Strong pairbreaking in anisotropic superconductors

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Abrikosov-Gor'kov work on pairbreaking in isotropic materials is generalized to anisotropic Fermi surfaces and order parameters. New scaling relations for states with a strong pairbreaking are found for the specific-heat jump $\Delta C \propto T_c^3$; for the penetration depth that deviates from the zero-T value as $\Delta\lambda = \beta T^2$ at low temperatures with $\beta \propto T_c^{-3}$, and for the slopes of the upper critical fields $H'_{c2}(T_c) \propto T_c$. A remarkably simple relation between these at first sight unrelated quantities is found: $\Delta C \beta^2 T_c^4 / |H'_{c2}| = \phi_0 / 16\pi^2$ is a universal constant. The predictions are checked on $CeCoIn_5$ and the possibility to apply them to iron-based materials is discussed.

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I. INTRODUCTION

A half century old and still relevant work of Abrikosov and Gor'kov (AG) on the pairbreaking in isotropic materials contains more than well known and broadly used results such as the famous formula for the critical temperature suppression by magnetic impurities,

$$\ln\frac{T_{c0}}{T_c} = \psi\left(\frac{1+\rho_s}{2}\right) - \psi\left(\frac{1}{2}\right), \tag{1}$$

T_{c0} is the critical temperature of the material free of magnetic impurities, ψ's are the digamma functions, $\rho_s = 1/2\pi T_c \tau_s$ with $\hbar = k_B = 1$, and τ_s is the spin-flip scattering time.[1] The AG description of the so-called "gapless state" that emerges at near-critical concentration of magnetic impurities when $T_c \ll T_{c0}$ is commonly considered as a peculiar case with not much experimental implications due to stringent conditions to be satisfied for such a state to exist.

However, it appears that in a number of new materials much more complex than elemental metals of AG concern, the gapless state is not that rare an occurrence. Recently, the AG idea has been applied to understand such basic properties of iron-based materials as the specific heat or the temperature behavior of the London penetration depth.[2,3] Complexity of these materials (few-bands Fermi surfaces, anisotropic order parameters of a not-yet-established symmetry, a complicated role of doping that cannot be considered as just a source of scattering) makes it difficult to apply the AG theory *per se* developed for a superconductor with isotropic order parameter on a Fermi sphere containing uncorrelated magnetic impurities.

In this paper, the AG idea on the gapless state is generalized to anisotropic Fermi surfaces and order parameters. Part of this work has been presented in short papers of Refs. 2 and 3 done for specific applications. Potentially, the results given below may have a broader value and deserve proper presentation. They show that physical properties of the state emerging in materials with a strong pairbreaking such as the specific heat, the London penetration depth, and the upper critical field are tightly bound and satisfy a few scaling relations which may serve as signatures for a strong pairbreaking. The most surprising of those is the relation

$$\frac{\Delta C \beta^2 T_c^4}{|H'_{c2}|} = \frac{\phi_0}{16\pi^2}, \tag{2}$$

where ΔC is the specific-heat jump at T_c, $\beta = d\lambda/d(T^2)$ is the slope of the London depth λ as a function of T^2 at low temperatures, $H'_{c2}(T_c)$ is the slope of the upper critical field at T_c (if λ is taken, e.g., for the ab plane of a uniaxial material, the corresponding H_{c2} is in the c direction). A remarkable feature of this relation is that while all quantities on the left-hand side (LHS) depend on the density of states, Fermi surface characteristics, scattering rates and the order parameter symmetry, no material parameters enter this formula. This suggests that although the derivation given below is done for the Fermi-surface averaged order parameter $\langle\Delta\rangle = 0$ (as for the d-wave or approximately for the $\pm s$ order parameter), the result may have a broader applicability. This conjecture is supported by the fact that Eq. (2) holds for the isotropic case of AG that can be checked by taking the needed quantities from Ref. 1.

It is also worth noting that the quantities on the LHS Eq. (54) are measured in independent experiments and the combination shown reduces to a universal number at the right-hand side (RHS). This property may serve as a stringent test for presence of a strong pairbreaking in a material.

In the next section, the formal scheme is described, some details of which are given elsewhere;[2,3] the main points are reproduced here to make the presentation tractable. Then the Ginzburg-Landau (GL) equations are derived to obtain the specific-heat jump $\Delta C \propto T_c^3$ and the slope of the upper critical field $H'_{c2} \propto T_c$. The derivation of the penetration depth follows. The section on new scaling relations and comparison with data available in Sec VIII conclude the paper.

