PHYSICAL REVIEW B **81**, 241102(R) (2010)

Reversed Doppler effect in double negative metamaterials

Sam Hyeon Lee,[1] Choon Mahn Park,[2] Yong Mun Seo,[3] and Chul Koo Kim[1]
[1]*Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea*
[2]*AEE Center, Anyang University, Anyang 430-714, Korea*
[3]*Department of Physics, Myongji University, Yongin 449-728, Korea*
(Received 13 April 2010; revised manuscript received 1 June 2010; published 14 June 2010)

Doppler shifts in double negative metamaterials have never been observed. This Rapid Communication presents experimental results on Doppler effect in a double negative acoustic metamaterial. We observed that frequency was downshifted when the source was approaching and upshifted when receding. Notably, while in ordinary media wavelengths corresponding to downshifted frequencies are longer, we demonstrate that in double negative metamaterials wavelengths increase as the frequencies increase. Consequently even though the frequencies were downshifted in front of the moving source, the wavelengths became shorter.

DOI: 10.1103/PhysRevB.81.241102 PACS number(s): 43.28.Py, 43.40.+s, 43.90.+v

Doppler effect is an important wave characteristics which is used in a wide range of technologies regarding distant sensing of moving objects such as radar speedometer, wind velocity sensing, blood-flow measurement, and astronomic researches. Metamaterials with simultaneously negative constitutive parameters (double negative, DNG) have recently been constructed from artificial unit-cell structures of sizes smaller than the wavelengths.[1–11] Waves in DNG media exhibit new set of characteristics which brought expectations for a new horizon of powerful applications which includes superlensing, subwavelength cavity resonator, and cloaking. Much attention has been paid to reversal of Doppler effect in the metamaterials,[1,5,12] yet so far, to the best of our knowledge, no experimental demonstration has been reported. Recently, reversed Doppler effects were observed in photonic crystals, phononic crystals, and transmission lines.[12–18] However, these systems are not double negative metamaterials, and physical origins of these results are fundamentally different from that of the Doppler effects in DNG metamaterials.

In this Rapid Communication we present observation of reversed Doppler effect in a DNG acoustic metamaterial, using a moving source. The frequency was downshifted when the source was approaching and upshifted when receding. The frequency shift was not linear with the source frequency. Also, while in ordinary media wavelengths corresponding to downshifted frequencies are longer, we demonstrate that in double negative metamaterials wavelengths increase as the frequencies increase. Even though the frequency shifts were reversed, the wavelength shifts were not reversed, and the wave in front was compressed and the wave in the rear was expanded in the DNG metamaterial, in the same fashion as in conventional media.

The acoustic DNG metamaterial used for the present measurement is based on a periodic array of interspaced tight membranes and side holes as shown schematically in Fig. 1(a). The tight membranes are responsible for the negative density,[10] and the side holes generate negative modulus.[11] Present structure, a composite of the two elements, is identical to that reported in Ref. 9; this metamaterial was double negative in the frequency range below 450 Hz, single negative (SNG) from 450 to 735 Hz, and double positive (DPS) above 735 Hz. As emphasized in Ref. 9, our system was not based on resonant elements which inevitably limit spectral width into a narrow range around the resonance frequency. Another example of DNG materials based on nonresonant-type building blocks is the electrical transmission line consisting of array of series inductors and shunt capacitors. Such transmission lines and our system theoretically do not have lower bound for DNG range, i.e., the negative index starts from zero frequency up to the beginning of the stop band. This was reflected in an unprecedented fractional bandwidth of 47% we experimentally achieved in Ref. 9, even though

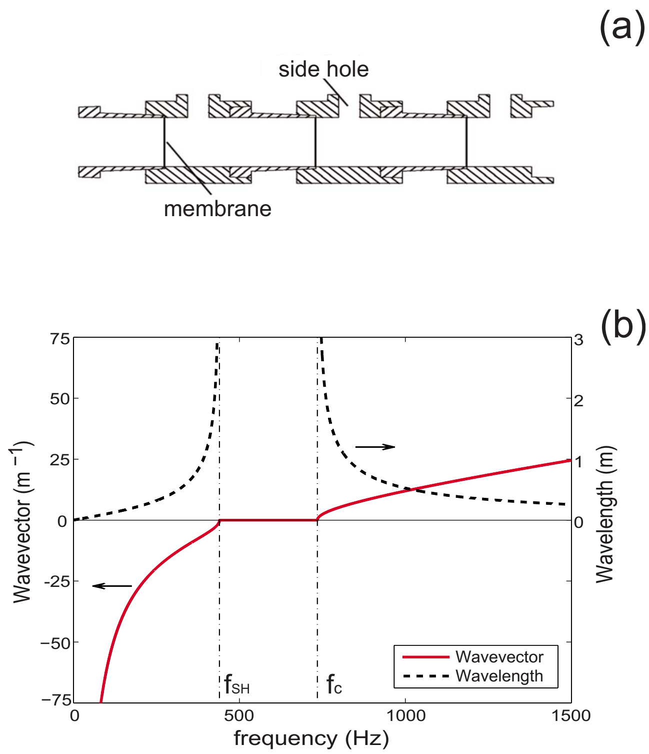

FIG. 1. (Color online) (a) Structure of the present DNG metamaterial consisting of interspaced membranes and side holes. This structure exhibits negative phase velocity in the frequency range below 440 Hz. (b) Dispersion curves from Eq. (1); wave vector (solid line) and wavelength (dashed line) as functions of frequency.

