

Superconducting cavity bus for single nitrogen-vacancy defect centers in diamond

J. Twamley¹ and S. D. Barrett^{1,2}

¹Centre for Quantum Computer Technology, Macquarie University, Sydney, New South Wales 2109, Australia

²Blackett Laboratory and Institute for Mathematical Sciences, Imperial College London, Prince Consort Road, London SW7 2BZ, United Kingdom

(Received 8 April 2010; published 11 June 2010)

Circuit-QED has demonstrated very strong coupling between light and matter, and has the potential to engineer large quantum devices. Hybrid designs have been proposed which couple large ensembles of atomic and molecular systems to the superconducting resonator. We show that one can achieve an effective strong coupling between light and matter for much smaller ensembles (and even a single electronic spin), through the use of an interconnecting quantum system: in our case a persistent current qubit. Using this interconnect we show that one can effectively magnify the coupling strength between the light and matter by over five orders of magnitude $g \sim 7 \text{ Hz} \rightarrow 100 \text{ kHz}$ and enter a regime where a single nitrogen-vacancy (NV) electronic spin can shift the cavity resonance line by over ~ 20 linewidths. With such strong coupling between an individual electronic spin in an NV and the light in the resonator, one has the potential build devices where the associated NV nuclear spins can be strongly coupled over centimeters via the superconducting bus.

DOI: [10.1103/PhysRevB.81.241202](https://doi.org/10.1103/PhysRevB.81.241202)

PACS number(s): 85.25.Hv, 03.67.Lx, 42.50.Pq, 42.79.Gn

I. INTRODUCTION

Achieving strong coupling between light and matter plays a vital role in the study of strongly correlated quantum dynamics. When strong coupling is achieved the matter portions of the hybrid light-matter system may act as a quantum memory and this can play an essential role in a quantum repeater or a quantum computer architecture. If the matter systems have long decoherence times and can be easily initialized, controlled, and selectively coupled/decoupled to the light bus, then one has the potential for the design of a scalable quantum device. Strong coupling is achieved when the coupling strength between the light and matter exceeds their respective decay rates $g > \kappa, \gamma$ and this means that the associated vacuum splittings can be resolved in a spectroscopic experiment. Circuit-QED has demonstrated very strong coupling between individual microwave photons trapped in a superconducting coplanar resonator and nearby superconducting qubits.^{1,2} With the recent demonstration of a two-qubit quantum algorithm,³ circuit-QED has the potential to engineer larger quantum devices. A number of theoretical proposals have been recently advanced for a hybrid circuit-QED/atomic system where one couples the microwave photons trapped in the superconducting cavity to a nearby ensemble of atoms or molecules. Examples include coupling to polar molecules,^{4–6} neutral atoms,^{7,8} and Rydberg atoms,⁹ to the superconducting resonator. Though technically demanding this type of design has the advantage of scaling-up the light-matter coupling strength for an ensemble consisting of N atomic/molecular systems by a factor \sqrt{N} the individual system light-matter coupling strength. A more convenient approach would be to couple to an ensemble of “atomiclike” solid-state systems which have long decoherence times.^{10,11} In such ensemble approaches it is typically difficult to implement single qubit unitaries without mapping the collective excitations to simpler systems to manipulate.¹¹ Further the spin coherence times of ensembles possessing dipole-dipole long-range interactions also improves with decreasing spin

concentration,¹² leading one to consider smaller and smaller ensembles. One promising solid-state atomiclike system which can couple magnetically to a superconducting cavity is a nitrogen-vacancy defect in diamond.¹⁰ In this paper, rather than consider engineering a large $\sim 10^{12}$, ensemble of near identical NV defects whose combined coupling scales as $\sqrt{N}g_{\text{indiv}} \sim 10^6 g_{\text{indiv}}$, which can reach strong coupling when $g_{\text{indiv}} \sim \text{Hz}$, we show that one can magnify the coupling to a single NV by 5 orders of magnitude to achieve strong coupling with a single NV. The system of a single NV strongly coupled to the stripline is directly analogous to an ion trap: where a single ion’s internal atomic state can be strongly coupled to the ion’s motional mode confined in a harmonic trap.

