



Strong orbital polarization in orthorhombic DyMnO₃: A combined x-ray linear dichroism and *ab initio* electronic structure study

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We present a combined experimental and theoretical study on the Mn 3*d* orbital occupation in orthorhombic DyMnO₃ single crystals. We have observed a very strong polarization dependence in the Mn *L*_{2,3} x-ray absorption spectra, indicative for a distinct anisotropic orbital occupation. *Ab initio* electronic-structure calculations clearly infer the existence of a strong orbital polarization in the *e_g* band with a staggered *d*_{3*x*²-*r*²/*d*_{3*y*²-*r*²-type ordering pattern in the *ab* plane. This finding is in excellent agreement with a quantitative analysis of the spectra using the atomic multiplet cluster approach.}}

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Correlated-electron materials such as the 3*d* transition-metal (TM) oxides exhibit an exceptionally rich variety of physical properties. These include, for example, metal-insulator transitions (MIT), colossal magnetoresistance, superconductivity, and multiferroicity. It is well recognized by now that the anisotropy in the TM 3*d* orbital occupation is a key factor for the extraordinary magnetic and electronic properties since it is intricately coupled to the charge, spin, and lattice dynamics.¹⁻⁷ The MIT in VO₂, V₂O₃, and Ca₂RuO₄ was found to be associated with a redistribution of the orbital occupation.⁸⁻¹⁰ The occupation of the *d*₂ and *d*_{0*d*₂ orbitals with minority spin gives rise to a giant orbital moment about 2μ_B for Ca₃Co₂O₆ and Ca₃CoRhO₆, respectively, elucidating the highly Ising-type magnetism.^{11,12} It was found that in CoO thin films the magnetic moments are oriented out of plane if the *t*_{2*g*} hole is situated in an orbital characterized by a linear combination of the *d*_{*xz*} and *d*_{*yz*} states and in plane if it is in the *d*_{*xy*} orbital.¹³ In LiVO₂, it was found that orbital ordering removes the spin frustration in the triangular lattice and drives the system into a spin-singlet state without any long-range magnetic order.¹⁴ In the manganese oxides, the orbital and charge ordering plays an important role for the colossal magnetoresistance and the metal-insulator transition.^{1,15} On top of this, orbital order could also lead to ferroelectricity.^{4,5,7}}

In the RMnO₃ (*R*=rare earth) system, the high-spin Mn ion has the *t*_{2*g*}³*e*_{*g*}¹ configuration. A strong Jahn-Teller (JT) distortion occurs to lift the degeneracy in the *e_g* orbital and leads to a pattern of orbital ordering through the crystal lattice. The spin ordering is coupled to the ordered orbital pattern according to the Goodenough-Kanamori rules.^{16,17} The different displacements of the oxygen ions around the Mn ions in RMnO₃ (*R*=Tb, Dy, and Gd) causes strong distortions and tilting of the MnO₆ octahedra as well as a reduction in Mn-O-Mn bond angles, thereby yielding a complicated

pattern of orbital and spin ordering, which in turn is thought of as the origin of the observed ferroelectricity.^{7,18-20} Thus, orbital polarization and ordering in orthorhombic RMnO₃ (*R*=Tb, Dy, and Gd) is an indispensable ingredient for the formation of complicated incommensurate spin structures at low temperatures.

Indeed, it was found recently that the Mn *L*_{2,3}-edge x-ray absorption spectra of hexagonal YMnO₃ and DyMnO₃ exhibit a strong polarization dependence, indicative for the presence of distinct orbital occupations in the *e_g* subshell. Yet, it is quite puzzling that such polarization dependence was not observed in the study on orthorhombic DyMnO₃.^{21,22} We note that thin films were used in that study. Our objective is now to resolve this issue and to reinvestigate the orbital occupations of the Mn 3*d* states in orthorhombic DyMnO₃ using bulk single crystals.

