PHYSICAL REVIEW B **81**, 220507(R) (2010)

Critical behavior of steady quantum turbulence generated by oscillating structures in superfluid ^{4}He

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(Received 10 May 2010; published 29 June 2010)

We report the critical behavior of steady quantum turbulence in superfluid ^{4}He. By using a thin vibrating wire with no bridge vortices in the superfluid, we find that lifetime of turbulent state for a given driving force reveals an exponential distribution. The mean lifetime estimated from the distribution decreases exponentially but greatly below a critical injection power. We estimate the vortex line density in the turbulent state and find that this critical behavior arises when the interdistance between vortex lines becomes as large as the turbulent region.

DOI: 10.1103/PhysRevB.81.220507 PACS number(s): 67.25.dk, 47.27.Cn, 05.70.Ln

Superfluid turbulence has attracted renewed experimental and theoretical interest as a result of recent investigations.[1] Turbulence in superfluid ^{4}He at very low temperatures consists only of a tangle of quantized vortex lines that defines superfluid flow. Hence, the energy flux in superfluid turbulence is described by the motions of the vortex lines. The energy of flows circulating vortex cores cascades from large vortex loops to smaller loops by reconnection (Richardson cascade); Kelvin waves with scales smaller than the vortex line spacing form along vortex lines and transfer energy from large wavelengths to smaller wavelengths (Kelvin wave cascade).[2–4] Consequently, the turbulence energy cascades from large scales to small scales, eventually dissipating at high wave numbers. This process can be observed in the decay of the density of vortex lines in superfluid helium[5,6] even at very low temperatures where the normal-fluid component is almost absent.

Motions of vortex lines are also manifested in the continuous generation of turbulence by oscillating structures in superfluid ^{4}He. Experimental studies using oscillating spheres,[7] wires,[8,9] grids,[10] and tuning forks[11,12] indicate that superfluid turbulence can be generated at oscillating velocities above a critical velocity of about 50 mm/s. Vortex lines form bridges between a structure and its surrounding boundaries and they are shaken by the oscillation, developing into turbulence at velocities exceeding the critical velocity.[13,14] In a previous study, we found that a vibrating wire in the absence of bridge vortices also generates turbulence after vortex rings are applied from a vortex ring generator to the wire.[15] In a turbulent state, the vibrating wire continues to generate vortex lines even when further vortex rings are not applied. At sufficiently high driving forces, the generation seems to continue indefinitely. The wire velocity remains constant during the generation of vortex lines. However, the generation will stop suddenly when the driving force is reduced. After that, the vibrating wire cannot generate turbulence even at high velocities. This behavior is in marked contrast to the responses of other oscillating structures with bridge vortices, with respect to intermittent switching between turbulent flow and laminar flow or, more precisely, potential flow.[16,17] A vibrating wire with no bridge vortices determines the lifetime of the turbulent state in the turbulent-to-laminar transition,[18] enabling the vortex dynamics in a turbulent state at the low-driving force limit to be studied.

In this Rapid Communication, we report the turbulence transition and the critical behavior of steady turbulence generated by a thin vibrating wire in superfluid ^{4}He. Thin oscillating structures can be used to define turbulence in restricted geometry at low driving forces, enabling the vortex dynamics at the transition to be studied. We find that the critical behavior arises when the interdistance between vortex lines becomes as large as the turbulent region. We also report that the fluctuation of vortices causes the lifetimes of turbulent states at the transition to have an exponential distribution.

Even at very low temperatures, oscillating structures can usually generate turbulence in superfluid ^{4}He.[7,8] In many cases, vortex lines remain attached to heterogeneous structures of an oscillator or form bridges between an oscillator and surrounding boundaries, causing the generation of turbulence.[13,14] This makes it difficult to study the turbulence transition caused by the motion of vortex lines in a turbulent flow. Recently, however, we managed to produce oscillating structures that effectively have no vortex lines attached in superfluid ^{4}He by using thin vibrating wires with smooth surfaces and slow filling of a container with liquid helium at low temperature.[13,19] In the absence of bridge vortices, a vibrating wire can generate turbulence after vortex rings are applied to the wire,[15] enabling the vortex dynamics and the energy flux in a turbulent state to be studied without the confusion caused by bridge vortices.

