf{u}(\mathbf{q}, \omega) e^{-i\omega t} \quad (5)$$

and acquires a transverse component

$$\langle \hat{\sigma}'_{\perp}(\mathbf{q}, \omega) \rangle_{\text{ne}} = M \chi_{\perp}^{\alpha\beta}(\mathbf{q}, \omega + i0) u^{\beta}(\mathbf{q}, \omega) \quad (6)$$

in the first order in \mathbf{u} in the momentum and frequency representation. Here, $\chi_{\perp}^{\alpha\beta}$ is the transverse spin susceptibility in a uniformly magnetized state with $\alpha, \beta = x, y$, and summing over β is implied.

Now, we study the ω -linear terms in the uniform ($\mathbf{q} = \mathbf{0}$) part of the transverse spin susceptibility, $\chi_{\perp}^{\alpha\beta}(\mathbf{q} = \mathbf{0}, \omega + i0)$.

We make the following transformation of the operator:

$$c = U \tilde{c} = \frac{1}{\sqrt{2\varepsilon(\varepsilon + M)}} \begin{pmatrix} v_F(k_y + ik_x) \\ \varepsilon + M \end{pmatrix} \tilde{c} \quad (7)$$

with $\varepsilon = \sqrt{(v_F k)^2 + M^2}$. Note $U^\dagger U = 1$, $U^\dagger \sigma^x U = v_F k_y / \varepsilon$, and $U^\dagger \sigma^y U = -v_F k_x / \varepsilon$. This transformation maps two component operator c into one component operator on the upper Dirac cone \tilde{c} . With this new operator, we calculate the transverse spin susceptibility in Matsubara form,

$$\begin{aligned} \chi_{\perp}^{\alpha\beta}(\mathbf{0}, i\omega_\lambda) &= \int_0^\beta d\tau e^{i\omega_\lambda \tau} \langle T_\tau \sigma^\alpha(\mathbf{0}, \tau) \sigma^\beta(\mathbf{0}, 0) \rangle \\ &= -T \sum_{\mathbf{k}, n} U^\dagger \sigma^\alpha U \tilde{G}(\mathbf{k}, i\varepsilon_n + i\omega_\lambda) U^\dagger \sigma^\beta U \tilde{G}(\mathbf{k}, i\varepsilon_n) \end{aligned} \quad (8)$$

with $\tilde{G}(\mathbf{k}, i\varepsilon_n) = [i\varepsilon_n - \varepsilon + \varepsilon_F + i\gamma \text{sgn}(\varepsilon_n)]^{-1}$. By symmetry consideration of the integrand in \mathbf{k} integral, we find $\chi_{\perp}^{\alpha\beta}(\mathbf{0}, i\omega_\lambda) \propto \delta_{\alpha\beta}$. After some calculations, we obtain the torque stemming from the time evolution,

$$\mathbf{t}_{\text{el}}^\alpha = M^2 \frac{i\omega}{2\pi} \frac{1}{2v_F^2} \left(\frac{v_F k_F}{\varepsilon_F} \right)^2 \varepsilon_F \boldsymbol{\tau} \times \mathbf{u} \quad (9)$$

$$= \frac{1}{2} \left(\frac{M v_F k_F}{\varepsilon_F} \right)^2 v_F \boldsymbol{\tau} \times \mathbf{n}. \quad (10)$$

This result fits the conventional Gilbert damping with

$$\alpha = \frac{1}{2} \left(\frac{M v_F k_F}{\varepsilon_F} \right)^2 v_F \tau \frac{a^2}{\hbar S}. \quad (11)$$