the lowest frequency data were not included due to weak signal problem. For the present Doppler measurements, we also could not collect data for frequencies below about 300 Hz—for the same reason. However, our data are sufficient as far as demonstration of reversed Doppler shift is concerned. The frequency dependence of the wave vector is given by,[9]

$$k(\omega) = s\frac{\omega}{v_0}\sqrt{(1-\omega_{SH}^2/\omega^2)(1-\omega_c^2/\omega^2)}, \qquad (1)$$

where $v_0=\sqrt{B/\rho'}\approx 320$ m/s is the speed of sound in the metamaterial at high-frequency limit with B and ρ' being the modulus and the average density of the fluid in the tube. The sign function $s=s(\omega)$ has the values -1 in the DNG frequency range and $+1$ for the DPS range. The critical frequencies are $f_{SH}=440$ Hz and $f_c=735$ Hz$(f=\omega/2\pi)$. The dispersion in Eq. (1) is shown in Fig. 1(b), where it can be seen that the wave vector is negative in the frequency range below f_{SH}.

Wavelength is the distance between adjacent wavecrests, and therefore is positive even when the wave vector is negative. Therefore, the wavelength λ can be expressed as $\lambda=2\pi/|k|=s(2\pi/k)$, which is plotted as a function of frequency in Fig. 1(b). Clearly, the slope of the wavelength curve is positive in the DNG frequency range, which means that the wavelength increases as the frequency increases. This may appear counterintuitive since we are accustomed to the ordinary media in which the wavelength decreases as the frequency increases. Note that in the DPS frequency range of our metamaterial, the slope is negative. However, positive wavelength slope occurs not only in our system but also in all DNG media, electromagnetic and acoustic media included. We prove this statement in the last part of this Rapid Communication. We note that reversal of the Doppler effect is a direct consequence of negative wave vector, regardless of whether wavelength slope is positive or negative. The slope of the wavelength curve is related to whether the wavelengths in front of a moving source are compressed or expanded.

In order to measure Doppler effect, both the sound source and the detector should, in principle, be placed inside of the metamaterial. The membranes inside the metamaterial, however, become a serious obstacle because anything moving inside the tube would puncture the membranes. This difficulty was overcome with the method schematically shown in Fig. 2(a). A sound source was made to move above the side holes so that the acoustic energy was coupled into the tube through the holes. The size of the moving source spans over several unit cells so that coupling into the tube was fairly continuous. Since the array of the side holes is a discrete structure, it is important to see if our method is equivalent to the case where the source is moving inside along the tube. We first consider the imaginary case that the source is moving inside the tube moving through the membranes without damaging them. Because each unit cell functions as a lumped element, the effect of moving source in the tube is to inject acoustic energy into one cell for a given period of time and next cell for the next period. The source motion within a cell makes ordinary Doppler shifts in the fluid, which creates

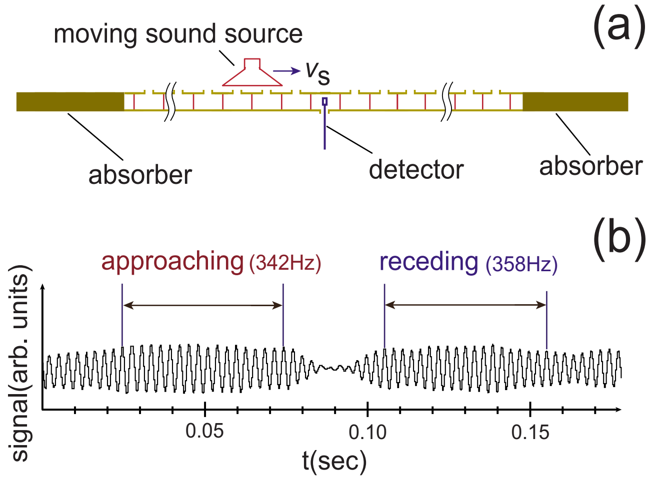

FIG. 2. (Color online) (a) Experimental setup for Doppler effect. (b) Recorded signal from the stationary detector in the metamaterial. In the case of the approaching source, the detected frequency was 342 Hz, which was 8 Hz downshifted from the source frequency. For the case of receding source, the detected frequency was 358 Hz.

many different frequencies. Since the source functions to excite vibration of the cell, the direction of the sound from the source does not matter, so that the cell is excited with the average frequency, which is the original frequency of the source. Essentially the same thing happens in our arrangement: When the source is moving outside just above the holes, the acoustic energy is injected into the cells through the side holes, from one cell to the next. Therefore, in case the unit-cell size is much smaller than the wavelengths, there is no difference between our method and the case of moving source inside the tube.