Polarization-dependent x-ray absorption spectroscopy (XAS) at the 3*d* TM *L*_{2,3} edge, complemented with multiplet cluster calculations, is a powerful method to determine the 3*d* orbital occupation in transition-metal oxides^{8,9,13,23} and have provided information of orbital ordering pattern in La_{1-*x*}Sr_{*x*}MnO₃ and La_{2-*x*}Sr_{*x*}MnO₄.²⁴⁻²⁹

In this Rapid Communication, we carried out our XAS study on orthorhombic DyMnO₃ single crystals using three independent polarizations, namely, *E*∥*a*, *E*∥*b*, and *E*∥*c*, where *E* denotes the electric field vector and *a*, *b*, *c* the crystallographic directions of orthorhombic DyMnO₃. In contrast with the results of previous experiments, we found that the Mn *L*_{2,3} x-ray absorption spectra of orthorhombic DyMnO₃ single crystals do show a strong polarization dependence. We also performed *ab initio* band-structure calculations and obtained a strong zigzag *e_g* orbital polarization in the *ab* plane with the *d*_{3*x*²-*r*² (or *d*_{3*y*²-*r*²) orbital occupation number close to 1. The corresponding orbital polarization is in excellent agreement with the analysis of the experimental spectra using the multiplet cluster calculations.}}

Untwined high-quality orthorhombic DyMnO_3 single crystals (space group: $Pbnm$) were grown by the high-temperature solution method with a PbF_2 flux in a Pt crucible. The rocking curve of the DyMnO_3 crystal has the full width at half maximum of $\sim 0.01^\circ$ for the (0 0 2) Bragg reflection, indicating the excellent crystal quality. Two crystal surfaces with crystallographic directions of (100), (010) (i.e., ab plane) and (100), (001) (i.e., ac plane), respectively, were aligned using an x-ray diffractometer. Total electron-yield Mn $L_{2,3}$ -edge x-ray absorption spectra, with a photon energy resolution of ~ 0.16 eV at 642 eV, were recorded at the Dragon beamline at National Synchrotron Radiation Research Center in Taiwan. The degree of linear polarization of the incident light was $\sim 99\%$. The crystals were mounted with the surface normal parallel to the Poynting vector of the light. Clean crystal surfaces were obtained by cleaving the crystals *in situ* in an ultrahigh-vacuum chamber with a base pressure $\sim 5 \times 10^{-10}$ Torr. A MnO single crystal was measured simultaneously in a separate chamber, which enabled us to achieve better than 0.02 eV accuracy in the relative energy alignment. Different DyMnO_3 crystals with the same axis showed excellent reproducibility of the experimental spectra, indicating the high quality of our crystals.

We performed band-structure calculations in a generalized gradient approximation plus on-site Coulomb interaction U (GGA+ U) for A-type antiferromagnetic (AFM) structures of DyMnO_3 . The band-structure calculations were performed using the full-potential projected augmented wave method as implemented in the Vienna *ab initio* simulation package (VASP). In the GGA+ U calculations, a Coulomb energy $U=5.0$ eV and an intra-atomic exchange parameter $J=0.87$ eV for the Mn 3d electrons were used.³⁰ The crystal-field potential due to the O^{2-} ligands within the Mn^{3+}O_6 octahedra in DyMnO_3 splits the fivefold Mn 3d orbitals into three lower-lying t_{2g} (d_{xy} , d_{yz} , and d_{xz}) orbitals and two higher-lying e_g orbitals. The e_g doublet is split further into two subbands $e_g^{\uparrow 1}$ and $e_g^{\uparrow 2}$ resulting from a strong JT distortion. Three d electrons are in the 1/2-filled t_{2g} shell and one in the e_g . The key issue is the distribution of this e_g electron between the in-plane $d_{3x^2-r^2}$ ($d_{3y^2-r^2}$) and out-of-plane $d_{y^2-z^2}$ ($d_{x^2-z^2}$) orbitals.

Figure 1(a) displays the orbital-projected partial density of states of fivefold Mn 3d states for DyMnO_3 using a local coordination on the Mn(4) site based on the GGA+ U calculations. There is a small indirect gap between the JT-split Mn $e_g^{\uparrow 1}$ and $e_g^{\uparrow 2}$ bands. Based on the GGA+ U calculations, the occupied $e_g^{\uparrow 1}$ and unoccupied $e_g^{\uparrow 2}$ bands are dominated by $d_{3x^2-r^2}$ and $d_{y^2-z^2}$ orbitals, respectively, in one coplanar Mn ion, whereas they exhibit predominantly $d_{3y^2-r^2}$ and $d_{x^2-z^2}$ character, respectively, for the other coplanar Mn ion. Figure 1(b) shows the electronic valence charge-density contour plot on the basal plane of DyMnO_3 . It clearly reveals the strong orbital polarization of the e_g band with a staggered $d_{3x^2-r^2}$ ($d_{3y^2-r^2}$)-type orbital ordering pattern in the ab plane.