We next examine the case of finite current by applying a dc electric field \mathbf{E} and calculate a linear response of σ_{\perp}^α to \mathbf{E} , i.e., $\langle \sigma_{\perp}^\alpha(\mathbf{q}) \rangle_{\text{ne}} = \chi_{\alpha i}(\mathbf{q}) E_i$. First, it is clear that $\chi_{\alpha i}(\mathbf{q} = \mathbf{0}) = -\varepsilon_{i\alpha} \sigma_{xx} / (e v_F)$, where $\varepsilon_{i\alpha}$ and σ_{xx} are the antisymmetric tensor and diagonal conductivity, respectively, because electron's spin is “attached” to its momentum. This represents the inverse spin-galvanic effect, i.e., charge current induces magnetic moment. Since we assume that Fermi level is far away from the surface gap, $\sigma_{xx} \gg \sigma_{xy}$, where σ_{xy} is the Hall conductivity. The dominant term in χ is thus $\chi_{xy} \propto \sigma_{xx}$. This is quite different from the case studied in Ref. 9, where Fermi level lies inside the surface gap and therefore σ_{xx} is vanishing. Hence, the only contribution to the inverse spin-galvanic effect is $\chi_{xx} \propto \sigma_{xy}$, which is much smaller than the effect proposed in this Rapid Communication. Compared with the inverse spin-galvanic effect in Rashba system,^{10–12} this effect is much stronger since the small Rashba coupling constant, i.e., the small factor $\alpha_R k_F / E_F$ in Eq. (16) of Ref. 10, does not appear in the present case. Taking into account the realistic numbers with $\alpha = 10^{-11}$ eV m and $v_F = 3 \times 10^5$ m/s, one finds that the inverse spin-galvanic effect in the present system is ~ 50 times larger than that in Rashba systems.

The next leading-order terms of the expansion in u^β and q_j can be obtained by considering the four-point vertices¹³ as

$$\langle \hat{\sigma}_\perp^\alpha(\mathbf{q}) \rangle_{\text{ne}} = -eM \frac{\pi}{4} \frac{5i}{8\pi\epsilon_F^2} \epsilon_{ik}\epsilon_{jl} [\delta_{\alpha\beta}\delta_{kl} + \delta_{\alpha k}\delta_{\beta l} + \delta_{\alpha l}\delta_{\beta k}] q_j u^\beta E_i \quad (12)$$

$$= -eM \frac{5i}{32\epsilon_F^2} [\mathbf{q} \cdot \mathbf{E} u^\alpha - \mathbf{q} \cdot (\mathbf{u} \times \hat{z})(\mathbf{E} \times \hat{z})_\alpha + \mathbf{u} \cdot (\mathbf{E} \times \hat{z})(\mathbf{q} \times \hat{z})_\alpha]. \quad (13)$$

Therefore, the spin-torque stemming from the spatial gradient has the form,

$$\mathbf{t}_{\text{el}}^\beta = -\beta \frac{1}{2e} \{ \mathbf{n} \times (\mathbf{j} \cdot \nabla) \mathbf{n} - [\mathbf{j} - (\mathbf{j} \cdot \mathbf{n}) \hat{z}] \nabla \cdot (\mathbf{n} \times \hat{z}) + [\nabla - (\mathbf{n} \cdot \nabla) \hat{z}] \mathbf{n} \cdot (\mathbf{j} \times \hat{z}) \}, \quad (14)$$

where $\mathbf{j} = \sigma_C \mathbf{E}$ with charge current \mathbf{j} and conductivity $\sigma_C = \frac{e^2}{4\pi} \left(\frac{v_F k_F}{\epsilon_F} \right)^2 \epsilon_F \tau$, and

$$\beta = \frac{5\pi}{4\epsilon_F \tau} \left(\frac{M}{v_F k_F} \right)^2. \quad (15)$$

From Eq. (14), one can find the following: (i) the spin-transfer torque of the form $(\mathbf{j} \cdot \nabla) \mathbf{n}$ is absent since we consider the upper Dirac cone only. (ii) The β term has a form essentially different from that in the conventional one.^{13,16,17} Considering $v_F k_F \cong \epsilon_F$, we get $\alpha/\beta \sim (\epsilon_F \tau)^2$ from Eqs. (11) and (15). Therefore, the β terms are negligible for a good surface metal, i.e., $\epsilon_F \tau \gg 1$.