In this experiment, we used two vibrating wires (A) with and (B) without bridge vortices as shown in Fig. 1(a). These wires were the same as those used in previous studies.[15,20] They were made from a thin superconducting NbTi wire with a 3 μm diameter fabricated from a commercial multifilament superconducting wire with dies. This fabrication technique produces wires with relatively smooth surfaces.[19] The wires were formed into a semicircle, the two arms of which were attached to columns mounted on a copper plate. These wires were located in a small chamber with a 0.1-mm-diameter pinhole. The wires and the chamber were placed in a cell and a magnetic field of 25 mT was applied. A heat exchanger made of sintered silver powder was mounted at the bottom of the cell to cool the helium. A RuO_2 thermometer was mounted on the cell wall. The resonance frequencies in vacuum of the vibrating wires (A) near and (B) far from

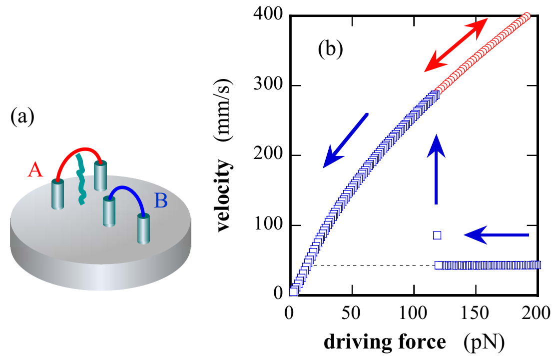

FIG. 1. (Color online) (a) Schematic drawing of vibrating wires (A) with bridge vortices and (B) without vortices. Wire A was used as a generator of vortex rings. (b) Peak velocity of vibrating wire B in superfluid ^{4}He at 30 mK for a potential-flow state (red circles) and for a down sweep of driving force after the transition to turbulence (blue squares). The broken line is extrapolated from the velocity in the turbulent state.

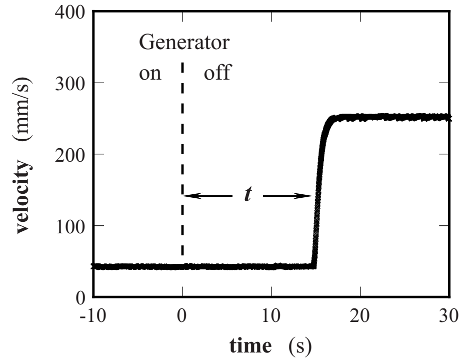

FIG. 2. Time series of the velocity of vibrating wire B for a driving force of 100 pN after stopping the generator. The time is measured from when the generator was stopped. The arrow indicates the lifetime t of the turbulent state.

the pinhole were 1030 Hz and 1590 Hz, respectively. After filling the cell with superfluid ^{4}He at a temperature below 100 mK over a period of 48 h, we found that there were bridge vortex lines attached to wire A near the pinhole but that wire B was effectively free of bridge vortices.[20]

The above conditions are useful for studying turbulence transitions generated by a vibrating wire with no bridge vortices.[18,21] The responses of vibrating wire B, which was effectively free of vortices, are reproducible for both up and down sweeps of the driving force in a laminar-flow state (more precisely, a potential-flow state). After entering a turbulent state at a high driving force as mentioned below, the vibrating wire maintains low velocity during the turbulent state on reducing the driving force and its velocity increases suddenly at the transition from turbulent to potential flow, as shown in Fig. 1(b).

To investigate the responses at the transition, we performed the following experiment. First, we applied vortex rings from vibrating wire A to vibrating wire B to generate turbulence for a given driving force. Second, we stopped applying the vortex rings. However, turbulence continued to be generated by vibrating wire B for a certain time; this indicates the lifetime of the turbulent state, as shown in Fig. 2. The velocity of the vibrating wire does not change during the turbulent state. At sufficiently high driving forces, the generation seems to continue indefinitely while the transition occurs from a turbulent state to a potential state at a low driving force.

