

Intrinsic structural distortion and superexchange interaction in the orthorhombic rare-earth perovskites $R\text{CrO}_3$

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High-resolution neutron powder diffraction has been applied to determine the structural evolution in the orthorhombic perovskite $R\text{CrO}_3$ family. The structural distortions observed have been found to be closely related to the dramatic variation in the magnetic ordering temperature T_N . In addition to the reduction in the orbital overlap integral that can account for the change in T_N in the $R\text{FeO}_3$ family, the effect of t - e hybridization due to the structural distortions are responsible for the dramatic change in T_N across the $R\text{CrO}_3$ family.

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I. INTRODUCTION

The magnetic properties of $R\text{CrO}_3$ are extensively reported in three consecutive reviews of perovskite oxides that cover a huge number of studies over a half century.¹ The magnetic structure of the orthochromites was also well-studied during the boom of neutron diffraction in 1960s–1970s. All kinds of magnetic structure in the orthorhombic perovskites can be well-described in connection with the crystal symmetry under the notation developed by Bertaut.² In most cases, the collinearly ordered spins are along the a or c axis of the $Pbnm$ cell. However, the coupling between the magnetic moment on the rare earth and the spins on Cr^{3+} is responsible for an easy-axis rotation below T_N in some orthochromites.³ Since the T_N of $R\text{CrO}_3$ changes monotonically from Lu to La independent of the rare-earth moment, the R - O - Cr coupling appears to have negligible influence on the Cr - O - Cr coupling. It has been widely accepted that the M - O - M bond angle (M =transition metal), which is reduced from 180° due to the cooperative octahedral-site rotations in the orthorhombic perovskite, is a major factor entering the perturbation formula $J \sim b^2/U$ of the M - O - M superexchange interaction; b is the orbital overlap integral over the M - O - M bond and U is the on-site Coulomb energy. This assumption has been shown to work properly in $R\text{FeO}_3$.⁴ In contrast, all Jahn-Teller (JT) active systems, such as $R\text{TiO}_3$,^{5,6} $R\text{VO}_3$,^{7,8} and $R\text{MnO}_3$,⁹ do not show a clear connection between the magnetic transition temperatures and the M - O - M bond angle. Instead, they exhibit transitions between different types of spin orderings as the rare earth R changes from La to Lu. As in $R\text{FeO}_3$, orbital ordering is not an issue in $R\text{CrO}_3$; both families exhibit the type- G spin ordering. Given the same superexchange coupling parameter J in the formula $k_B T_N = 4S(S+1)J$, a huge difference between $T_N = 760$ K in LaFeO_3 and $T_N = 298$ K in LaCrO_3 can be well accounted for by the total spin $S = 5/2$ for $\text{Fe}^{3+}: t^3 e^2$ and $S = 3/2$ for $\text{Cr}: t^3 e^0$. However, a dramatic change in T_N from LaCrO_3 to LuCrO_3 appears troublesome for applying the same overlap integral reduction as in $R\text{FeO}_3$. Due to incomplete structural data, no attempt has been made so far to answer whether the M - O - M bond-angle evolution over R^{3+} ion can justify the

change in Néel temperature in the $R\text{CrO}_3$ family. In this paper, we report results of high-resolution neutron diffraction on all $R\text{CrO}_3$ members except a few neutron-absorbing rare earths. By comparison with the structural work of the $R\text{FeO}_3$ family in the literature, we are able to identify the local structural distortion that is intrinsic to the orthorhombic perovskite. We have shown that the scenario of overlap integral versus T_N , which has been assumed to be generally applicable in the superexchange interaction, is limited to some special cases like in $R\text{FeO}_3$ where both t and e orbitals are half filled. The observed overall change in T_N across the $R\text{CrO}_3$ family can be explained well only after the effect of t - e hybridization due to the local site distortion and cooperative octahedral-site rotation is taken into account.