A typical signal from this setup is shown in Fig. 2(b), as recorded from the stationary detector in the metamaterial, with a 350 Hz source moving at a speed of 5 m/s. For the frequency of 350 Hz, phase velocity was negative, -230 m/s. The amplitude dip in the middle of the detected signal was the result of the opposite direction of the side hole at the position of the detector and was used as a landmark showing the moment the source passed the position of the detector. The signal before the dip represents Doppler shift for an approaching source, and that after the dip corresponds to a receding source. When we connected the signal into a loud speaker, we heard the sound changing from low to high pitch as the sound source passed the detector. The pattern is reversed compared to our daily experience of Doppler effects on the street. In the case of the approaching source, the detected frequency was 342 Hz, which was 8 Hz downshifted from the source frequency. In plain air, the shifted frequency would be 355 Hz, upshifted by 5 Hz. The magnitudes of the shifts, 8 Hz and 5 Hz for the metamaterial and the air, respectively, are inversely proportional to the wave speeds, 230 and 340 m/s, within the experimental error. For the case of receding source, the detected frequency was 358 Hz.

Doppler-shift formula for DNG materials is given by[1,5]

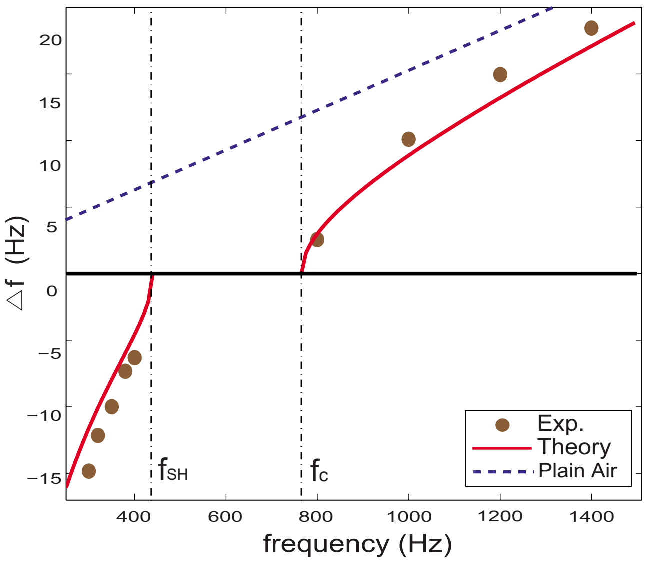

FIG. 3. (Color online) Theoretical curve from Eq. (3) agrees with the experimental data. Dashed line is the Doppler shifts for plain air, which is positive and nondispersive for all frequencies.

$$\Delta\omega = \omega' - \omega = \frac{v_S}{v_{ph}}\omega, \tag{2}$$

where v_S is the velocity of the approaching source. The shift is negative for the frequencies below ω_{SH} because the phase velocity, $v_{ph}=\omega/k$, is negative. The shift as a function of source frequency can be obtained from Eqs. (1) and (2),

$$\Delta\omega = s\frac{v_S}{v_0}\omega\sqrt{(1-\omega_{SH}^2/\omega^2)(1-\omega_c^2/\omega^2)}. \tag{3}$$

This is shown in Fig. 3 as the theoretical curve with $v_S = 5$ m/s. Clearly in the DNG frequency range, the frequency shift $\Delta\omega$ is negative, and Doppler shift is reversed. In the SNG range, $f_{SH} \leq f \leq f_c$, Doppler shift cannot be defined since the waves are evanescent. In the DPS region above 735 Hz, both the wave vector and the shift are positive, yielding positive Doppler shift.

The experimental data shown in Fig. 3 were obtained using an approaching source moving at 5 m/s speed. Positive and negative values represent upshift and downshift, respectively. The Doppler shift for plain air is also shown with the dotted line for comparison. In air, the Doppler shift is positive and proportional to the source frequency. In the metamaterial, the shifts both in the DNG range and in the DPS range are highly dispersive. Doppler shift data agreed well with the theoretical curves, and were all negative in the DNG frequency range, and all positive for the DPS frequencies.