We repeatedly measured the lifetime at a temperature of 40 mK with a fixed driving force and found that it is distributed over a wide range (0.2–64 s) at a driving force of 100 pN, as shown in Fig. 3(a). The transition from turbulent flow to potential flow seems to occur randomly. We estimate the time dependence of the transition probability for the turbulent state having a period longer than a specified time, and find that the probability has an exponential distribution, as shown in Fig. 3(b); this demonstrates that it has a memoryless property. This distribution is consistent with that observed in studies of intermittent switching between turbulent flow and potential flow for a sphere.[16] The lifetime observed here seems to reflect the same transition mechanism involved in switching between turbulent flow to potential flow. Switching in the opposite direction is caused by bridge vortex lines attached to the sphere because we did not observe switching in the responses of a vibrating wire with no bridge vortices, whereas we did observe it for a wire with bridge vortices.[13] The probability at low lifetimes differs slightly in each study. It has an exponential distribution over the whole range for a wire with no bridge vortices, but it does not have an exponential distribution for a sphere at low lifetimes, suggesting that the bridge vortices affect the lifetime. A vibrating wire with no bridge vortices is suitable for studying the turbulence transition.

To study the transition mechanism, we estimated the mean lifetime τ found by fitting the data with the function $\exp(-t/\tau)$, where t is the lifetime. The mean lifetime τ is

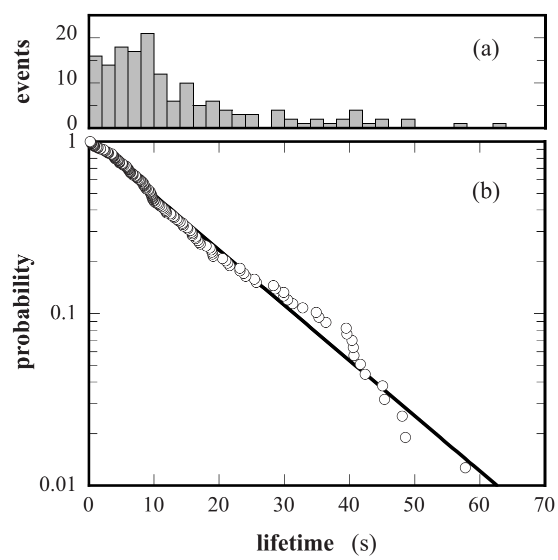

FIG. 3. Distribution of lifetime of turbulent state measured at a driving force of 100 pN: (a) histogram and (b) cumulative distribution, which shows the time dependence of the transition probability for the turbulent state for a period exceeding the specified time. The probability is well fitted by an exponential function (see text).

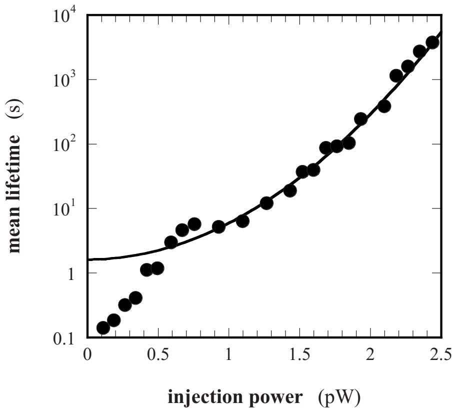

FIG. 4. Mean lifetime as a function of power injected into the turbulence region. The data above a power of 0.9 pW are well fitted by Eq. (1) (solid curve). Below this power, however, the mean lifetime decreases greatly with decreasing power.

13.4 s for a driving force of 100 pN, as shown in Fig. 3(b). We estimated the mean lifetimes for other driving forces and plotted them as a function of power injected into the turbulence region in Fig. 4. Here, the injection power is equal to the total power minus the power dissipated in the wire and the power lost in the scattering of thermally excited phonons. Above an injection power of 0.9 pW, the mean lifetimes are well fitted by the function

$$\tau = \tau_0 \exp\left(\frac{P^2}{P_0^2}\right), \tag{1}$$

where P is the injection power and τ_0 and P_0 are the fitting parameters. We estimated τ_0 and P_0 to be 1.6 s and 0.88 pW, respectively. This result is consistent with that of a previous study for an oscillating sphere,[16] suggesting that the lifetime is due to fluctuations in the vortex lines in the turbulent region. Intermittent turbulence transition has been also observed in pipe flow of classical fluids in a limited range of Reynolds numbers.[22] It is characterized by an exponential distribution of lifetimes and the mean lifetime increases superexponentially with Reynolds number, similar to the behaviors seen in the quantum-turbulence transition. There might be common features between the turbulence transitions of classical and quantum fluids.[23]