II. METHOD

Perhaps, the simplest for our purpose is the Eilenberger quasiclassical version of the weak-coupling Gor'kov's theory that holds for a general anisotropic Fermi surface and for any gap symmetry,[4]

$$\boldsymbol{v}\boldsymbol{\Pi} f = 2\Delta g - 2\omega f + \frac{g}{\tau_-}\langle f\rangle - \frac{f}{\tau_+}\langle g\rangle, \tag{3}$$

$$-\boldsymbol{v}\boldsymbol{\Pi}^* f^+ = 2\Delta^* g - 2\omega f^+ + \frac{g}{\tau_-}\langle f^+\rangle - \frac{f^+}{\tau_+}\langle g\rangle, \tag{4}$$

$$g^2 = 1 - ff^+, \tag{5}$$

$$\Delta(\boldsymbol{r},\boldsymbol{k}_F) = 2\pi TN(0)\sum_{\omega>0}^{\omega_D}\langle V(\boldsymbol{k}_F,\boldsymbol{k}_F')f(\boldsymbol{k}_F',\boldsymbol{r},\omega)\rangle_{\boldsymbol{k}_F'}. \tag{6}$$

Here, $\boldsymbol{v}$ is the Fermi velocity, $\boldsymbol{\Pi}=\nabla+2\pi i\boldsymbol{A}/\phi_0$, $\boldsymbol{A}$ is the vector potential, and ϕ_0 is the flux quantum. $\Delta(\boldsymbol{r},\boldsymbol{k}_F)$ is the order parameter that in general depends on the position $\boldsymbol{k}_F$ at the Fermi surface of other than the isotropic s-wave symmetry. The functions $f(\boldsymbol{r},\boldsymbol{v},\omega)$, f^+, and g originate from Gor'kov's Green's functions integrated over the energy variable near the Fermi surface. Further, $N(0)$ is the total density of states at the Fermi level per one spin; the Matsubara frequencies are $\omega=\pi T(2n+1)$ with an integer n. The averages over the Fermi surface are shown as $\langle\cdots\rangle$.

The scattering in the Born approximation is characterized by two scattering times, the transport scattering time τ responsible for conductivity in the normal state, and τ_m for spin-flip processes,

$$\frac{1}{\tau_\pm} = \frac{1}{\tau} \pm \frac{1}{\tau_m}. \tag{7}$$

The strong scattering in unitary limit is not considered here. Usually, two dimensionless scattering parameters are employed,

$$\rho = \frac{1}{2\pi T_c\tau} \quad \text{and} \quad \rho_m = \frac{1}{2\pi T_c\tau_m}, \tag{8}$$

or equivalently $\rho_\pm=\rho\pm\rho_m$. $\hbar$ and k_B are set unities, except when comparisons with data are discussed.

Commonly, the effective coupling V is assumed factorizable, $V(\boldsymbol{k}_F,\boldsymbol{k}_F')=V_0\Omega(\boldsymbol{k}_F)\Omega(\boldsymbol{k}_F')$.[5] This assumption is quite restrictive as far as complicated Fermi surfaces and interactions are concerned. E.g., within two-band schemes with four coupling constants V_{ij}, the factorizable model implies $V_{11}V_{22}-V_{12}V_{21}=0$. Still, the assumption appears to work well for one-band materials and it simplifies the algebra considerably. One then looks for the order parameter in the form

$$\Delta(\boldsymbol{r},T;\boldsymbol{k}_F) = \Psi(\boldsymbol{r},T)\Omega(\boldsymbol{k}_F). \tag{9}$$

The function $\Omega(\boldsymbol{k}_F)$ describes the variation in Δ along the Fermi surface and is conveniently normalized,

$$\langle\Omega^2\rangle = 1. \tag{10}$$

Then, the self-consistency Eq. (6) takes the form

$$\Psi(\boldsymbol{r},T) = 2\pi TN(0)V_0\sum_{\omega>0}^{\omega_D}\langle\Omega(\boldsymbol{k}_F)f(\boldsymbol{k}_F,\boldsymbol{r},\omega)\rangle. \tag{11}$$

Instead of dealing with the effective microscopic electron-electron interaction V and with the energy scale ω_D, one can use within the weak-coupling scheme the critical temperature T_{c0} of the (hypothetic) clean material utilizing the identity

$$\frac{1}{N(0)V_0} = \ln\frac{T}{T_{c0}} + 2\pi T\sum_{\omega>0}^{\omega_D}\frac{1}{\omega}, \tag{12}$$

which is equivalent to the BCS relation $\Delta_0(0)=\pi T_{c0}e^{-\gamma}=2\omega_D\exp[-1/N(0)V_0]$; γ is the Euler constant. Substitute Eq. (12) in Eq. (11) and replace ω_D with infinity due to fast convergence,

$$\frac{\Psi}{2\pi T}\ln\frac{T_{c0}}{T} = \sum_{\omega>0}^{\infty}\left(\frac{\Psi}{\omega} - \langle\Omega f\rangle\right). \tag{13}$$

GL domain and $T_c(\tau,\tau_m)$

Near T_c, $f\ll 1$, $g=1-ff^+/2$ and Eq. (3) reads

$$\frac{1}{2}\boldsymbol{v}\boldsymbol{\Pi}f = \Delta - \omega_+ f + \frac{\langle f\rangle}{2\tau_-} - \frac{ff^+}{2}\left(\Delta + \frac{\langle f\rangle}{2\tau_-}\right) + \frac{f\langle ff^+\rangle}{4\tau_+}. \tag{14}$$

Here,

$$\omega_+ = \omega + 1/2\tau_+, \tag{15}$$

and the terms at the RHS are arranged according to their order in powers of $\delta t=1-T/T_c$: the terms on the upper line are of the order $\delta t^{1/2}$ whereas on the lower line $\sim\delta t^{3/2}$. Note that on the LHS, $\Pi f\sim f/\xi\sim\delta t$.