Figure 2(a) shows our experimental polarization-dependent Mn $L_{2,3}$ -edge absorption spectra of bulk DyMnO_3 single crystals for the three polarizations: $E\parallel a$ (red line), $E\parallel b$ (black line), and $E\parallel c$ (blue line). The experimental linear dichroic (LD) spectra, namely, the difference between two polarizations, i.e., $(E\parallel c)-(E\parallel a)$, $(E\parallel b)-(E\parallel c)$, and

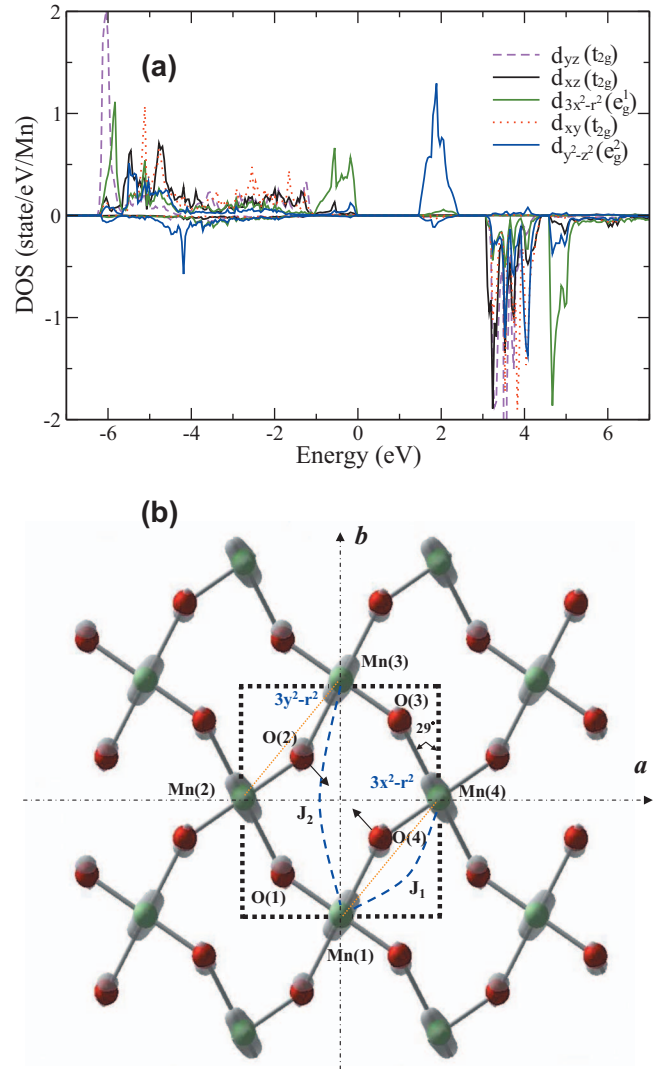


FIG. 1. (Color) (a) Orbital-projected partial density of states of fivefold Mn 3d states on the Mn(4) site for an A-type antiferromagnetic structure of orthorhombic DyMnO_3 calculated with the GGA+ U method. The parameters in these calculations are described in the text. For each panel the upper half denotes the and the lower half the minority-spin states. The energy zero is at the Fermi energy (E_f). (b) Electronic valence charge-density contour plot on the basal plane of DyMnO_3 . The green and red balls denote Mn and O ions, respectively. Gray lobes on Mn ions indicate occupied $e_g^{\uparrow 1}$.

$(E\parallel b)-(E\parallel a)$ (green line) are depicted in Figs. 2(c)–2(e). It is clear that our spectra exhibit a very strong polarization dependence along the three different crystallographic directions in sharp contrast to the results of previous experiments on DyMnO_3 thin films which showed practically only isotropic spectra (possibly due to strain effect on substrates).^{21,22} In our spectra, for example, the main peak for the $E\parallel b$ polarization (i.e., the direction of spiral spin) lies at a lower energy than for polarizations $E\parallel a$ and $E\parallel c$. Our spectra therefore, provide a clear indication for a large anisotropic orbital occupation of Mn e_g states as expected for the strongly distorted MnO_6 octahedron.³¹

To extract information on the orbital occupation from the polarization-dependent Mn $L_{2,3}$ -edge XAS spectra of