The density of the vortex lines decreases with decreasing injection power, causing the fluctuations to increase and thereby shortening the lifetime. Below an injection power of 0.9 pW, however, the mean lifetime decreases greatly with decreasing power. It is notable that the power of 0.9 pW is equal to the fitting parameter P_0=0.88 pW. This is an observation of the critical behavior of the transition from turbulent flow to potential flow at the critical power P_0. Since vortex lines in the path of the wire vibration are essential for sustaining turbulence generation,[15] it is plausible that the vortex line density is too low in the path below the critical power, i.e., the vortex line spacing of the tangle becomes larger than the dimension of the swept path.

In order to examine the critical behavior of the transition, we discuss the energy flux in the turbulent region. In the turbulent state, the velocity and the resonance frequency of the vibrating wire are constant for a fixed driving force until turbulence ceases to be generated. This reveals that the vortex line density in the wire path remains constant during turbulence generation. Simulations of vortex loops attached to an oscillating object[15,24] suggest that the motions of the wire create a vortex tangle by stretching vortex loops in the path while vortex loops cascade to smaller loops by reconnection, namely, a Richardson cascade, and annihilate to a dissipation regime at high Kelvin wave numbers or sufficiently small vortex rings can escape from the wire path. Consequently, the energy flux from vortex creation to dissipation maintains a constant vortex line density in the path. It is plausible that the constant density is associated with the energy transfer from the Richardson cascade to the Kelvin-wave cascade.[25] The energy flux ε per unit mass is given by

$$\varepsilon = a\kappa^3 L^2, \tag{2}$$

where κ and L are the circulation quantum and the vortex line density, respectively. The parameter a depends on models[3,4] but nevertheless is on the order of unity. The turbulent region is expected to be limited to the thickness and the peak-to-peak amplitude of the wire at low driving forces. We assume a=1 and estimate the vortex line spacing $\ell = L^{-1/2}$ to be 7 μm from the critical injection power P_0=0.88 pW, which is close to the peak-to-peak amplitude 9 μm of the wire at the critical power. This result indicates that the critical behavior arises due to the limited dimensions of the turbulent region. The vibrating amplitude is larger than the wire thickness of 3 μm; however, the critical behavior seems to be insensitive to the wire thickness. This can be explained because the turbulent region parallel to the wire axis is sufficiently long and vortex loops are able to spread in this direction. However, the thickness should affect the critical behavior at large wire-vibration amplitudes. We discuss this in another paper[26] with respect to the frequency dependence of the critical behavior of the turbulent state.

In summary, we report the transition to steady quantum turbulence generated by a thin vibrating wire with smooth surfaces. Since the turbulence is restricted in the path of the wire, the fluctuation of vortex lines is reflected in the turbulence transition. We discuss energy flux in the turbulent state and find that the critical behavior arises at the interdistance between vortex lines reaching the size of the turbulent region. Thus, quantum turbulence can be used to investigate the turbulence transition with respect to the energy flux governed by vortices in the turbulent state.

The authors are very grateful to M. Tsubota, M. Kobayashi, S. Fujiyama, and R. Hänninen for theoretical discussions and to W. Schoepe, A. I. Golov, V. B. Eltsov, and M. Krusius for stimulating discussions. This research was supported by a Grant-in-Aid for Scientific Research on Priority Areas (Grant No. 17071008) from the Ministry of Education, Culture, Sports, Science, and Technology of Japan.

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[1] W. F. Vinen and R. J. Donnelly, Phys. Today **60**(4), 43 (2007).

[2] M. Tsubota and M. Kobayashi, in *Progress in Low Temperature Physics*, edited by W. P. Halperin and M. Tsubota (North-Holland, Amsterdam, 2008), Vol. 16, p. 1.

[3] V. S. L'vov, S. V. Nazarenko, and O. Rudenko, Phys. Rev. B **76**, 024520 (2007).