One looks for the solution $f=f_1+f_2+\cdots$, where $f_1\sim\delta t^{1/2}$, $f_2\sim\delta t$, etc. Hence, in the lowest order

$$0 = \Delta - \omega^+ f_1 + \langle f_1\rangle/2\tau_-. \tag{16}$$

Taking the average over the Fermi surface one obtains

$$\langle f_1\rangle = \langle\Delta\rangle/\omega_m, \quad \omega_m = \omega + 1/\tau_m \tag{17}$$

(note the difference in definitions of ω_+ and ω_m). Hence,

$$f_1 = \frac{1}{\omega^+}\left(\Delta + \frac{\langle\Delta\rangle}{2\tau_-\omega_m}\right). \tag{18}$$

Similarly, comparing terms of the order δt one gets

$$\langle f_2\rangle = 0, \quad f_2 = -\frac{1}{2\omega_+^2}\boldsymbol{v}\boldsymbol{\Pi}\left(\Delta + \frac{\langle\Delta\rangle}{2\tau_-\omega_m}\right). \tag{19}$$

Evaluation of higher-order corrections for arbitrary Δ anisotropy is increasingly cumbersome unlike the case $\langle\Delta\rangle=0$ for which one finds for the uniform state,

$$f_3 = -\frac{\Delta}{2\omega_+^3}\left(\Delta^2 - \frac{\langle\Delta^2\rangle}{2\tau_+\omega^+}\right), \quad \langle f_3\rangle = 0. \tag{20}$$

The critical temperature of materials with anisotropic order parameter is suppressed by scattering. In zero field, all quantities are coordinate independent; besides, as $T\to T_c$, $g\to 1$. Therefore, one can utilize f of Eq. (18) in the lowest order and substitute it in the self-consistency Eq. (13) to obtain for T_c,

$$\frac{1}{2\pi T_c}\ln\frac{T_{c0}}{T_c}=\sum_{\omega>0}^{\infty}\left(\frac{1}{\omega_c}-\frac{1}{\omega_c^+}-\frac{\langle\Omega\rangle^2}{2\omega_c^m\omega_c^+\tau_-}\right), \tag{21}$$

where the subscript c shows that ω's are taken at T_c. The sums here are expressed in terms of digamma functions,

$$\ln\frac{T_{c0}}{T_c}=\psi\left(\frac{1+\rho_+}{2}\right)-\psi\left(\frac{1}{2}\right)-\langle\Omega\rangle^2\times\left[\psi\left(\frac{1+\rho_+}{2}\right)-\psi\left(\frac{1}{2}+\rho_m\right)\right]. \tag{22}$$

In this form, this generalization of the AG Eq. (1) for the T_c suppression for any (Born) scattering and for an arbitrary symmetry of the order parameter is due to Openov.[6] Within a two-band scheme, the T_c suppression has been given by Golubov and Mazin.[7]

If $T_c\to 0$, one can use asymptotic expansion $\psi(z)=\ln z-1/2z$ for large arguments since $\rho,\rho_m\to\infty$. The leading term then gives that $T_c=0$ when scattering times satisfy the relation

$$\frac{1}{\tau_m}\left(\frac{\tau_m}{2\tau_+}\right)^{1-\langle\Omega\rangle^2}=\frac{\Delta_0(0)}{2}. \tag{23}$$

Here, $\Delta_0(0)=\pi T_{c0}e^{-\gamma}$ is the zero-temperature gap of the scattering-free material. Clearly, this reduces to the AG critical rate $1/\tau_m=\Delta_0(0)/2$ for isotropic order parameters. If $\langle\Omega\rangle=0$ (e.g., for the d wave), one has the critical combined rate: $1/\tau_+=\Delta_0(0)$.

For a general anisotropy $\langle\Omega\rangle\neq 0,1$ in the absence of spin-flip scattering ($\tau_m\to\infty$) the LHS is zero and Eq. (23) has no solutions for τ, i.e., T_c does not turn zero for any τ. However, a finite τ at which $T_c=0$ does exists for any finite τ_m. One can show that near the critical value $\tau_{+,crit}^{\langle\Omega\rangle^2-1}=\Delta_0(0)(\tau_m/2)^{\langle\Omega\rangle^2}$, the critical temperature behaves similarly to the AG gapless case: $T_c\propto(\tau_+-\tau_{crit}^+)^{1/2}$.