II. EXPERIMENT

Polycrystalline samples of $R\text{CrO}_3$ were prepared by standard solid-state reactions. The mixtures of rare-earth oxides and chromium oxide in a stoichiometric ratio were sintered in air at 980 – 1450 °C with several intermediate grindings. We prepared about 7 g of powder sample for each member in the $R\text{CrO}_3$ family to ensure high quality neutron-diffraction data. These materials were checked to be single phase by x-ray powder diffraction (XRD). Neutron powder-diffraction (NPD) patterns were collected at the high-resolution powder diffractometer (HRPT) (Ref. 10) of the SINQ spallation source at the Paul Scherrer Institute, Villigen, Switzerland. The samples were packed in cylindrical vanadium holders of 6 mm diameter excepting the DyCrO_3 , which was contained in a double-walled sample holder to minimize neutron absorption. The patterns were collected at room temperature with a wavelength of 1.494 Å. In all cases the high-intensity mode was used; the collection time was about 4 h per diagram (20 h for DyCrO_3). The NPD patterns were analyzed with the Rietveld method with the FULLPROF (Ref. 11) program. The magnetization measurements were carried out in a superconductor quantum interference device magnetometer (Quantum Design). All crystal-structure and magnetization data will be published separately. We present here our major results.

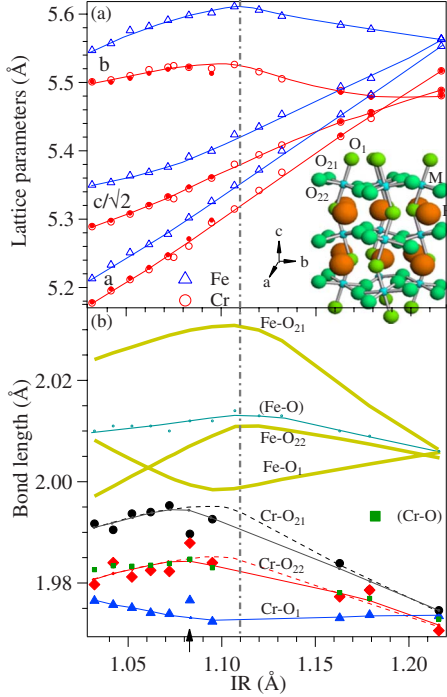


FIG. 1. (Color online) The IR dependences crossing all rare-earth ions from Lu^{3+} to La^{3+} of (a) lattice parameters (solid symbols are from neutron diffraction) and (b) M -O bond lengths of $R\text{CrO}_3$. Error bars are comparable to the size of symbols. The anomalous bond lengths of DyCrO_3 , shown by an arrow, are attributed to the Dy neutron absorption. The inset shows the structural model of the $Pbnm$ perovskite.

III. RESULTS AND DISCUSSION

A precise knowledge of the crystal structure is always essential for interpretation of the physical properties. We first made a thorough analysis of the crystal structure of $R\text{CrO}_3$ in comparison with that of $R\text{FeO}_3$ since the latter is a model system in which the T_N variation can be well accounted for by the M -O- M bond angle. We start by analyzing the lattice parameters of $R\text{CrO}_3$ and $R\text{FeO}_3$ as a function of the ionic radius (IR) of the R^{3+} ions, Fig. 1(a). The lattice parameters in both systems show the same trend as IR increases. Major features include (1) $b > c/\sqrt{2} > a$ for most R as required by the crystal symmetry of the $Pbnm$ space group provided that the octahedra are rigid and (2) the curve b versus IR shows an anomaly near $\text{IR}=1.11 \text{ \AA}$ in contrast to the straight line predicted by the software SPUDS (Ref. 12) for rigid octahedra. The anomaly in feature (2) is universal for all the orthorhombic perovskites¹³ as the O_{21} - M - O_{22} bond angle shown in Fig. 2(a) starts to decrease from 90° for $\text{IR} > 1.11 \text{ \AA}$. The resulting decrease in the rotation b axis leads eventually to a $b < a$ as is seen in LaCrO_3 . As a general character of the $Pbnm$ perovskites, the long M -O bond is always within the ab plane. The Fe-O_{21} and Fe-O_{22} bond lengths reach their maximum near $\text{IR}=1.11 \text{ \AA}$; at the same IR, the apical Fe-O_1 is located at a minimum. Although no data are available for the $R\text{CrO}_3$ with the neutron absorbers $R=\text{Sm}$, Eu , and Gd , we have good reasons to believe that bond lengths versus IR follow the dashed lines in Fig. 1(b) similar to that of $R\text{FeO}_3$

