Strong influence of nonlocal nonequilibrium effects on the dynamics of the order parameter in a phase-slip center: Ring studies

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(Received 25 December 2009; revised manuscript received 25 March 2010; published 19 May 2010)

We study the influence of the inelastic relaxation time $\tilde{\tau}_E$ of the quasiparticle distribution function $f(E)$ on the phase slip process in quasi-one-dimensional superconducting rings at a temperature close to the critical temperature T_c. We find that the initial time of growth of the order parameter $|\Delta|$ in the phase slip core after the phase slip is a nonmonotonic function of $\tilde{\tau}_E$ which has a maximum at $\tilde{\tau}_E \simeq \tilde{\tau}_{GL} = \pi\hbar/8k_B(T_c-T)$ and has a tendency to saturate for large $\tilde{\tau}_E \gg \tilde{\tau}_{GL}$. The effective "heating" of the electron subsystem due to the increase in $|\Delta|$ in the phase slip center together with the above effect result in a nonmonotonic dependence of the number of subsequent phase slips on $\tilde{\tau}_E$ in rings of relatively large radius (in which each phase slip reduces the current density to a small fraction of its initial value). During the phase slip process the order parameter distribution has two peaks near the phase slip core due to the diffusion of the nonequilibrium quasiparticles from that region.

DOI: 10.1103/PhysRevB.81.184521 PACS number(s): 74.25.Op, 74.20.De, 73.23.−b

I. INTRODUCTION

More than 30 years ago it was found that the superconducting state can partially survive in quasi-one-dimensional superconducting wire/bridge [with width/thickness less or compared with the coherence length $\xi(T)$)] even if the current density becomes larger than the depairing current density $j_{\rm dep}$. In this state, a so-called phase slip center (PSC) appears—a finite region of size of about $\xi(T)$ where the order parameter oscillates with the Josephson frequency $\omega_J = 2eV/\hbar$ and a voltage drop V occurs over a large region around the PSC [with size $L_{\rm PSC} \gg \xi(T)$]. When the order parameter $\Delta = |\Delta|e^{i\phi}$ passes through zero in one point inside the phase slip core the phase difference of the order parameter changes(slips) by 2π near this point—which is responsible for the name of this phenomena. Such a state was intensively studied experimentally in the 1970s and 1980s (for reviews, see Refs. 1–3 and the book of Tinkham[4]) and its existence was also confirmed in wide thin superconducting films with a current density distribution which is nearly uniform over the width $W \gg \xi(T)$ of the sample[5–8] (it was called a phase slip line).

The interest to this subject was renewed recently when new phenomena were observed experimentally in superconducting nanowires. We may mention the S behavior of current-voltage characteristics in voltage driven regime,[9] negative magnetoresistance,[10–14] and enhancement of the critical current in weak magnetic fields.[13,15,16] The origin of the last phenomena is still under debate[17] and one of the possible explanation is the dependence of the order parameter dynamics in the phase slip center on the applied magnetic field and the length of the wire.[18] Besides there is practical interest in the understanding of the physics of the phase slip process because new devices for the detection of single photons[19] and some organic molecules[20] are based on the local destruction of superconductivity in a current carrying wire and the appearance of voltage pulses due to phase slips.

The first theoretical model (see Ref. 21) of the phase slip process appeared soon after the discovery of this phenomena[22] and the authors proposed a phenomenological description of the *time-averaged* properties of the phase slip process (see for example[4]). Its validity was verified directly in the experiment of Dolan and Jackel[23] and showed good agreement with the majority of experiments but it did not reveal the conditions for the existence of the order parameter oscillations. Numerical solution of the full set of time-dependent microscopic equations describing this process at arbitrary temperature (see for example Ref. 24) looks problematic even now and the main tool for theoretical investigation was the time-dependent Ginzburg-Landau equations in the so-called local approximation (the time and space variation of the order parameter are much larger than the inelastic relaxation time of the quasiparticle distribution function $\tilde{\tau}_E$ and its decay length $\tilde{L}_E = \sqrt{D\tilde{\tau}_E}$, where D is the diffusion coefficient). They were derived in the 'dirty' limit from the quasiclassical equations for the Green functions in Ref. 25 and contain the time $\tilde{\tau}_E$ as a parameter (in the limit of $\tilde{\tau}_E|\Delta|/\hbar \ll 1$ they pass to ordinary time-dependent Ginzburg-Landau equations—see Ref. 4). Numerical solution of these equations[2,25] qualitatively supported the main features of the phenomenological model: i.e., the existence of the order parameter oscillations in the phase slip core and the finite normal current near the phase slip core on a scale much larger than the coherence length. But no quantitative agreement with the experimental results was found[2] because the majority of the experiments were made at temperatures where the local approximation model is inapplicable.