II. COPLANAR WAVEGUIDE RESONATOR

We consider a coplanar waveguide resonator (CPW), and the magnetic coupling between such a resonator and a nearby magnetic spin system. The Hamiltonian for microwave photons in a CPW resonator is $\hat{H}_r = \hbar \omega_r (\hat{a}^\dagger \hat{a} + 1/2)$, and recent devices,¹³ have reported $\omega_r/2\pi \sim 6 \text{ GHz}$ with a quality factor $Q \sim 2.3 \times 10^5$, giving a cavity decay rate of $\kappa/2\pi \sim 26 \text{ kHz}$. The total equivalent inductance of these resonators near their resonant frequency is typically a few nano-Henry $L_r \sim 2 \text{ nH}$. We now estimate the size of the magnetic field generated by the vacuum fluctuations of the photons within the resonator. This will be used to estimate the size of the coupling directly to the NV when placed next to the central conductor of the resonator. The RMS current flowing through the resonator when the photon mode is in the ground state can be estimated to be $I_{\text{rms}} = \sqrt{\hbar \omega_r/2L_r}$. Assuming that in the superconducting state that the current in the central conductor flows in a thin strip at the surface we can estimate the magnetic field strength a distance d away to be

$$B_{0,\text{rms}}(d) = \mu_0 I_{\text{rms}}/2\pi d. \quad (1)$$

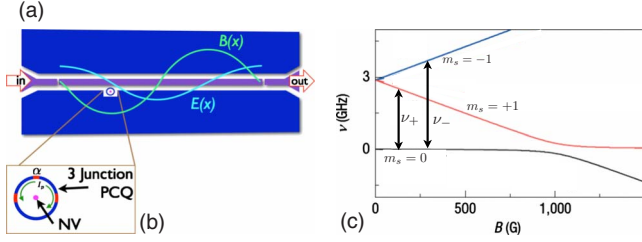


FIG. 1. (Color online) PCQ loop interconnect. (a) superconducting coplanar resonator with PCQ loop located at an antinode of $B(x)$ of the resonator. The PCQ loop encircles an individual magnetic spin system—in this case a nitrogen-vacancy defect in diamond. The PCQ loop couples via mutual inductance to the coplanar resonator and couples to the magnetic spin via the B field at the center of the loop generated by the persistent currents in the loop. (b) Detail of the PCQ made up of three Josephson junctions, two identical and the other smaller by a factor α , encircling the magnetic spin system coupled via the persistent circulating currents I_p . (c) Energy levels of ground state triplet 3A_2 , of the NV as a function of applied magnetic field.

III. COUPLING OF THE NV DIRECTLY TO THE COPLANAR RESONATOR

To estimate the size of the magnetic coupling between the electrons in the NV and a nearby CPW we take, for simplicity, the NV dipole axis to be along the \hat{z} direction and describe the Hamiltonian for the ground state triplet (spin 1), 3A_2 electronic system of the NV by

$$\hat{H}_{NV}/\hbar = g_e \beta_e B_z \hat{S}_z + D \left(\hat{S}_z^2 - \frac{2}{3} \right), \quad (2)$$

where in the first Zeeman term B_z is the z component of the magnetic field at the NV, \hat{S}_z is the z -spin 1 operator, $g_e = -2$, and $\beta_e/2\pi \sim 1.4 \times 10^4$ MHz/T. The second term is the so-called zero-field splitting (ZFS), term with $D/2\pi \sim 2870$ MHz for an NV. From Fig. 1(c), for $B_z \sim$ several gauss the selection rules $\Delta m_s = \pm 1$, hold and $\delta\nu_{\pm}/\delta B_z \sim \pm 28$ GHz/T. Let us consider now an NV placed a distance $d=50$ nm away from the central conductor of the CPW resonator where $B_{0,rms} \sim 2.5$ milligauss. It will couple magnetically via the Zeeman term in Eq. (2), and using Eq. (1), the size of this coupling will be $|\bar{g}|/2\pi \sim 2.5 \times 10^{-7} \times 28$ GHz ~ 7 kHz, while for $d \sim 5$ μm , we have $|\bar{g}|/2\pi \sim 70$ Hz. These couplings are far below the linewidth of the best CPW resonators fabricated to date and thus the direct magnetic coupling to a single NV so close to the resonator will not be resolved. If we could achieve strong coupling between the single electronic spin and the cavity electromagnetic field, then it would be easier to control the coupled light-matter system. The apparently tiny size of the CPW-single NV coupling strength gives one little hope that strong coupling could be possible. In the following sections we describe how this *is possible*, by encircling the magnetic spin system with a persistent current qubit (PCQ).