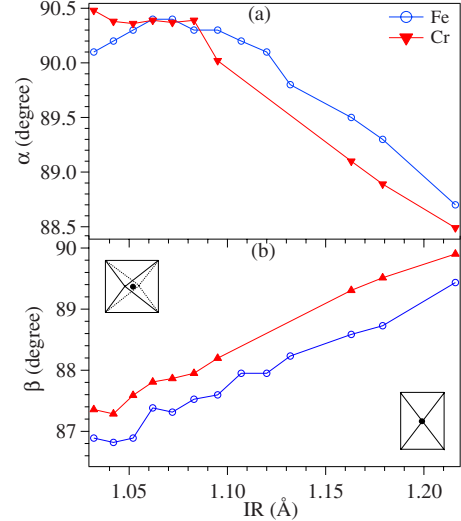


FIG. 2. (Color online) The $\text{IR}(R^{3+})$ dependence of (a) the O_{21} - M - O_{22} bond angle α and (b) the angle β between M - O_1 bond and the O_{21} - M - O_{22} plane. Insets: octahedron projected along the axis normal to the O_{21} - M - O_{22} plane.

since they all share the same features of lattice parameters. The largest bond-length splitting in both systems occurs near $\text{IR}=1.11 \text{ \AA}$. The classic case of $R\text{FeO}_3$ was accomplished by using x-ray single-crystal diffraction¹⁴ in which the Fe-O bond lengths were determined within large error bars. We have reproduced here in Fig. 1(b) the guide lines that go through scattered data points of the Fe-O bond lengths versus IR in $R\text{FeO}_3$.¹⁴ The results of our NPD data trace down the profile of Cr-O bonds as a function of IR with high precision. Since there is no orbital order to have an influence on the magnetic interactions in these two JT-inactive systems, we focus on the superexchange interaction that is modulated by structural distortions.

We turn to the Cr-O-Cr bond angle, the so-called “superexchange angle” in the orthorhombic perovskites. As shown in Fig. 3(a), the averaged Cr-O-Cr bond angle θ is progressively reduced from 180° as the IR decreases. As a matter of fact, the profile of θ versus IR is nearly identical in several orthorhombic perovskites $R\text{MO}_3$ ($M=\text{Ti}, \text{V}, \text{Cr}, \text{Mn}, \text{Fe}, \text{Ga}$). For a given R , $R\text{CrO}_3$ has the smallest octahedral-site tilting angle among these perovskite families because of the relatively smaller bond-valence-sum radius of the Cr^{3+} ion and therefore larger geometric tolerance factor t . Without a rigorous proof so far, the empirical relationship between the superexchange coupling J and the superexchange angle for the $R\text{FeO}_3$ family has undergone an evolution from $J \sim \cos \theta$ (Ref. 15) to $J \sim \cos^2 \theta$ (Ref. 16) and finally to $J \sim \cos^4(\omega/2)/d^7$ (Ref. 4), where d denotes the M -O bond length and $\omega=180-\theta$. As shown in Fig. 1(b), d is essentially independent of IR for $R\text{FeO}_3$ and can be taken as an averaged bond length. In order to make a quantitative comparison of the calculated T_N based on the structural data and the observed T_N , we have normalized T_N by that of LaMO_3 . The calculation based on the formula $J \sim \cos^4(\omega/2)/d^7$ can indeed produce the essential feature of the observed T_N versus IR for the $R\text{FeO}_3$ family in Fig. 3(b). However, this angle dependence can only account for about 30% of the overall