[4] E. Kozik and B. Svistunov, Phys. Rev. B **77**, 060502(R) (2008).

[5] S. R. Stalp, L. Skrbek, and R. J. Donnelly, Phys. Rev. Lett. **82**, 4831 (1999).

[6] P. M. Walmsley, A. I. Golov, H. E. Hall, A. A. Levchenko, and W. F. Vinen, Phys. Rev. Lett. **99**, 265302 (2007).

[7] J. Jäger, B. Schuderer, and W. Schoepe, Phys. Rev. Lett. **74**, 566 (1995).

[8] H. Yano, N. Hashimoto, A. Handa, M. Nakagawa, K. Obara, O. Ishikawa, and T. Hata, Phys. Rev. B **75**, 012502 (2007).

[9] D. I. Bradley, S. N. Fisher, A. M. Guénault, R. P. Haley, V. Tsepelin, G. R. Pickett, and K. L. Zaki, J. Low Temp. Phys. **154**, 97 (2009).

[10] H. A. Nichol, L. Skrbek, P. C. Hendry, and P. V. E. McClintock, Phys. Rev. Lett. **92**, 244501 (2004).

[11] M. Blažková, D. Schmoranzer, L. Skrbek, and W. F. Vinen, Phys. Rev. B **79**, 054522 (2009).

[12] D. I. Bradley, M. J. Fear, S. N. Fisher, A. M. Guénault, R. P. Haley, C. R. Lawson, P. V. E. McClintock, G. R. Pickett, R. Schanen, V. Tsepelin, and L. A. Wheatland, J. Low Temp. Phys. **156**, 116 (2009).

[13] N. Hashimoto, R. Goto, H. Yano, K. Obara, O. Ishikawa, and T. Hata, Phys. Rev. B **76**, 020504(R) (2007).

[14] R. Hänninen, M. Tsubota, and W. F. Vinen, Phys. Rev. B **75**, 064502 (2007).

[15] R. Goto, S. Fujiyama, H. Yano, Y. Nago, N. Hashimoto, K. Obara, O. Ishikawa, M. Tsubota, and T. Hata, Phys. Rev. Lett. **100**, 045301 (2008).

[16] W. Schoepe, Phys. Rev. Lett. **92**, 095301 (2004).

[17] N. Hashimoto, A. Handa, M. Nakagawa, K. Obara, H. Yano, O. Ishikawa, and T. Hata, J. Low Temp. Phys. **148**, 299 (2007).

[18] H. Yano, Y. Nago, R. Goto, K. Obara, O. Ishikawa, and T. Hata, J. Phys.: Conf. Ser. **150**, 032126 (2009).

[19] Y. Nago, T. Ogawa, A. Mori, Y. Miura, K. Obara, H. Yano, O. Ishikawa, and T. Hata, J. Low Temp. Phys. **158**, 443 (2010).

[20] H. Yano, T. Ogawa, A. Mori, Y. Miura, Y. Nago, K. Obara, O. Ishikawa, and T. Hata, J. Low Temp. Phys. **156**, 132 (2009).

[21] H. Yano, Y. Nago, R. Goto, K. Obara, O. Ishikawa, and T. Hata, J. Phys.: Conf. Ser. **150**, 032125 (2009).

[22] B. Hof, C. W. H. van Doorne, J. Westerweel, F. T. M. Nieuwstadt, H. Faisst, B. Eckhardt, H. Wedin, R. R. Kerswell, and F. Waleffe, Science **305**, 1594 (2004); B. Hof, A. de Lozar, D. J. Kuik, and J. Westerweel, Phys. Rev. Lett. **101**, 214501 (2008).

[23] M. Avila, A. P. Willis, and B. Hof, J. Fluid Mech. **646**, 127 (2010).

[24] S. Fujiyama and M. Tsubota, Phys. Rev. B **79**, 094513 (2009).

[25] W. F. Vinen and J. J. Niemela, J. Low Temp. Phys. **128**, 167 (2002).

[26] H. Yano, S. Mio, N. Chiba, Y. Nago, K. Obara, O. Ishikawa, and T. Hata (unpublished).