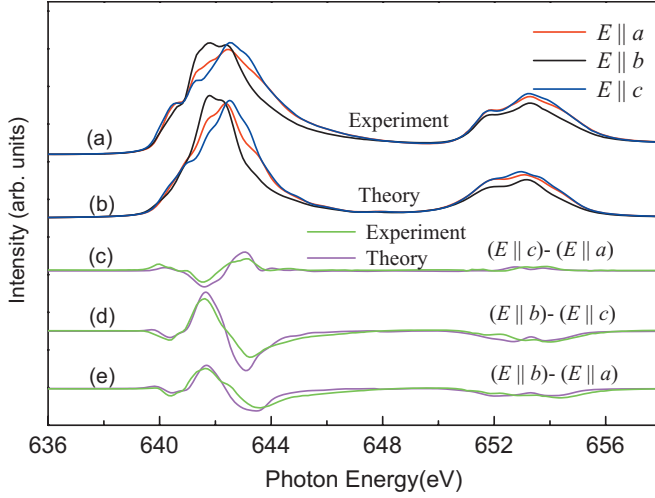


FIG. 2. (Color) (a) Experimental and (b) theoretical polarization-dependent Mn $L_{2,3}$ -edge absorption spectra of single-crystalline DyMnO₃ for (a) $E\parallel a$ (red line), $E\parallel b$ (black line), and $E\parallel c$ (blue line). Experimental (green line) and theoretical (magenta line) LD spectra for $(E\parallel c)-(E\parallel a)$, $(E\parallel b)-(E\parallel c)$, and $(E\parallel b)-(E\parallel a)$ are shown in (c), (d), and (e), respectively.

DyMnO₃, we have simulated the experimental spectra using the well-proven cluster model.^{32,33} The method uses for each Mn site a MnO₆ cluster which includes the full atomic multiplet theory and the local effects of the solid. It accounts for the intra-atomic $3d-3d$ and $2p-3d$ Coulomb interactions, the atomic $2p$ and $3d$ spin-orbit couplings, the O $2p$ -Mn $3d$ hybridization, and the proper local crystal-field parameters. The parameters for the multipole part of the Coulomb interactions were given by the Hartree-Fock values while the monopole parts $U_{cd}-U_{dd}=1.5$ eV and O $2p$ -Mn $3d$ charge-transfer energy $\Delta=4$ eV were estimated from photoemission experiments on LaMnO₃.³⁴ The Mn $3d$ -O $2p$ transfer integrals were adapted to account for the various Mn-O bond lengths according to Harrison's prescription.³⁵ The crystal splitting between e_g and t_{2g} states, $10Dq$, the splitting of e_g states, Δe_g , and the splitting of t_{2g} states, Δt_{2g} , are tuned to reproduce the experimental spectra.

The simulated spectra for the three polarizations $E\parallel a$ (red line), $E\parallel b$ (black line), and $E\parallel c$ (blue line) are given in Fig. 2(b). Figures 2(c)–2(e) show a comparison of the experimental (green line) with the theoretical LD spectra (magenta line). One can clearly see that we have been able to achieve a very good agreement between the simulation and the experiment. From this set of best fits we obtain values of 0.9 eV, 0.7 eV, and 0.2 eV for the crystal splitting parameters $10Dq$, Δe_g , and Δt_{2g} , respectively, and we have used $pd\sigma=-2.10$, -1.76 and -1.43 eV and $pd\pi=-pd\sigma/2.15$ for the three Mn-O bonds.

We have also extracted the orbital occupation numbers from the set of best simulations. These numbers are presented in Table I, where one can see the very large $d_{3x^2-r^2}$ orbital occupation number of 1.06. This is even larger than the 0.81 value in LaMnO₃.^{36,37} The result demonstrates that the e_g electron is constrained to the $d_{3x^2-r^2}$ or $d_{3y^2-r^2}$ orbital. The small occupation number of 0.38 in $d_{y^2-z^2}$ orbital in Table I is originating from the Mn-O covalent mixing. The

TABLE I. Occupation numbers of Mn $3d$ states and ligand molecular orbitals on the Mn(4) site in orthorhombic DyMnO₃ deduced from configuration-interaction cluster simulation.