IV. PCQ

A PCQ,¹⁴ is formed when a superconducting loop is interrupted by three Josephson junctions (Fig. 1), where all junctions are identical except that one is smaller by a factor $\alpha > 0.5$, than the other two. When the loop is biased by a magnetic flux which is close to half a flux quantum, the device is an effective two level system,¹⁵ with the qubit made up of two countercirculating persistent currents. The effective two level system (or PCQ), is described by the Hamiltonian $\hat{H}_q = \frac{\hbar}{2}(\epsilon \hat{\sigma}_z + \Delta \hat{\sigma}_x)$, with $\epsilon = \frac{2I_p}{\hbar}(\Phi_x - \Phi_0/2)$, and Φ_x is the external magnetic flux through the loop. Going to an eigenbasis we can write $\hat{H}_q = \frac{\hbar}{2}\omega_0 \hat{\sigma}_z$, with $\omega_0 = \sqrt{\Delta^2 + \epsilon^2}$. Recent work,¹³ has $\alpha=0.7$, and $I_p \sim 450$ nA, while $\Delta/2\pi \sim 5.2$ GHz. Persistent currents as large as $I_p \sim 800$ nA have been observed,¹⁶ while the area of PCQ loops are typically $A \sim 1-2$ μm^2 .

V. COUPLING OF THE NV INDIRECTLY TO THE COPLANAR RESONATOR VIA THE PCQ

The strong coupling of a coplanar resonator to a PCQ has been recently demonstrated.¹⁷ To estimate the coupling strength we note $\hat{H}_{CPW-PCQ} = -\hat{\mu} \cdot \hat{B}$, where $\hat{\mu}$ is the magnetic dipole of the PCQ induced by the circulating persistent currents of magnitude I_p , $|\hat{\mu}| = I_p A$, and where A is the area of the PCQ loop. From Eq. (1), for a PCQ a distance d from the central CPW conductor we find

$$|g| \sim \frac{I_p A \mu_0 I_{rms}}{\hbar \pi d} = \frac{I_p \mu_0}{\hbar} \left(\frac{r_{loop}^2}{d} \right) \sqrt{\frac{\hbar \omega_r}{2L_r}}, \quad (3)$$

where we have assumed a circular PCQ loop of radius r_{loop} . Taking $r_{loop} = 0.8$ μm , $I_p = 600$ nA, and $L_r = 2$ nH, we get $|g|/2\pi \sim 28.7$ MHz. The Hamiltonian describing this coupling in the case where $\omega_0 \sim \omega_r$, can be written as $\hat{H}_{CPW-PCQ} = \hbar g (\hat{a}^\dagger \hat{\sigma}^- + \hat{a} \hat{\sigma}^+)$, where \hat{a} destroys a photon in the CPW while $\hat{\sigma}^-$ excites the qubit states of the PCQ.

We now consider placing the circular PCQ loop around an NV so that the NV is at the center of the loop. As has been noted previously,¹⁴ the persistent currents present in a PCQ generate sizable changes in magnetic flux within the loop $\Delta\Phi \sim 10^{-3}\Phi_0$. Typically one surrounds the PCQ with a sensitive superconducting quantum interference device (SQUID) detector to measure the PCQ qubit via these small flux changes. In what follows we use the PCQ (without the SQUID), as a magnetic interconnect, coupling the NV through to the CPW resonator. We first note that the PCQ must be nominally biased by a magnetic flux $\Phi = \Phi_0/2$ to operate in the regime where the states corresponding to counter circulating currents are degenerate. This yields a static $B_s \sim \Phi_0 A/2$, magnetic field at the center of the loop and $B_s \sim 5$ gauss for $A = 2$ μm^2 . We now estimate the small changes in magnetic field at the center of the loop generated by the persistent countercirculating currents, and from these, the small changes in the NV transition frequencies as these alter the Zeeman term in the NV's Hamiltonian. The magnetic field at the center of the loop due to the persistent currents I_p , is $\vec{B}_{I_p} = \pm 2\mu_0 A I_p / (4\pi r_{loop}^3) \hat{z}$. Further, exact