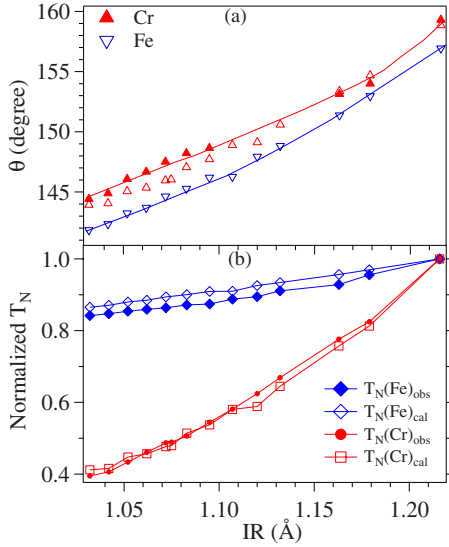


FIG. 3. (Color online) The IR(R^{3+}) dependences of (a) the averaged M - O - M bond angle θ (hollow symbols are from XRD) and (b) normalized T_N of $R\text{FeO}_3$ and $R\text{CrO}_3$ by $T_N=740$ K for LaFeO_3 and 289 K for LaCrO_3 , respectively. T_N of $R\text{FeO}_3$ are after Ref. 15.

change in T_N across the $R\text{CrO}_3$ family. Some important spin-spin interaction term in addition to the change in the superexchange angle is missing in our understanding of the $R\text{CrO}_3$ family.

What has been ignored is the hybridization between intersite t - e orbitals which are orthogonal for a 180° M - O - M bond. As illustrated in Fig. 4, octahedral-site tilting not only reduces the t orbital overlap integral considered but also introduces the orbital overlap between the π and the σ bonds. The influence on the overall exchange interaction of the intersite t - e orbital hybridization is sharply different for $\text{Fe}^{3+}:t^3e^2$ versus $\text{Cr}:t^3e^0$. The superexchange interaction between two adjacent transition-metal ions is delivered by a virtual charge transfer (VCT). Figure 5 illustrates the case for

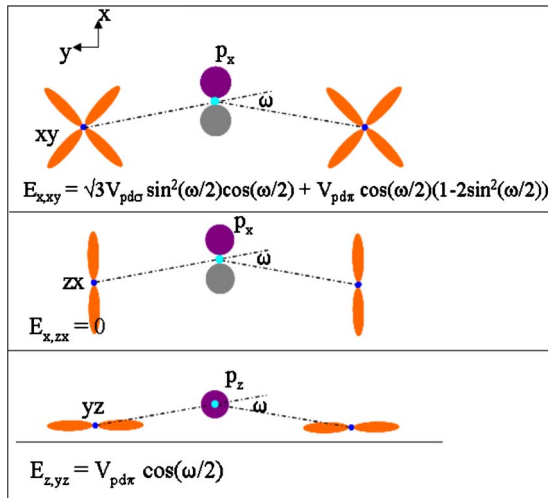


FIG. 4. (Color online) Schematic diagram of orbital overlap integrals. These are the examples of 15 two-center orbital overlap integrals. Formulas inside diagram show the angle dependence of the overlap integral. See text for the bond-length dependence V_{pd} .

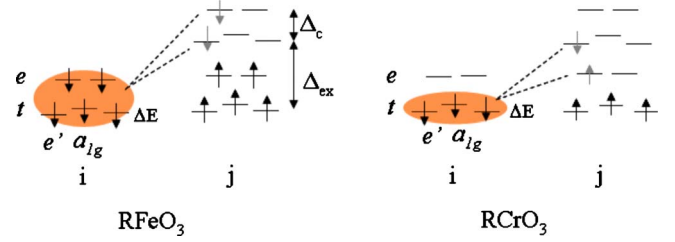


FIG. 5. (Color online) Schematic diagram of the hybridization effect on the virtual charge transfer for the superexchange interaction in perovskites $R\text{CrO}_3$ and $R\text{FeO}_3$. The VCT occurs for all occupied spin states; only those for the representative magnetic couplings are shown in the figure. Δ_c stands for the crystal-field splitting, Δ_{ex} is the exchange splitting, $\Delta E = E_{a_{1g}} - E_c$.