A first attempt to go beyond the local approximation was made in Ref. 26. The author assumed that the deviation from equilibrium of the odd in energy (also called longitudinal) $f_L(E) = f(-E) - f(E)$ part of $f(E)$ is negligible and concentrated on the effect of even in energy (transverse) $f_T(E) = 1 - f(-E) - f(E)$ part of $f(E)$. In the present paper we show that this assumption is invalid and f_L plays a significant role in the dynamics of the order parameter. Besides even in the local approximation the deviation of f_L from equilibrium leads to an important effect—it considerably increases the relaxation time of the order parameter.[27]

The majority of the theoretical works on these phenomena were made in the regime of constant applied current. However, it is rather difficult to find a solution of the microscopic equations (describing the dynamics of Δ) satisfying the condition $j=\text{const}$ because in the boundary conditions for the quasiparticle distribution function enters the voltage, not the current[28,29] (for further discussion see text below Eq. (4) in Sec. II). Because of that, as a model system, we study the phase slip process in a superconducting ring (with radius R) placed in an applied magnetic field H_{appl} (in this case we may use periodical boundary conditions for all physical variables). If we start from the state with zero vorticity $L_v = \oint \nabla \phi dx / 2\pi = 0$ then at some critical magnetic field the current density in the ring reaches j_{dep} and the superconducting state becomes unstable. The starting phase slip process decreases j approximately by $\delta j \simeq j_{\text{dep}} \xi(T)/R$ after each phase slip (because $j \sim |\nabla \phi - 2eA/c|$, A is the vector potential and has the same sign as $\nabla \phi$ and each phase slip increases $|\nabla \phi|$ by $1/R$). The phase slip process stops when j is below some critical value j_{c1} and we are interested to obtain the number of phase slips $N_{ps} \sim (j_{\text{dep}} - j_{c1})/\delta j$ during the transition period. As we show below j_{c1} and hence N_{ps} are defined by the dynamics of the order parameter in the phase slip core which depends on $\tilde{\tau}_E$.

The paper is organized as follows. In Sec. II, we present our theoretical model. In Secs. III and IV we discuss the phase slip process in small and large rings, correspondingly. Finally, in Sec. V, we discuss our results and their relation with the phase slip process in a current carrying wire and recent experiments on transport measurements in superconducting nanowires.

II. MODEL

To simulate the phase slip process at a temperature close to T_c we use a set of equations first derived in Refs. 25 and 30 (for a comprehensive derivation of these equations see Ref. 24) for "dirty" superconductors. They consist of the Usadel equation for the normal $\alpha(E) = \cos\Theta = N_1(E) + iR_1(E)$ and anomalous $\beta(E) = \sin\Theta = N_2(E) + iR_2(E)$ Green's functions

$$\frac{d^2\Theta}{dx^2} + [(2iE - 1/\tau_E) - Q^2 \cos\Theta]\sin\Theta + 2|\Delta|\cos\Theta = 0, \tag{1}$$

(where $Q = \partial\phi/\partial x - 2eA/c$ is the superfluid velocity), the Boltzman-like equations for the longitudinal $f_L(E)$ and transverse $f_T(E)$ parts of the quasiparticle distribution function $2f(E) = 1 - f_L(E) - f_T(E)$

$$N_1 \frac{\partial f_L}{\partial t} - \nabla[(N_1^2 - R_2^2)\nabla f_L] = -\frac{N_1}{\tau_E}(f_L - f_L^0) - R_2 \frac{\partial f_L^0}{\partial E}\frac{\partial|\Delta|}{\partial t} + 2N_2R_2Q\nabla f_T, \tag{2a}$$

$$N_1 \frac{\partial}{\partial t}\left(f_T + \varphi\frac{\partial f_L^0}{\partial E}\right) - \nabla((N_1^2 + N_2^2)\nabla f_T) = -\frac{N_1}{\tau_E}\left(f_T + \varphi\frac{\partial f_L^0}{\partial E}\right) - N_2|\Delta|\left(2f_T - \frac{\partial f_L^0}{\partial E}\frac{\partial\phi}{\partial t}\right) + 2N_2R_2Q\nabla f_L, \tag{2b}$$