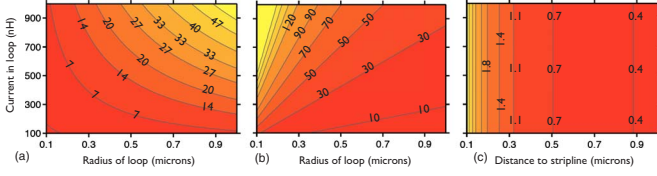


FIG. 2. (Color online) Comparison of coupling strengths: (a) strength of coupling between the persistent current qubit and the coplanar resonator as a function of r_{loop} and I_p , i.e., $g/2\pi$, in megahertz where the loop center is placed a distance $d=r_{loop}$ away from the central electrode; (b) coupling between the NV and PCQ, i.e., $\eta/2\pi$, in kHz; and (c) direct coupling between the NV and CPW in kHz.

placement of the NV at the center of the loop is not required as the induced magnetic field varies slowly there. This magnetic field leads to a small shift in the NV's microwave transition frequencies ($m_s \rightarrow \pm 1$), of $\eta/4\pi \equiv \Delta\nu \sim \pm |\vec{B}_I| \times 28 \text{ GHz/T}$ and for small loops and large currents $\eta/2\pi \sim 50\text{--}100 \text{ kHz}$ [see Fig. 2(b)]. Obviously as one reduces r_{loop} while retaining relatively large persistent currents I_p , one can increase η . Through this small change in magnetic field the PCQ qubit state can thus couple to the NV through the Zeeman term and we can now write the full NV Hamiltonian with the coupling to the PCQ as

$$\hat{H}_{\text{NV-PCQ}} = \frac{1}{2} \hbar \eta \hat{\sigma}_z \hat{S}_z + \hbar g_e \beta_e B_s \hat{S}_z + \hbar D \left(\hat{S}_z^2 - \frac{2}{3} \mathbb{I} \right), \quad (4)$$

where $\hat{\sigma}_z$, the PCQ Pauli z operator, couples directly to the NV triplet \hat{S}_z operator.

VI. FULL HAMILTONIAN

Using the above we are now able to describe the Hamiltonian of the coupled CPW-PCQ-NV system as

$$\begin{aligned} \hat{H} = & \hbar \omega_r \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \hbar \frac{\omega_0}{2} \hat{\sigma}_z + \hbar g_e \beta_e B_s \hat{S}_z + \text{ZFS} \\ & + \hbar g (\hat{a}^\dagger \hat{\sigma}^- + \hat{a} \hat{\sigma}^+) + \hbar \frac{\eta}{2} \hat{\sigma}_z \hat{S}_z + \hbar \zeta (e^{-i\omega t} \hat{a}^\dagger + e^{i\omega t} \hat{a}), \end{aligned} \quad (5)$$

where ZFS is zero-field splitting (second line), of Eq. (4), and where we have included a term which drives the CPW resonator at rate ζ . Driving the cavity resonantly, $\omega = \omega_r$, we can move to an interaction picture defined by the first line in Eq. (6), with $\omega_0 \sim \omega_r$, to find

$$\hat{H}_I = \hbar \frac{\delta}{2} \hat{\sigma}_z + \hbar \zeta (\hat{a} + \hat{a}^\dagger) + \hbar g (\hat{a}^\dagger \hat{\sigma}^- + \hat{a} \hat{\sigma}^+) + \hbar \frac{\eta}{2} \hat{\sigma}_z \hat{S}_z + H_{\text{decay}}, \quad (6)$$

where the detuning between the PCQ and CPW resonator is $\delta = \omega_0 - \omega_r$, and where H_{decay} , (which we model more specifically below), denotes decay and dephasing from the cavity, PCQ and NV.