Combining Eqs. (13) and (21) one excludes the unphysical T_{c0},

$$\frac{\Psi}{2\pi T}\ln\frac{T_c}{T}=\sum_{\omega>0}^{\infty}\left(\frac{\Psi}{t\omega_c^+}+\frac{\Psi\langle\Omega\rangle^2}{2t\omega_c^m\omega_c^+\tau_-}-\langle\Omega f\rangle\right), \tag{24}$$

where $t=T/T_c$.

III. STRONG PAIRBREAKING, $T_c\ll T_{c0}$

Situations of interest here are of T_c strongly suppressed relative to T_{c0}. It is convenient for this purpose to rearrange Eq. (24) by adding and subtracting Ψ/ω_+ under the sum. The following manipulation is self-evident:

$$2\pi T\sum_{\omega>0}\left(\frac{1}{t\omega_c^+}-\frac{1}{\omega^+}\right)=\sum_{n=0}^{\infty}\left(\frac{1}{n+1/2+\rho_+/2}-\frac{1}{n+1/2+\rho_+/2t}\right)=\psi\left(\frac{\rho_+}{2t}+\frac{1}{2}\right)-\psi\left(\frac{\rho_+}{2}+\frac{1}{2}\right)$$

$$\approx-\ln t-\frac{1-t^2}{6\rho_+^2}. \tag{25}$$

The parameter ρ_+ is large if $T_c\to 0$ and one can use large arguments asymptotics of the digamma functions. Combining Eqs. (24) and (25) one obtains the self-consistency equation in the form

$$\frac{\Psi(1-t^2)}{12\pi T\rho_+^2}=\sum_{\omega>0}^{\infty}\left(\frac{\Psi}{\omega_+}+\frac{\Psi\langle\Omega\rangle^2}{2t\omega_c^m\omega_c^+\tau_-}-\langle\Omega f\rangle\right). \tag{26}$$

A. GL equation, $\langle\Delta\rangle=0$

The GL equations are obtained by utilizing smallness of Δ/ω and of $\boldsymbol{v}\boldsymbol{\Pi}\Delta/\omega^2$ near T_c. Hence, one can use Eqs. (18)–(20) for f and the self-consistency Eq. (26). Since f_3 is calculated above for $\langle\Delta\rangle=0$, only this case will be considered below. For the case of exclusively transport scattering ($\tau_m=\infty$), the GL equations have been derived in Ref. 8. It is done here for a finite τ_m.

To write the self-consistency Eq. (26) near T_c one has to express $\langle\Omega f\rangle$ with the help of Eq. (14). To this end, one applies $\langle\Omega/\omega_+\ldots\rangle$ to Eq. (14) keeping terms up to the order $\delta t^{3/2}$,

$$\langle\Omega f\rangle=\frac{\Psi}{\omega_+}-\left\langle\frac{\Omega}{2\omega_+}\boldsymbol{v}\boldsymbol{\Pi}f\right\rangle-\left\langle\frac{\Delta f_1^2\Omega}{2\omega_+}\right\rangle+\frac{\langle f_1^2\rangle\langle\Omega f_1\rangle}{4\tau_+\omega_+} \tag{27}$$

and substitutes the result in Eq. (26),

$$\frac{\Psi\delta t}{6\pi T\rho_+^2}=\sum_{\omega>0}\Psi^3\left(\frac{\langle\Omega^4\rangle}{2\omega_+^3}-\frac{1}{4\tau_+\omega_+^4}\right)-\frac{\langle\Omega^2 v_i v_k\rangle\Pi_i\Pi_k\Psi}{4}\sum_{\omega>0}\frac{1}{\omega_+^3}. \tag{28}$$

All terms here are of the order $\delta t^{3/2}$ so that ω's are taken at T_c.

The sums on RHS are expressed in terms of ψ functions of parameters ρ which are large in the situation of interest. One obtains after straightforward algebra,

$$\Psi\left(1-\frac{\Psi^2}{\Psi_0^2}\right)=-(\xi^2)_{ik}\Pi_i\Pi_k\Psi, \tag{29}$$

where the zero-field order parameter is given by

$$\Psi_0^2=\frac{4\pi^2T_c^2\delta t}{3\langle\Omega^4\rangle-2} \tag{30}$$

and the tensor of squared coherence length is

$$(\xi^2)_{ik}=\frac{3\langle\Omega^2 v_i v_k\rangle}{8\pi^2T_c^2\delta t}. \tag{31}$$

B. Slopes of the upper critical field at T_c

For a uniaxial material, the slope of the upper critical field along the c direction near T_c is given by

$$\frac{dH_{c2,c}}{dT} = -\frac{4\pi\phi_0 k_B^2}{3\hbar^2\langle\Omega^2 v_a^2\rangle}T_c \tag{32}$$

(in common units). Although the pairbreaking parameters do not enter this result explicitly, they affect $H_{c2,c}$ and its slope via $T_c(\rho_+)$.