It is desired to verify the negative shifts by a control experiment with a DPS medium. The measurement from the same experimental setup with the source moving outside should give positive Doppler shifts for DPS media. A natural trial to provide the required DPS medium would be to fill the tube with plain air without any structure inside. However, this does not work because the presence of the side holes makes the air in the tube to become a metamaterial with negative modulus.[11] Fortunately, a way around this difficulty was found in the present system itself, because our system becomes DPS for the frequencies higher than f_c. Therefore, nonreversed Doppler results in the DPS frequency range provide the control for the reverse Doppler shifts in the DNG range. Indeed, our Doppler data for the frequencies higher than f_c were positive and approached asymptotically the line given by the velocity of the metamaterial at high frequencies. Therefore, it is evident that the present reversed Doppler effect is the proper experimental result and not an artifact of our experimental setup.

In all DNG materials including electromagnetic and acoustic media, the wavelengths increase as the frequencies increase. This intriguing phenomenon can be proved as follows. Backward propagation means that the direction of the wave vector k is opposite to the group velocity $d\omega/dk$. In a DNG band of any metamaterial k is negative, and thus the slope $d\omega/dk$ is positive: If $d\omega/dk$ is positive, so is $dk/d\omega$, which is the slope of the wave vector curve as a function of frequency. Therefore, the slope of the wavelength curve given by

$$d\lambda/d\omega = -s(2\pi/k^2)(dk/d\omega) \tag{4}$$

is positive for DNG frequency range, where $s=-1$. Since we derive Eq. (4) with general double negative conditions only, the slopes of wavelength curves for all materials in the DNG band as functions of the frequency are positive, inverted from the "negative slopes" of the ordinary media. The positive slope means that the wavelength increases as the frequency increases. Because of this, downshift of the frequency in front of the source results in compression of the wavelength. As a consequence, the wavelength change pattern is not reversed in DNG metamaterials and remain the same fashion as in DPS media despite of reversal of the frequency shifts. For example, in case of a 350 Hz source moving with 5 m/s speed, it can be calculated from Eqs. (1) and (3) that the wavelengths in front and in rear of the source are 0.6 m and 0.7 m, respectively. In all negative refractive index materials, when a source is moving, the wavelengths in front is shorter than those in rear.

In conclusion, we measured Doppler shifts in the present acoustic metamaterial. We observed that the Doppler shifts were reversed in the DNG material. Additionally, we demonstrated that the distribution of the wavecrests in the space around the source in any DNG medium looks qualitatively the same as that in a conventional media: the waves in front of the source are compressed and those in the rear are expanded. The results presented in this Rapid Communication will provide useful basis for future applications of DNG media.

The research was partially supported by the National Research Foundation of Korea (Grant No. NRF F01-2009-000-10028-0).

[1] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[2] J. B. Pendry, A. J. Holden, W. J. Stewart, and I. Youngs, Phys. Rev. Lett. **76**, 4773 (1996).
[3] J. B. Pendry, A. J. Holden, D. J. Robin, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).
[4] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[5] C. Caloz and T. Itoh, *Electromagnetic Metamaterials—Transmission Line Theory and Microwave Applications* (Wiley, New York, 2006) and references therein.
[6] N. Engheta, IEEE Antennas Wireless Propag. Lett. **1**, 10 (2002).
[7] J. Li and C. T. Chan, Phys. Rev. E **70**, 055602(R) (2004).
[8] N. Fang *et al.*, Nature Mater. **5**, 452 (2006).
[9] S. H. Lee, C. M. Park, Y. M. Seo, Z. G. Wang, and C. K. Kim, Phys. Rev. Lett. **104**, 054301 (2010).
[10] S. H. Lee, C. M. Park, Y. M. Seo, Z. G. Wang, and C. K. Kim, Phys. Lett. A **373**, 4464 (2009).
[11] S. H. Lee, C. M. Park, Y. M. Seo, Z. G. Wang, and C. K. Kim, J. Phys.: Condens. Matter **21**, 175704 (2009).
[12] K. M. K. H. Leong, A. Lai, and T. Itoh, Microwave Opt. Technol. Lett. **48**, 545 (2006).
[13] N. Seddon and T. Bearpark, Science **302**, 1537 (2003).
[14] E. J. Reed, M. Soljacic, M. Ibanescu, and J. D. Joannopoulos, Science **305**, 778b (2004).
[15] E. J. Reed, M. Soljacic, and J. D. Joannopoulos, Phys. Rev. Lett. **91**, 133901 (2003).
[16] A. B. Kozyrev and D. W. van der Weide, Phys. Rev. Lett. **94**, 203902 (2005).
[17] E. J. Reed, M. Soljacic, and J. D. Joannopoulos, Phys. Rev. Lett. **96**, 069402 (2006).
[18] X. Hu, Z. Hang, J. Li, J. Zi, and C. T. Chan, Phys. Rev. E **73**, 015602(R) (2006).