Up to now, we consider only one branch of the band where the Fermi energy is sitting. When we consider the two-band structure, i.e., the 2×2 matrix Hamiltonian $\mathcal{H} = v_F [(k_y - \frac{Mn_x}{v_F}) \sigma^x - (k_x + \frac{Mn_y}{v_F}) \sigma^y]$, we have the correction to

the Berry phase term. In analogy with the minimal coupling of electromagnetic field, $\mathbf{A} = -\frac{M}{e v_F} (-n_y, n_x)$ plays the same role as the $U(1)$ gauge. By integrating the fermions out, one can get a Chern-Simons term in terms of the magnetization $L_{CS} = \sigma_{xy} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$, where $\mu, \nu, \rho = t, x, y$. When the gradient of magnetization vanishes, it can be rewritten as

$$L_{CS} = \sigma_{xy} \left(\frac{M}{e v_F} \right)^2 (n_x \dot{n}_y - n_y \dot{n}_x). \quad (16)$$

This additional term can be interpreted as an additional Berry phase for the magnetization. In fact, as n_z remains constant in the present case, we have $[n_x, n_y] = i n_z$. Therefore, n_x and n_y become conjugate variables up to a factor, which naturally leads to a Berry phase: $n_x \dot{n}_y - n_y \dot{n}_x$. This term is exactly equivalent to the Chern-Simons term.

Including all the terms derived above, we finally arrive at a modified LLG equation,

$$\dot{\mathbf{n}} - 2\sigma_{xy} \left(\frac{M}{e v_F} \right)^2 \dot{\mathbf{n}} / (s_0 N) = \gamma_0 \mathbf{H}_{\text{eff}} \times \mathbf{n} + \left(\frac{M}{e v_F s_0 N} \right) [-\mathbf{j} + (\mathbf{n} \cdot \mathbf{j}) \hat{z}] + (\alpha_0 + \alpha/N) \dot{\mathbf{n}} \times \mathbf{n} + \mathbf{t}_{\text{el}}^\beta / (s_0 N), \quad (17)$$

where N is the number of ferromagnetic layers. Note that α , β , and Berry phase terms originate from the interplay between Dirac fermions and local magnetization which persists over a few layers of the ferromagnet. Therefore, the overall coefficients are divided by the number of ferromagnetic layers N .

Ferromagnetic resonance. Observing the small value of β , the spatial gradient of magnetization can be neglected for the time being. Only one uniform domain in the absence of current is taken into account for simplicity. Without loss of generality, assume that an external magnetic field is applied along z direction, and consider the magnetization precession around that field. $\dot{n}_z = 0$ is kept in the first-order approximation, namely, n_z is a constant in the time evolution. By inserting the ansatz $n_{x(y)}(t) = n_{x(y)} e^{-i\omega t}$ into the modified LLG equation, one obtains

$$\Re\omega = \frac{\xi}{\xi^2 + \eta^2} \omega_0, \quad \Im\omega = -\frac{\eta}{\xi^2 + \eta^2} \omega_0, \quad (18)$$

where $\eta = \alpha_0 + \alpha/N$, $\omega_0 = \gamma_0 H_{\text{eff}}$, and $\xi = 1 - 2\sigma_{xy} \left(\frac{M}{e v_F} \right)^2 / (s_0 N)$. Expanding up to the first order in σ_{xy} and η , one gets $\Re\omega = \omega_0 + 2\sigma_{xy} \left(\frac{M}{e v_F} \right)^2 \omega_0 / (s_0 N)$ and $\Im\omega = \eta \omega_0$. Therefore, the precession frequency acquires a shift proportional to σ_{xy} in the presence of interplay between Dirac fermions and the ferromagnetic layer. The relative shift of $\Re\omega$ is $2\sigma_{xy} \left(\frac{M}{e v_F} \right)^2 / (s_0 N) = \frac{1}{\pi s N} \frac{M}{\epsilon_F} \left(\frac{M a}{v_F} \right)^2 \sim \frac{1}{N} \left(\frac{M}{\epsilon_F} \right)^3$.¹⁸ By tuning the Fermi level, this shift can be accessible experimentally.