It is worth recalling that in isotropic s-wave materials, the slope $H'_{c2} \propto T_c$ in the clean limit because

$$H_{c2} = -\frac{\phi_0(1-T/T_c)}{2\pi\xi_0^2}, \quad \xi_0 \sim \frac{\hbar v}{\Delta_0} \propto \frac{1}{T_c}. \tag{33}$$

For the dirty case H'_{c2} is T_c independent; indeed,

$$H_{c2} \propto \frac{1-T/T_c}{\xi_0 \ell}, \tag{34}$$

where ℓ is the mean-free path. The proportionality of H'_{c2} to T_c is a property of the AG gapless state. In our case, the result (32) is obtained for a strong pairbreaking in materials with anisotropic order parameter.

IV. ARBITRARY TEMPERATURES

For a strong T_c suppression, as was shown by AG, the formalism used for derivation of the GL equations near T_c applies at *all temperatures*.[1] Physically, this is because the pairbreaking suppresses the order parameter so that the expansion in powers of Δ and its derivatives can be done at any T. The calculation then proceeds in a manner similar to that near T_c.

In the zero-field state, we look for solutions of Eilenberger equations as $f_0 = f^{(1)} + f^{(2)} + \cdots$, where $f^{(1)} \sim \Delta$, $f^{(2)} \sim \Delta^2$, etc. We then obtain

$$f_0 = \frac{\Delta}{\omega_+} + \frac{\Delta}{2\omega_+^3}\left(\frac{\langle\Delta^2\rangle}{2\tau_+\omega_+} - \Delta^2\right) + \mathcal{O}(\Delta^5). \tag{35}$$

One can see that even at low temperatures $f_{0,max} \sim \tau_+ T_c \sim 1/\rho_+ \ll 1$ because for a strong pairbreaking $T_c \to 0$. This is a quasiclassical justification for the AG statement that $f \ll 1$ at all T's.

The T dependence of Δ (or Ψ) is obtained from the self-consistency equation. For a strong pairbreaking, this is Eq. (26) with $\langle\Delta\rangle = 0$ in the case considered. Using f of Eq. (35), one obtains for the field-free state,

$$\Psi^2 = \frac{2\pi^2(T_c^2 - T^2)}{3\langle\Omega^4\rangle - 2}, \tag{36}$$

this reduces to the result (30) near T_c and to AG form for $\Omega = 1$.

V. SPECIFIC HEAT

Eilenberger Eqs. (3) and (13) in zero field can be obtained minimizing the functional[4]

$$\mathcal{F} = N(0)\left[\Psi^2 \ln\frac{T}{T_{c0}} + 2\pi T\sum_{\omega>0}\left(\frac{\Psi^2}{\omega} - \langle I\rangle\right)\right], \tag{37}$$

$$I = 2\Delta f + 2\omega(g-1) + \frac{f\langle f\rangle}{2\tau^-} + \frac{(g\langle g\rangle - 1)}{2\tau^+}. \tag{38}$$

The function g here is an abbreviation for $\sqrt{1-f^2}$. Taking account of the self-consistency Eq. (13), we obtain the energy difference between the normal and superconducting states,

$$-\frac{F_s - F_n}{2\pi TN(0)} = \sum_{\omega>0}\left\langle \Delta f + 2\omega(g-1) + \frac{f\langle f\rangle}{2\tau^-} + \frac{g\langle g\rangle - 1}{2\tau^+}\right\rangle. \tag{39}$$

One can check that this reduces to the known result for isotropic s-wave cases with or without pairbreaking.[9] This offers a straightforward way to calculate the specific heat near T_c. The calculation, in general, is tedious because one has to keep track of terms up to $\Delta^4 \propto \delta t^2$. Again, only the case $\langle\Delta\rangle = 0$ is considered.

Up to the fourth order in Δ we have with the help of Eq. (35),

$$g = 1 - \frac{\Delta^2}{2\omega_+^2} + \frac{3\Delta^4}{8\omega_+^4} - \frac{\Delta^2\langle\Delta^2\rangle}{4\tau_+\omega_+^5}, \tag{40}$$

where all ω's are taken at T_c. Substituting these in the energy difference one finds for large ρ_+,

$$-\frac{F_s - F_n}{2\pi TN(0)} = \frac{\Psi^4}{4}\sum\left(\frac{\langle\Omega^4\rangle}{\omega_+^3} - \frac{1}{2\tau_+\omega_+^4}\right) = \frac{\Psi^4(3\langle\Omega^4\rangle - 2)\tau_+^2}{12\pi T}. \tag{41}$$

Thus, with Eq. (30) for the zero-field order parameter, the energy difference between the normal and superconducting states reads

$$F_n - F_s = \frac{2\pi^4 N(0)\tau_+^2}{3\hbar^2(3\langle\Omega^4\rangle - 2)}k_B^4(T_c^2 - T^2)^2 \tag{42}$$

in common units. The electronic specific heat follows

$$\frac{C_s}{T} = (\gamma_e - \mu T_c^2/3) + \mu T^2, \tag{43}$$

$$\mu = \frac{8\pi^4 k_B^4 N(0)\tau_+^2}{\hbar^2(3\langle\Omega^4\rangle - 2)}. \tag{44}$$

Here, $\gamma_e T$ is the electronic heat capacity of the normal phase. Hence, not only C_s/T is linear in T^2 but the linear part of C_s is reduced,

$$\gamma_{eff} = \gamma_e - \mu T_c^2/3 \tag{45}$$

so that the reduction is proportional to T_c^2.