[where φ is an electrostatic potential and $f_L^0(E) = \tanh(E/2k_BT)$ is the equilibrium Fermi-Dirac distribution function of the quasiparticles] and a time dependent equation for the complex order parameter $\Delta = |\Delta|e^{i\phi}$ (we write them separately for the dynamics of $|\Delta|$ and ϕ)

$$a_1\frac{\partial|\Delta|}{\partial t} = a_1\frac{\partial^2|\Delta|}{\partial x^2} + \left(1 - \frac{T}{T_c} - a_2|\Delta|^2 - a_1Q^2 - \Phi_1(x,t)\right)|\Delta|, \tag{3a}$$

$$a_1|\Delta|\frac{\partial\phi}{\partial t} = 2a_1\frac{\partial|\Delta|^2Q}{\partial x} - \Phi_2(x,t), \tag{3b}$$

which are similar to the ordinary time dependent Ginzburg-Landau equations (see chapter 10 in the book of Tinkham, for example[4]) but with the additional nonequilibrium terms

$$\Phi_1(x,t) = -\int_0^\infty R_2(f_L - f_L^0)dE/|\Delta|$$

and

$$\Phi_2(x,t) = -\int_0^\infty N_2 f_T dE/|\Delta|.$$

In Eqs. (1), (2a), (2b), (3a), and (3b), the order parameter and energy are scaled by Δ_0 ($\Delta_0 \simeq 1.76k_BT_c$ is the zero temperature order parameter value in the weak-coupling limit), distance is in units of the zero temperature coherence length $\xi_0 = \sqrt{\hbar D/\Delta_0}$ and time in units of $t_0 = \hbar/\Delta_0$. Because of this choice of scaling the numerical coefficients in Eqs. (3a) and (3b) are $a_1 \simeq 0.69$ and $a_2 \simeq 0.33$. The current is scaled in units of $j_0 = \Delta_0/(\xi_0\rho_n e)$, the superfluid velocity in units of $Q_0 = \hbar c/2e\xi_0$, and the electrostatic potential is in units of $\varphi_0 = \Delta_0/e$ (where ρ_n is the normal state resistivity and e is the electric charge). The dimensionless time $\tau_E = \tilde{\tau}_E\hbar/\Delta_0$ defines the relaxation time of the nonequilibrium *uniform* distribution of the quasiparticles and the dimensionless length $L_E^2 = D\tilde{\tau}_E/\xi_0^2 = \tilde{\tau}_E\Delta_0/\hbar$ defines the range over which the nonequilibrium *nonuniform* distribution of the quasiparticles decay in the sample.

The current in the ring can be found using the following equation:

$$j = 2a_1|\Delta|^2Q + \int_0^\infty ((N_1^2 + N_2^2)\nabla f_T + 2N_2R_2f_LQ)dE. \tag{4}$$

From Eqs. (1), (2a), (2b), (3a), (3b), and (4) it becomes clear why there is a problem with a wire carrying a constant current. One needs to find a solution of Eqs. (1), (2a), (2b), (3a), and (3b) (by sorting the boundary conditions for f_L and f_T, which contains the voltage as a parameter) that satisfies the condition $j = \text{const}$—which is an implicit problem and has to be solved at each time step.

Assuming that the effect of the free charges is negligible in the superconductor the electrostatic potential is determined by the following expression:

$$\varphi = -\int_0^\infty N_1 f_T dE. \quad (5)$$

We use periodic boundary conditions for the system of Eqs. (1), (2a), (2b), (3a), and (3b)

$$\Theta(-S/2) = \Theta(S/2), \quad f_L(-S/2) = f_L(S/2),$$

$$f_T(-S/2) = f_T(S/2), \quad \Delta(-S/2) = \Delta(S/2),$$

where $S=2\pi R$ is the circumference of the ring.

The characteristic inelastic scattering time (due to electron-phonon collisions) is typically in the range $\tau_E = 0.5-16\,000$ for all existing low-temperature superconductors [for example, in Nb $\tau_E \simeq 10^2$ and in Zn $\tau_E \simeq 10^4$ (Ref. 24)].