Meanwhile, the Gilbert damping constant α can be measured directly without referring to the theoretical expression in Eq. (11). One can investigate the ferromagnetic layer thickness dependence of FMR linewidth. While increasing the thickness N of ferromagnet, the Gilbert damping constant

stemming from the Dirac fermions decreases inversely proportional to the thickness. Taking into account the realistic estimation with $\varepsilon_F\tau \sim 100$ and $M/\varepsilon_F \sim 0.3$, one has $\alpha/s_0 \sim 1$, while $\alpha_0 \sim 0.001$ usually. Therefore, even for a hundred of layers of ferromagnet, the contribution from the proximity effect is still significant compared to the one coming from the ferromagnet itself. Observing that the imaginary part of resonance frequency in Eq. (18) is proportional to η , one may plot the relation between the FMR peak broadening, namely, $\mathcal{J}\omega$ and $1/N$. The broadening is a linear function of $1/N$ and approaches the value of the ferromagnet at large thickness limit. We can find the value of α from the slope of the plot.

On the other hand, the real part of FMR frequency provides rich physics as well. Since in the presence of additional Berry phase, the frequency shift is proportional to the Hall conductivity on the surface of TI, it leads to a new method to measure the Hall conductivity without four-terminal probe. In an ideal case when the Fermi level lies inside the surface gap, this quantity is quantized as $\sigma_{xy}^0 = \frac{e^2}{2h}$. However, in realistic case, Fermi level is away from the surface gap, and therefore the Hall conductivity is reduced to $\sigma_{xy} = \frac{e^2}{2h} \frac{Mn_z}{\varepsilon_F}$.¹⁸ As a result, the shift of resonance frequency is proportional to $n_z^2 \propto \cos^2 \theta$ and the FMR isotropy is broken. Here, θ is the angle between effective magnetic field and the normal to the surface of TI. One can perform an angle-resolved FMR measurement. The signal proportional to $\cos^2 \theta$ comes from the additional Berry phase.

Since parameters α and β depend on M and τ , it is quite important to measure these quantities directly. Molecular-beam epitaxy method can be applied to grow TI coated by a thin layer of soft ferromagnet. As is required in the above calculation, Fermi level of TI should lie inside the bulk band

gap. Also, the soft ferromagnet should be an insulator or a metal with proper work function. One may employ angular-resolved photoemission spectroscopy (ARPES) or scanning tunneling microscope techniques to measure the surface gap Δ opened by the ferromagnet, which is given by $\Delta = Mn_z$. As the easy axis n_z can be found experimentally, M can be fixed as well. On the other hand, the lifetime τ is indirectly determined by measuring the diagonal conductivity σ_{xx} via $\sigma_{xx} = \frac{e^2}{4\pi} \left(\frac{v_{Fk_F}}{\varepsilon_F}\right)^2 \varepsilon_F \tau$. Finally, Fermi surface can be determined by ARPES and all parameters in LLG equation Eq. (17) can be obtained.

In summary, we have investigated theoretically the dynamics of magnetization on the surface of a three-dimensional topological insulator. We have derived the Landau-Lifshitz-Gilbert equation in the presence of charge current, and analyzed the inverse spin-galvanic effect and ferromagnetic resonance predicting anomalous features of these phenomena.

One may expect that topological surface state is affected by the ferromagnet due to strain and the exchange coupling. However, topology of the system guarantees that the surface state at low energy is described by Dirac Hamiltonian. Thus, our results should be qualitatively unaltered even in the presence of these effects.

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