The specific-heat jump at T_c is

$$\Delta C = C_s - C_n = \frac{2\mu}{3} T_c^3. \tag{46}$$

Within a weak-coupling scheme, ΔC for an arbitrary Δ anisotropy has been obtained in Ref. 10.

VI. LONDON PENETRATION DEPTH

Let us now consider the response to a small current

$$\boldsymbol{j} = -4\pi|e|N(0)T \,\mathrm{Im} \sum_{\omega>0} \langle \boldsymbol{v} g \rangle. \tag{47}$$

Weak currents leave the order-parameter modulus unchanged but cause the condensate to acquire an overall phase $\theta(\boldsymbol{r})$. One then looks for perturbed solutions as

$$\Delta = \Delta_0 e^{i\theta}, \quad f = (f_0 + f_1)e^{i\theta},$$

$$f^+ = (f_0 + f_1^+)e^{-i\theta}, \quad g = g_0 + g_1, \tag{48}$$

where the subscript 1 denotes small corrections to the uniform state f_0, g_0. In the London limit, the only coordinate dependence is that of the phase θ, i.e., f_1, g_1 are $\boldsymbol{r}$ independent. The Eilenberger equations provide the corrections among which we need only g_1,

$$g_1 = \frac{i f_0^2 \boldsymbol{v}\boldsymbol{P}}{2\omega_+} = \frac{i\Delta_0^2 \boldsymbol{v}\boldsymbol{P}}{2\omega_+^3}. \tag{49}$$

Here, $\Delta_0 = \Psi_0 \Omega$ is the zero-field order parameter, see Eq. (30), and $\boldsymbol{P} = \nabla\theta + 2\pi \boldsymbol{A}/\phi_0 \equiv 2\pi \boldsymbol{a}/\phi_0$ with the "gauge-invariant vector potential" $\boldsymbol{a}$.

Substitute $g_0 + g_1$ in Eq. (47) and compare the result with $4\pi j_i/c = -(\lambda^2)_{ik}^{-1} a_k$ to obtain

$$(\lambda^2)_{ik}^{-1} = \frac{8\pi^2 e^2 N(0) T}{c^2} \langle v_i v_k \Omega^2 \rangle \Psi^2 \sum_{\omega>0} \frac{1}{\omega_+^3}$$

$$= \frac{16\pi^3 e^2 N(0) k_B^2 \tau_+^2}{c^2 \hbar^2 (3\langle \Omega^4 \rangle - 2)} \langle v_i v_k \Omega^2 \rangle (T_c^2 - T^2), \tag{50}$$

where the second line is for $\rho_+ \gg 1$.

It is now easy to obtain the low-T behavior of $\Delta\lambda_{ab} = \lambda_{ab}(T) - \lambda_{ab}(0)$ for a uniaxial material,

$$\Delta\lambda_{ab} = \eta \frac{T^2}{T_c^3}, \quad \eta = \frac{c\hbar}{8\pi k_B \tau_+} \sqrt{\frac{3\langle \Omega^4 \rangle - 2}{\pi e^2 N(0) \langle v_a^2 \Omega^2 \rangle}}. \tag{51}$$

One readily obtains for $T=0$,

$$\lambda_{ab}(0) = 2\eta/T_c. \tag{52}$$

VII. SCALING RELATIONS

The relations (43) and (44) for the specific heat, Eq. (32) for the slope of H_{c2}, and Eqs. (50)–(52) for the penetration depth contain material parameters: the density of states $N(0)$, Fermi velocities v_i, the parameter Ω of the gap anisotropy, and the combined scattering time τ_+. One can express the ratio $(3\langle \Omega^4 \rangle - 2)/\tau_+^2$ in terms of the coefficient μ of Eq. (43) and substitute the result in the expression for $\lambda(0)$, thus establishing a relation between the behavior of the specific heat and the penetration depth. $\langle v_a^2 \Omega^2 \rangle$ can in turn be expressed in terms of the H_{c2} slope of Eq. (32). Surprisingly, all material parameters drop out of the expression

$$\frac{\mu \lambda_{ab}^2(0) T_c^3}{|H'_{c2,c}|} = \frac{3\phi_0}{8\pi^2} \tag{53}$$

with a universal quantity on the RHS.