the Fe^{3+} ions. The superexchange interactions over the half-filled π bond t^3 - O - t^3 and the half-filled σ bond e^2 - O - e^2 are all antiferromagnetic (AF) according to the superexchange rules. The t - e hybridization does not bring in any new component to the overall superexchange interaction in this case. The reduction in the σ bond overlap integral b^σ due to octahedral-site rotation appears not to be compensated by a small gain of the overlap integral through the intersite t - e hybridization. Therefore, the angular dependence of $J \sim \cos^4(\omega/2)/d^7$, which considers only the σ bond, is still a good approximation. In the case of $R\text{CrO}_3$, however, only the t orbitals are half filled. A t - e hybridization similar to that in $R\text{FeO}_3$ introduces a VCT to the empty e component of the hybridized orbital at site j , i.e., t^3 - O - e^0 , which leads to a ferromagnetic coupling J_{hb}^σ according to the superexchange rules. The cartoon for $R\text{CrO}_3$ in Fig. 5 shows the t - e hybridization that leads to the ferromagnetic VCT to the empty e orbital on site j whereas the t^3 - O - t^3 transfer gives a dominant AF coupling J^π . The final superexchange interaction for the type G antiferromagnetic ordering is $J = J^\pi - J_{\text{hb}}^\sigma$. The progressively enhanced J_{hb}^σ becomes comparable to the angle- θ -dependent J^π as IR decreases. In order to check whether this argument can account quantitatively for the observed T_N versus IR, we have derived J^π and J_{hb}^σ by including all possible two-center orbital overlap integrations between (p_x, p_y, p_z) on O^{2-} and $(xy, yz, zx, x^2 - y^2, z^2 - r^2)$ on Cr^{3+} based on the Slater-Koster tables of interatomic matrix elements,¹⁷ which gives the following angle dependence for the orbital overlap integrals b^π and b_{hb}^σ

$$b_{\text{hb}}^\sigma = [\cos(\omega/2) + \sin(\omega/2)] \left\{ \sqrt{3} \cos(\omega/2) \sin(\omega/2) + \frac{\sqrt{3}}{2} [\cos^2(\omega/2) - \sin^2(\omega/2)] - \frac{1}{2} \right\} V_{pd\sigma},$$

$$b^\pi = \{2[\sin(\omega/2) + \cos(\omega/2)] - 4 \sin(\omega/2) \cos^2(\omega/2)\} V_{pd\pi}.$$

Therefore, the superexchange coupling J can be expressed by

$$J = J_0[(b^\pi)^2 - \eta(b_{\text{hb}}^\sigma)^2], \quad \eta = \frac{U + \Delta_{\text{ex}}}{U + \Delta_c} \left(\frac{V_{pd\sigma}}{V_{pd\pi}} \right)^2.$$

J_0 here is a constant that determines the absolute value of T_N but it has no effect on the calculation of a normalized T_N in

Fig. 3(b). A match between the observed T_N and the calculated value like that shown in Fig. 3(b) requires an $\eta=1.5$. That is a highly reasonable number since the bond-length-dependent integral $V_{pd\sigma}$ is slightly larger than $V_{pd\pi}$ in oxides as has been shown by Harrison¹⁸ and Δ_{ex} is slightly larger than Δ_c in octahedral-site Cr^{3+} .

Another local site distortion is the angle β between the apical $M-O_1$ bond and the $O_{21}-M-O_{22}$ plane. As seen from Fig. 2(b), this angle is near 90° for $LaFeO_3$ but it is reduced monotonically as IR decreases. The same angle change can also be seen in $RCrO_3$. Landron and Lepetit¹⁹ have shown that the intrasite hybridization between t and e orbitals is highly sensitive to the angle β (they have defined the trigonal distortion of an octahedron by a different angle). In our case, the intrasite $t-e$ hybridization vanishes for $\beta=90^\circ$. The component of e orbital mixing into the t orbital due to the trigonal distortion of the octahedral site is extremely small in our case but the intrasite hybridization splits the degenerated t_2 orbitals into $2 \times e'$ and a_{1g} . The intrasite hybridization strengthens the ferromagnetic coupling $J_{hb}^\sigma = (b_{hb}^\sigma)^2 / (U + \Delta_c)$ by offsetting a $\Delta E = E_{e'} - E_{a_{1g}}$ in the energy difference $U + \Delta_c$ associated with this VCT.