From the above analysis it is clear that as one reduces the size of the PCQ loop the coupling to the CPW decreases

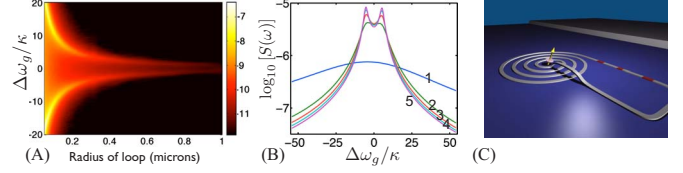


FIG. 3. (Color online) (a) *Splitting of the cavity spectrum due to the NV*: steady state power spectrum of the cavity $S(\omega)$, as a function of the PCQ loop radius. We consider only one of the vacuum Rabi peaks for $\delta=0$, centered at $\omega = \omega_g \equiv g$, and plot $\log_{10}[S(\Delta\omega_g)]$, where $\Delta\omega_g \equiv \omega - \omega_g$. We choose $\zeta = 2\kappa$, $I_p = 800 \text{ nA}$, $T_{1\text{NV}} = 4 \text{ ms}$, $T_{2\text{NV}} = 600 \mu\text{s}$, $\kappa/2\pi = 26 \text{ kHz}$, $\omega_0 = \omega_r = 2\pi \times 6 \text{ GHz}$, and $E_p = 2\kappa$. We omit the dephasing terms in Eq. (7), corresponding to the case when $T_{2\text{PCQ}} = 2T_{1\text{PCQ}}$. (b) *Dependence of the NV splitting of the PCQ vacuum Rabi line on the decoherence rates*: we plot $\log_{10}[S(\omega)]$ with the $I_p = 800 \text{ nA}$ and $r_{loop} = 0.2 \mu\text{m}$ and set $T_{1\text{PCQ}} = T_{2\text{PCQ}} = \tau = [0.5, 5.0, 10.0, 15.0, 20.0] \mu\text{s}$ (curves 1–5). We see that we would require $\tau > 5 \mu\text{s}$ to begin to resolve the splitting; (c) *multiturn PCQ interconnect*: by creating an n -looped spiral inductor incorporating the three Josephson junction PCQ one can amplify the PCQ coupling strengths to the resonator (in the distance), and the NV (diamond with arrow) n times.

while the coupling to the NV increases. In Fig. 2, we plot the dependence of the couplings as a function of loop radius and persistent current. From this we obtain the central result of this paper: that if one can fabricate PCQ loops with $r_{loop} \sim 0.1 \mu\text{m}$ (or smaller), and $I_p \sim 800 \text{ nA}$ (or larger), then the couplings $[g_{\text{CPW-PCQ}}, \eta, g_{\text{NV-CPW}}]/2\pi \sim [5 \text{ MHz}, 280 \text{ kHz}, 4 \text{ kHz}]$, while $\kappa/2\pi \sim 26 \text{ kHz}$.¹³ From Fig. 3(A), at $r_{loop} \sim 0.1 \mu\text{m}$, we observe a splitting of the cavity spectrum by ~ 20 cavity linewidths due to the NV coupling. This indicates that the NV-PCQ coupling will be resolvable through the spectroscopy of the CPW and that the NV will be effectively strongly coupled through the PCQ interconnect into the stripline resonator.

To examine how this coupling alters when we include realistic decay models, we write the full phenomenological quantum Master equation $\dot{\hat{\rho}} = \hat{L}\hat{\rho} = -i[\hat{H}_I, \hat{\rho}] + \bar{\mathcal{L}}\hat{\rho}$, where

$$\bar{\mathcal{L}}\hat{\rho} = \sum_{j=1}^5 \left[\hat{C}_j \hat{\rho} \hat{C}_j^\dagger - \frac{1}{2} \{ \hat{C}_j^\dagger \hat{C}_j, \hat{\rho} \} \right] \quad (7)$$

and $\hat{C}_j = \{ \sqrt{\kappa} \hat{a}, \sqrt{\gamma_{\text{PCQ}}} \hat{\sigma}^+, \sqrt{\gamma_{\text{NV}}} \hat{S}^+, \sqrt{\gamma_{\phi\text{PCQ}}} \hat{\sigma}_z, \sqrt{\gamma_{\phi\text{NV}}} \hat{S}_z \}$, where we have damping of the CPW resonator at rate κ , decay of the PCQ/NV qubits $\gamma_{\text{PCQ/NV}}/2\pi = 1/T_{1\text{PCQ/NV}}$, and their associated dephasing times $\gamma_{\phi\text{PCQ/NV}}/2\pi = 1/T_{\phi\text{PCQ/NV}} = 1/T_{2\text{PCQ/NV}} - 1/2T_{1\text{PCQ/NV}}$. We compute the power spectrum of the cavity under the small driving ζ , from