The relation (53) involves the slope $\mu = d(C_s/T)/d(T^2)$ and the zero-T penetration depth $\lambda(0)$, both of which are difficult to access experimentally [the first due to the necessity to subtract the phonon contribution from the measured C/T, the second because usually only the deviation of $\lambda(T)$ from $\lambda(0)$ is measured]. Fortunately, one can avoid these difficulties using another scaling relation. Writing Eq. (51) as $\Delta\lambda_{ab} = \beta T^2$ with $\beta = \eta/T_c^3$ one finds

$$\frac{\Delta C \beta^2 T_c^4}{|H'_{c2,c}|} = \frac{\phi_0}{16\pi^2} = 1.27 \times 10^{-9} \text{ G cm}^2. \tag{54}$$

Measurement of the jump ΔC does not require the phonon contribution be subtracted and determination of $\beta = d(\Delta\lambda)/d(T^2)$ does not require knowledge of $\lambda(0)$.

VIII. DISCUSSION

The scaling relations (53) and (54) are derived for a single band materials with an order parameter satisfying $\langle \Delta \rangle = 0$. However, the very fact that no material parameters enter these relations suggests that they might be applicable for a broader class of materials with a strong pairbreaking. An indication of such a possibility comes from AG work on the gapless state in *isotropic* materials with magnetic impurities. One can easily deduce from their paper the quantities of interest here,

$$\Delta C = \frac{4\pi^4 k_B^4 N(0) \tau_m^2}{3\hbar^2} T_c^3, \quad H'_{c2} = \frac{\pi \phi_0 k_B^2 \tau_m}{2\hbar^2 v^2 \tau} T_c \tag{55}$$

and

$$\beta = \left. \frac{d\lambda}{d(T^2)} \right|_{T=0} = \frac{c\hbar^2}{8\pi^4 k_B^3 T_c^3} \sqrt{\frac{m}{\pi N e^2 \tau \tau_m}}, \tag{56}$$

where N is the density of carriers of a mass m. Direct substitution shows that the relation (54) is satisfied.

Four quantities on the LHS of Eqs. (53) and (54) are measured in independent experiments so that these results might be used as a stringent test of the pairbreaking scenario. Materials in which the scaling relations discussed here have a good chance to hold are suggested below.

Clearly, $\langle \Delta \rangle = 0$ for the d-wave compounds. Hence, the underdoped cuprates with a strongly suppressed T_c may be among materials satisfying the above scaling formulas and the relations (53) and (54), in particular. It is worth noting that the scaling Eq. (52) is supported by the surface

resistance[11] and optical data[12] for a series of samples of $YBa_2Cu_3O_{6+x}$ with T_c varying from 3 to 17 K.

1. $CeCoIn_5$

Another example where the model developed here may work is $CeCoIn_5$. This is a clean heavy-fermion d-wave superconductor[13,14] with $T_c=2.3$ K; in the normal phase, the material is a paramagnet.[15] Note that quantities entering the relation (53) are either for zero field [μ and $\lambda(0)$] or for temperatures near T_c (H'_{c2}) so that the domain of interest here is not affected by complications related to paramagnetic constrains or to possible Fulde-Ferrell-Larkin-Ovchinnikov phase.

The penetration depth for this material is found to behave according to

$$\lambda_{ab} = 358 \text{ nm}/\sqrt{1-t^2} \tag{57}$$

practically at all temperatures that agrees with the strong pairbreaking prediction, Eq. (50).[16] Moreover, C_s/T (where C_s is the electronic part of the specific heat) is close to being linear in T^2 at low temperatures[17] in agreement with Eq. (43). Reading the data on C_s/T from Fig. 2b of Ref. 17, one estimates the coefficient of the T^2 term as $\mu \approx 3.5 \times 10^4$ erg/cm^3 K^3. The slope of the upper critical field is also known: $H'_{c2,c} \approx 11.5$ T/K as reported, e.g., in Ref. 16. Then, the LHS of the scaling relation (53) is estimated as 4.8×10^{-9} G cm^2. This is reasonably close to the universal number $3\phi_0/8\pi^2 = 7.6\times 10^{-9}$ G cm^2 on the RHS of this relation.

Similar agreement is found if one tries to check the scaling relation (54). According to Ref. 17, for the material of interest $\Delta C/T_c \approx 2$ J/mol K^2 that translates to $\Delta C \approx 3.6$ erg/cm^3 K. The low-temperature behavior of the penetration depth following from Eq. (57) $\Delta\lambda_{ab} \approx \beta T^2$ with $\beta \approx 3.4\times 10^{-6}$ cm/K^2. Using these numbers one obtains

$$\frac{\Delta C \beta^2 T_c^4}{|H'_{c2,c}|} \approx 1.2 \times 10^{-9} \text{ G cm}^2, \tag{58}$$

that is notably close to $\phi_0/16\pi^2 = 1.27\times 10^{-9}$ G cm^2.