The intersite $t-e$ hybridization should influence physical properties in all orthorhombic perovskites. The $RCrO_3$ family with t^3e^0 exemplifies clearly the effect. The same mechanism should also be applicable to another system with t^3e^0 , for example, the perovskites $AMn^{4+}O_3$ ($A=Ca, Sr,$ and $Sr_{1-x}Ba_x$) (Ref. 20) where T_N is found to be very sensitive, though less dramatically than that in $RCrO_3$, to IR. However, other factors such as the size variance and the change in A-O covalency have to be included in order to account for the curve of T_N versus IR quantitatively in the $AM^{4+}O_3$ perovskites. In the JT-active family $RTiO_3$, orbitals remain disordered in the paramagnetic phase. Fitting to a Curie-Weiss law of the paramagnetic susceptibility gives an antiferromagnetic coupling for the $RTiO_3$ family with larger IR and a ferromagnetic coupling for those with smaller IR. The intrinsic local site distortions are certainly playing a role for the transition of the magnetic interaction by confining orbital fluctuations in such a way that the AF t^1-O-t^1 coupling is preferred for the $RTiO_3$ with large IR and the ferromagnetic t^1-O-t^0 for those with smaller IR. However, long-range magnetic order and orbital order occur at the same temperature,⁶ which indicates that orbital order is a prerequisite to long-range magnetic order. The increasing ferromagnetic t^1-O-e^0 coupling as IR decreases contributes to the transition from the antiferromagnetic phase to the ferromagnetic phase shown in the phase diagram of the $RTiO_3$ family.⁵ In another JT-active family, RVO_3 , the orbital flipping transition in the magnetically ordered phase occurring for $IR \leq r_{Dy}$ cannot be explained only by the change in geometric tolerance factor t .

A careful comparison between the effects of hydrostatic pressure versus uniaxial pressure has clarified the effect to be due to the $t-e$ hybridization in these highly distorted perovskites.²¹

IV. CONCLUSION

The structural evolution of orthorhombic $RCrO_3$ has been resolved by high-resolution neutron diffraction. A side-by-side comparison with that of $RFeO_3$ gives a solid proof that the following local structural distortions are intrinsic to orthorhombic perovskites RMO_3 : (a) the longest $M-O$ bond length is within the $O_{21}-M-O_{22}$ plane, (b) the bond-length splitting undergoes a maximum at a middle IR near 1.11 Å, (c) at the same IR the $O_{21}-M-O_{22}$ bond angle α starts to deviate from 90° , and (d) the angle β between the apical $M-O_1$ bond and the $O_{21}-M-O_{22}$ plane, which is near 90° for $LaFeO_3$ and $LaCrO_3$, reduces monotonically as IR decreases. These intrinsic structural distortions are in addition to the cooperative octahedral-site rotations, widely called the $GdFeO_3$ -type distortion, and are essential elements to interpret physical properties in these perovskites. As a consequence of structural distortions that include the cooperative octahedral-site rotation and local distortions, the $t-e$ hybridization is inevitable. For $RFeO_3$, where the AF coupling consists of the superexchange interactions of the t^3-O-t^3 and e^2-O-e^2 orbitals, the hybridization does not bring any new component. Therefore the empirical angle dependence of the superexchange coupling J remains a good approximation. The more dramatic change in T_N in the $RCrO_3$ family highlights the importance of the $t-e$ hybridization effect. The ferromagnetic coupling through the superexchange interaction t^3-O-e^0 is comparable to the antiferromagnetic coupling through the superexchange interaction t^3-O-t^3 . By considering all possible two-center orbital overlap integrals involved in the virtual charge transfer for the superexchange interaction in the orthorhombic perovskite structure, we are able to account quantitatively for the observed T_N versus IR in the $RCrO_3$ family.

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