Having in mind a real experimental situation we introduced a pointlike defect (through the local suppression of the critical temperature by less than 0.6%) located at $x=0$. As a result the critical current density j_c (and critical magnetic field H_c) at which the superconducting state becomes unstable in an inhomogeneous ring is smaller (by less than 2%) than the depairing current density in the homogenous ring. In our simulations we use such a value of H_{appl} (inducing the screening current $j_c < j < j_{\text{dep}}$) which suppresses the order parameter only in the neighborhood of the defect. As an initial state we take $|\Delta|^2 = (1 - T/T_c - a_1 A^2)/a_2$, $\phi(x)=0$ and $A = H_{\text{appl}} R/2$, which corresponds to the homogenous solution of Eqs. (3a) and (3b) with the condition $L_v=0$ (state with zero vorticity).

We used the implicit Crank-Nicolson method for the numerical solution of Eqs. (1), (2a), (2b), (3a), and (3b). The coordinate step of the grid was equal to ξ_0 [which is much smaller than $\xi(T)$ for the studied temperature interval $0.9 < T/T_c < 0.98$] and the time step varied from $0.25t_0$ up to $2t_0$ depending on the variation of Δ in time.

III. RING WITH SMALL RADIUS

In Fig. 1, we present the time dependence of $|\Delta|$ in the phase slip center (at $x=0$) for a ring with circumference $S = 60\xi_0$ ($R \sim 9.5\xi_0$) at $T=0.9T_c$ with $\tau_E=500$ after turning on the overcritical magnetic field $H_{\text{appl}} = 1.018H_c$. Because of the small radius even one phase slip considerably lowers the current density (see Fig. 1) and we found only a transition with a change of vorticity of $\delta L_v = 1$.

The decay time of the order parameter (t_{init}) from the initial state to the moment when $|\Delta|=0$ in the phase slip core strongly depends on τ_E (see inset in Fig. 1). It could be understood if we look at Eq. (3a), which describes the dynamics of $|\Delta|$. Because $|\Delta|$ decreases the right hand side (RHS) of Eq. (3a) is negative. Nonequilibrium effects enter this equation via the potential Φ_1, which is negative in this regime ($\partial|\Delta|/\partial t < 0$) and roughly proportional to τ_E [see Eq. (2a) in the limit when one can neglect diffusion and time dependence for simplicity]. Therefore, potential Φ_1 makes the RHS of Eq. (3a) less negative and it may considerably increase the decay time of $|\Delta|$. From a physical point of view the negative sign of Φ_1 means cooling of the electron subsystem. This local dynamical cooling effectively makes the current in the system closer to j_{dep} [because $j_{\text{dep}}(T)$ increases with decreasing temperature] and it enhances the decay time of $|\Delta|$. The detailed dependence of t_{init} on τ_E is not the subject of the present paper and we presented these results just to point out that the initial time decay is not linearly proportional to τ_E as was expected from the local equilibrium approximation with an uniformly decaying order parameter.[4,27] Although the order parameter decreases in the phase slip center on a time scale much larger than τ_E the diffusion of the nonequilibrium quasiparticles from the phase slip core seems to play an important quantitative role on the value of t_{init} (see also discussion on page 254 in Ref. 27).

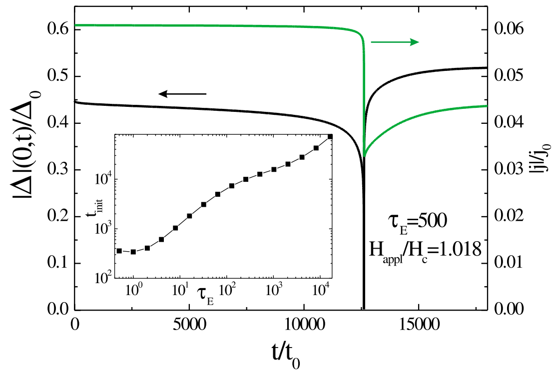

FIG. 1. (Color online) Time dependence of the order parameter (black curve) in the phase slip center and current density (green curve) for a ring with circumference $S=60\xi_0$ at $T=0.9T_c$ for $H_{\text{appl}} = 1.018H_c$ and $\tau_E=500$. In the inset we show the decay time of $|\Delta|(0)$ from the initial value $|\Delta|^2 = (1 - T/T_c - a_1 A^2)/a_2$ to zero as function of τ_E at the same applied magnetic field $H_{\text{appl}} = 1.018H_c$.