	$3x^2-r^2$	y^2-z^2	yz	zx	xy
Mn $3d$	1.06	0.39	1.03	1.02	1.03
O $2p$	1.92	1.64	1.97	1.98	1.97

negative integrals in the LD spectra $(E\parallel b)-(E\parallel a)$ and $(E\parallel b)-(E\parallel c)$ are indicative for a relatively large projection of the occupied $d_{3x^2-r^2}$ and $d_{3y^2-r^2}$ orbitals along the b direction. This is in good agreement with the orbital ordering pattern shown in Fig. 1(b), in which the angle between the $d_{3x^2-r^2}$ (or $d_{3y^2-r^2}$) and the b axis is 29° (40° in LaMnO₃), reflecting the significant GdFeO₃-type distortion and the decrease in the Mn-O-Mn bond angle.³⁸

For the orbital polarization of the e_g -JT electron, the occupied e_g -orbital wave function is generally described as

$$\phi(\theta) = \cos(\theta/2)|3z^2 - r^2\rangle \pm \sin(\theta/2)|x^2 - y^2\rangle. \quad (1)$$

Here θ defines the respective orbital component.^{3,39} The simplest case of an alternative $d_{3z^2-r^2}$ and $d_{x^2-y^2}$ ordering corresponds to $\theta=0^\circ$ and 180° , respectively. The angle θ can be estimated from the actual magnitude of the JT distortion and is given by the expression

$$\tan(\theta) = \frac{\sqrt{3}(l-s)}{2m-l-s}, \quad (2)$$

where l , m , and s denote the three Mn-O unit distances.³ We obtain a θ angle of $\sim 113^\circ$ for Mn-O distances of 2.24, 1.94, and 1.89 Å in orthorhombic DyMnO₃. This is close to the 120° scenario corresponding to a staggered $d_{3x^2-r^2}/d_{3y^2-r^2}$ type of orbital ordering, consistent with the present experimental results and GGA+ U calculations. The larger differences between the three Mn-O distances in DyMnO₃ lead to the formation of a staggered $d_{3x^2-r^2}/d_{3y^2-r^2}$ type of orbital ordering as compared with the smaller differences in the Mn-O distances of 2.00, 1.98, and 1.84 Å in La_{0.5}Sr_{1.5}MnO₄ exhibiting a cross-type orbital ordering of $d_{x^2-z^2}/d_{y^2-z^2}$.²⁶

As shown in Fig. 1(b), the distortion leads to a shorter distance between O₂ and O₄, which in turn slightly changes the next-nearest-neighbor (NNN) AFM superexchange interaction through the Mn(1)-O₄-O₂-Mn(3) paths (J_2 on the order of a few millielectron volts), since the occupied e_g orbitals in the Mn(1) and Mn(3) are parallel. At the same time, this distortion strongly suppresses the predominant NN ferromagnetic (FM) superexchange interaction ($J_1=-13$ meV for LaMnO₃ and ~ -2 meV for DyMnO₃).^{18,40} Under the staggered $d_{3x^2-r^2}/d_{3y^2-r^2}$ type of orbital ordering as shown in Fig. 1(b), the AFM NNN superexchange interaction between Mn(1) and Mn(3) (along the b axis) is stronger than that between Mn(2) and Mn(4) (along the a axis). Thus, a subtle balance of the competition between the NN FM and the NNN AFM superexchange interactions along the b axis leads to a frustrated spin system and gives rise to a noncollinear spin arrangement in the spiral phase at low temperatures.^{18,19}

In conclusion, we have observed a strong polarization dependence in the soft-x-ray absorption spectra at the Mn $L_{2,3}$ edge in orthorhombic DyMnO₃ single crystals. *Ab initio* electronic-structure calculations clearly reveal a strong orbital polarization in the e_g band with a staggered $d_{3x^2-r^2}/d_{3y^2-r^2}$ type of ordering pattern in the ab plane. Configuration-interaction cluster calculations revealed that the e_g electron is indeed constrained to the $d_{3x^2-r^2}$ or $d_{3y^2-r^2}$ orbital. The negative integral intensity of linear dichroic spectra ($E\parallel b$)–($E\parallel a$) and ($E\parallel b$)–($E\parallel c$) points toward a relatively large projection of the occupied $d_{3x^2-r^2}$ and $d_{3y^2-r^2}$

orbitals along the b direction. The coexistence and competition between NN FM and NNN AFM interactions along the b axis lead to a frustrated spin system giving rise to the complex magnetic structure.

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