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega\tau} \langle \hat{a}^\dagger(\tau+t) \hat{a}(t) \rangle d\tau, \quad (8)$$

where we use the quantum regression theorem $\langle \hat{a}^\dagger(\tau+t) \hat{a}(t) \rangle = \text{Tr}[\hat{a}^\dagger e^{\mathcal{L}\tau} \hat{a} \hat{\rho}_{ss}]$, where $\hat{\rho}_{ss}$ is the steady state of the Master Eq. (7). With just the CPW coupled to the PCQ we expect to see a very large vacuum Rabi splitting and these peaks will be further slightly split due to the interaction of the PCQ with the NV. Flux qubits fashioned to date suffer

from relatively large decay and dephasing times $T_{1\text{PCQ}} = 10T_{2\text{PCQ}} = 20 \mu\text{s}$. However these times might be increased by engineering the devices in a more symmetric layout as proposed by¹⁸ and with this in mind we take the following decoherence parameters for our simulations: $\{\kappa/2\pi, T_{1\text{PCQ}}, T_{1\text{NV}}, T_{2\text{PCQ}}, T_{2\text{NV}}\} = [26 \text{ kHz}, 20 \mu\text{s}, 4000 \mu\text{s}, 2 \mu\text{s}, 600 \mu\text{s}]$. We make a few comments about these values: at low temperatures, $T_{1\text{NV}}$ due to spin lattice relaxation is very long (see Fig. 2 in Ref. 19), but its precise value is not significant for this paper as long as $T_{1\text{NV}}$ is long compared with the PCQ and cavity decay times and for computational efficiency we take it to be 4 ms. Without using active refocusing techniques, to achieve $T_{2\text{NV}}^* \sim 600 \mu\text{s}$ is challenging and currently the best result so far has reached $T_{2\text{NV}}^* \sim 20 \mu\text{s}$.¹² To reach longer $T_{2\text{NV}}^*$ times one must apply refocusing schemes such as the Uhrig scheme,^{20,21} to the NV on time scales much faster than the effective coupling strength of the NV to the CPW [as seen in Fig. 3(A) to be $\leq 10\kappa \sim 120 \text{ kHz}$]. Although one can apply MW pulses of length $\sim 10 \text{ ns}$ and thus execute Uhrig-type schemes, these refocusing schemes may also decouple the NV from the PCQ and a detailed study to optimize the refocusing so as to preserve the desired coupling to the PCQ may be required. In Fig. 3(A), we plot $S(\omega)(r_{\text{loop}})$, and see that with no dephasing we begin to observe very large splittings due to the NV for $r \leq 1 \mu\text{m}$ (comparable with those seen in Ref. 2). However when we include dephasing effects we observe that the NV splitting in the resonator spectrum is quite sensitive [see Fig. 3(B)].

VII. MULTITURN INTERCONNECTS

In the previous section we showed that one can amplify the magnetic coupling of the NV by encompassing the NV

with a PCQ loop. The coupling between the NV and PCQ increases with decreasing loop radius but this also decreases the coupling between the PCQ and the resonator. There may also be technical difficulties in fabricating very small PCQ structures. To circumvent this one can consider multiturn PCQ loops [see Fig. 3(C)], where the circular loop of the PCQ winds multiple times around the NV, thus scaling up the resulting B field induced by the PCQ circulating currents I_p , and thus scaling up the strength of the PCQ-NV coupling, and also the PCQ-CPW coupling. Such a structure may require a free air bridge and the increased sensitivity of the multiloop may come at the expense of shorter dephasing times.