2. Iron-based materials

The model used here is restricted to materials with one Fermi-surface sheet. Clearly, multiband iron-based materials do not belong to this group. However, quite a few experimentally measured properties of these materials behave in agreement with the results derived above for a strong pairbreaking. Perhaps, the most remarkable of those properties is the specific-heat jump ΔC proportional to T_c^3 reported by Bud'ko, Ni and Canfield for "122" series of $Ba(Fe_{1-x}Co_x)_2As_2$ and $Ba(Fe_{1-x}Ni_x)_2As_2$ with T_c ranging from a few up to about 40 K.[18] The data shown in Fig. 1 are in a profound agreement with the strong pairbreaking result (46).

Moreover, the data for slopes of H_{c2} compiled in Ref. 2 for a number of "1111" compounds are clearly show $H'_{c2} \propto T_c$. A similar compilation for "122" series is also consistent with the scaling Eq. (32) albeit with a considerable scatter.

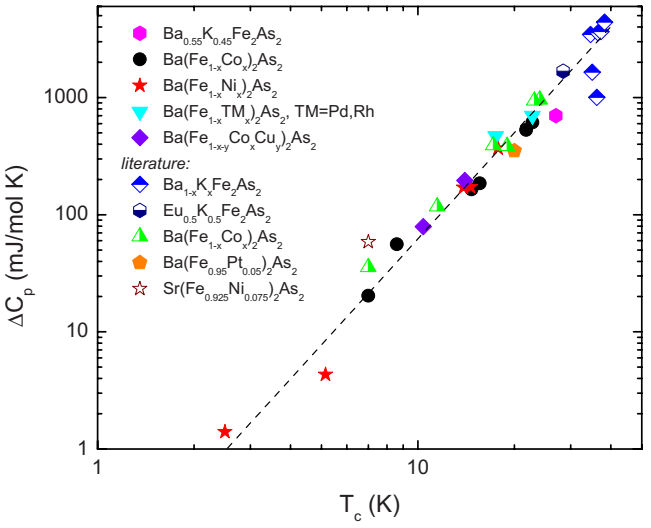

FIG. 1. (Color online) The specific-heat jump versus T_c for a few 122 compounds shown on a log-log plot. The dashed line corresponds to $\Delta C \propto T_c^3$. The data for the upper four entries in the legend are from Ref. 18; the new data points for mixed Co-Cu doping are taken by the same group, but have not been included in the original publication. The literature data are taken from Refs. 19–25.

The third piece of evidence in favor of the strong pairbreaking in iron-pnictides is the low-temperature behavior of the London penetration depth $\lambda(T)$. The power-law behavior of $\Delta\lambda_{ab} \propto T^n$ with $n \approx 2$ has been seen in many iron-based materials.[19] Moreover, the prefactor η/T_c^3 in Eq. (51) extracted from the data varies from one material to another approximately as $1/T_c^3$ that further supports the pairbreaking scenario.

At this point, there are not yet enough data for a reliable check of the scaling relation (53) or (54) for which one needs all quantities entering these relations for the same sample. Still, an attempt to roughly estimate, e.g., the RHS of Eq. (54) for the optimally doped $Ba(Fe_{1-x}Co_x)_2Fe_2As_2$ gives a number $\sim 5\times 10^{-9}$ G cm^2 of a correct order of magnitude.

Notwithstanding a good agreement of the data with the strong pairbreaking results $\Delta C \propto T_c^3$, $H'_{c2} \sim T_c$, and $\Delta\lambda_{ab} \propto T^2/T_c^3$, applicability of the model proposed here to iron-based materials is still in question. Besides simplifications mentioned above of a one-band weak coupling with the scattering in a weak Born limit, the question of what physical meaning, if any, one should associate with the parameter T_{c0} which at least implicitly present in any AG-based model. The point is that the specific-heat data of Fig. 1 suggest that even in the materials with $T_c \sim 40$ K the pairbreaking is still strong. Adopting a rough estimate of $N(0)$ and the experimental ΔC, one can evaluate τ_+ along with the scattering parameter ρ_+, that allows one to estimate T_{c0} from Eq. (1). This order-of-magnitude estimate places T_{c0} well in the room-temperature range. Does this mean that by a clever choice of dopants one can get rid of the pairbreaking scattering and, therefore, push critical temperatures of iron-based materials to much higher values than those currently observed? The answer is, probably, negative. The point is that

the spin fluctuations in iron pnictides might be responsible for the effective coupling forming the Cooper pairs and, at the same time, cause the pairbreaking. If so, one cannot get rid of the pairbreakers without destroying the superconductivity itself.

To conclude: scaling relations for anisotropic superconductors with a strong pairbreaking are proposed. The formulas relating independently measured specific-heat jumps, slopes of the upper critical field, and the low-temperature behavior of the penetration depth are derived within a one-band weak coupling scheme but, being independent of material parameters, may hold for a broader class of materials.

Note added in proof: Recently reported μSR data,[26] confirm the scaling $\lambda(0) \propto 1/T_c$ of Eq. (52).

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