VIII. SUMMARY

Circuit-QED has already demonstrated strong coupling between solid-state qubits and a superconducting bus and this heralds a route toward the future construction of large scale quantum devices. Although there have been proposals for the strong coupling of ensembles to cavities we present a completely new technique to achieve strong coupling of single atomic systems to superconducting cavities, which is reminiscent to trapped ions or cavity-QED systems which operate with true individual atomic systems. Through our PCQ interconnect one has the potential to strongly couple individual, long lived, electronic or perhaps, nuclear, spins into the superconducting bus. This will allow one to use such systems in quantum devices for information processing or metrology as long lived quantum memories, or to deterministically entangle individual atomic solid-state systems over centimeter length scales, or, in the case of an individual NV, optically readout and reset the quantum state of the system. Recently, a related work appeared which considers coupling an ensemble of NV defects to a flux qubit.²²

¹A. Blais, R. S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, *Phys. Rev. A* **69**, 062320 (2004).

²A. Wallraff, D. Schuster, A. Blais, and L. Frunzio, *Nature (London)* **431**, 162 (2004).

³L. DiCarlo *et al.*, *Nature (London)* **460**, 240 (2009).

⁴A. André, D. Demille, J. Doyle, and M. Lukin, *Nat. Phys.* **2**, 636 (2006).

⁵P. Rabl, D. DeMille, J. M. Doyle, M. D. Lukin, R. J. Schoelkopf, and P. Zoller, *Phys. Rev. Lett.* **97**, 033003 (2006).

⁶K. Tordrup and K. Molmer, *Phys. Rev. A* **77**, 020301(R) (2008).

⁷J. Verdú, H. Zoubi, C. Koller, J. Majer, H. Ritsch, and J. Schmiedmayer, *Phys. Rev. Lett.* **103**, 043603 (2009).

⁸D. Petrosyan and M. Fleischhauer, *Phys. Rev. Lett.* **100**, 170501 (2008).

⁹D. Petrosyan, G. Bensky, G. Kurizki, I. Mazets, J. Majer, and J. Schmiedmayer, *Phys. Rev. A* **79**, 040304(R) (2009).

¹⁰A. Imamoglu, *Phys. Rev. Lett.* **102**, 083602 (2009).

¹¹J. H. Wesenberg, A. Ardavan, G. A. D. Briggs, J. J. L. Morton, R. J. Schoelkopf, D. I. Schuster, and K. Molmer, *Phys. Rev. Lett.* **103**, 070502 (2009).

¹²G. Balasubramanian *et al.*, *Nature Mater.* **8**, 383 (2009).

¹³T. Niemczyk, F. Deppe, M. Mariani, E. P. Menzel, E. Hoffmann, G. Wild, L. Eggenstein, A. Marx, and R. Gross, *Supercond. Sci. Technol.* **22**, 034009 (2009).

¹⁴T. P. Orlando, J. E. Mooij, L. Tian, C. H. van der Wal, L. Levitov, S. Lloyd, and J. J. Mazo, *Phys. Rev. B* **60**, 15398 (1999).

¹⁵J. E. Mooij, *Science* **285**, 1036 (1999).

¹⁶F. G. Paauw, A. Fedorov, C. J. P. M. Harmans, and J. E. Mooij, *Phys. Rev. Lett.* **102**, 090501 (2009).

¹⁷A. A. Abdumalikov, O. Astafiev, Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, *Phys. Rev. B* **78**, 180502(R) (2008).

¹⁸M. Steffen, F. Brito, D. Divincenzo, S. Kumar, and M. Ketchen, *New J. Phys.* **11**, 033030 (2009).

¹⁹S. Takahashi, R. Hanson, J. van Tol, M. S. Sherwin, and D. D. Awschalom, *Phys. Rev. Lett.* **101**, 047601 (2008).

²⁰G. S. Uhrig, *Phys. Rev. Lett.* **98**, 100504 (2007).

²¹M. J. Biercuk, H. Uys, A. P. VanDevender, N. Shiga, W. M. Itano, and J. J. Bollinger, *Nature (London)* **458**, 996 (2009).

²²D. Marcos, M. Wubs, J. Taylor, R. Aguado, M. Lukin, and A. Sorensen, [arXiv:1001.4048](https://arxiv.org/abs/1001.4048) (unpublished).