It is also clear that the nonequilibrium effects play an important role only when the term $\Phi_1|\Delta|$ is comparable to the other terms in the RHS of Eq. (3a). When $|\Delta|$ becomes relatively small in the phase slip region the main terms in Eq. (3a) are $a_1|\Delta|Q^2$ (which could be rewritten as $j_s^2/|\Delta|^3 4a_1$ using the equilibrium relation $j_s = 2a_1|\Delta|^2 Q$) and $a_1 \partial^2|\Delta|/\partial x^2$. As a result for relatively small values of $|\Delta|$ (see Fig. 2) the dynamics of the order parameter becomes independent of τ_E. The potential Φ_1 in the phase slip center[31] stays finite and reaches a maximal negative value when the order parameter goes to zero (see Fig. 3). This maximal value monotonically increases in absolute value for small τ_E (see inset in Fig. 3) and does not depend on τ_E when $\tau_E \gtrsim 15$ (which scales with τ_{GL} at $T=0.9T_c$). The last property is connected with the fact that the fast changes of the order parameter in the phase slip center occurs mainly on a time scale $\sim \tau_{GL}$ (see Fig. 2) and in case $\tau_E \gg \tau_{GL}$ the deviation from equilibrium of the quasiparticle distribution function $f_L - f_L^0 \sim \int_{-\tau_{GL}}^{\tau_{GL}} d(t - t_{\text{init}})\, \partial|\Delta|/\partial t$ does not depend on τ_E.

After the phase slip the order parameter starts to grow and the sign of $\partial|\Delta|/\partial t$ changes. It leads to a positive value of Φ_1 (see Fig. 3) which impedes the growth of the order parameter in a similar way as a negative Φ_1 value impedes the order parameter decay (compare different curves in Fig. 2 for $t - t_{\text{init}} > 10$ and $t - t_{\text{init}} < -10$). But there is a finite time interval

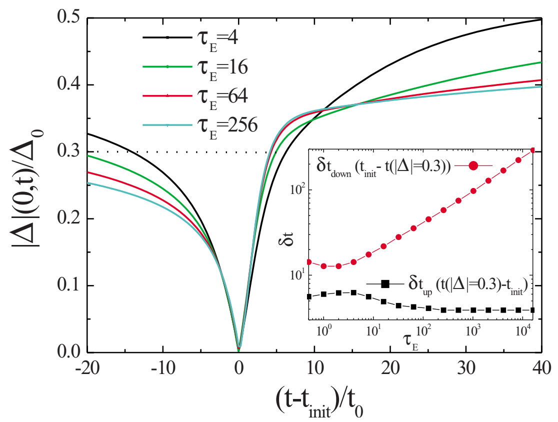

FIG. 2. (Color online) Time dependence of the order parameter $|\Delta|$ in the phase slip center near the moment of the phase slip ($t=t_{\mathrm{init}}$) for different values of τ_E. In the inset we present the dependence of the decay time of $|\Delta|$ from $|\Delta|=0.3$ to zero (red circles) and time of growth of $|\Delta|$ from zero to $|\Delta|=0.3$ (black squares) as function of τ_E.

just after the phase slip when the order parameter already increases but Φ_1 is still negative (see Fig. 3). In this case nonequilibrium effects accelerate the growth of the order parameter [the RHS of Eq. (3a) becomes larger for negative Φ_1]. But this effect is noticeable only for $\tau_E > \tau_{GL}$ because in the local approach (when $\tau_E \ll \tau_{GL}$) the potential $\Phi_1 \sim \tau_E \partial|\Delta|^2/\partial t$ (Ref. 25) is positive just after the phase slip. This delay time in the sign change of Φ_1 results in a nonmonotonic dependence on τ_E of the initial time of growth of $|\Delta|(\delta t_{\mathrm{up}})$ after the phase slip (see inset in Fig. 2). It increases at small $\tau_E < \tau_{GL}$ (following predictions of the local approach), reaches the maximal value at $\tau_E \sim \tau_{GL}$ and then decreases. For large $\tau_E \gg \tau_{GL}$ it does not depend on τ_E because the nonequilibrium potential Φ_1 saturates for large τ_E.

A positive sign of Φ_1 (at $t \gtrsim t_{\mathrm{init}} + \tau_{GL}$) means an effective "heating" of the electron subsystem and the "heated" state relaxes on a time scale τ_E because in a ring of *small circumference* (comparable with L_E) the diffusion from the most "heated" (phase slip core) region is not effective. Below we show that this effect together with the acceleration of the

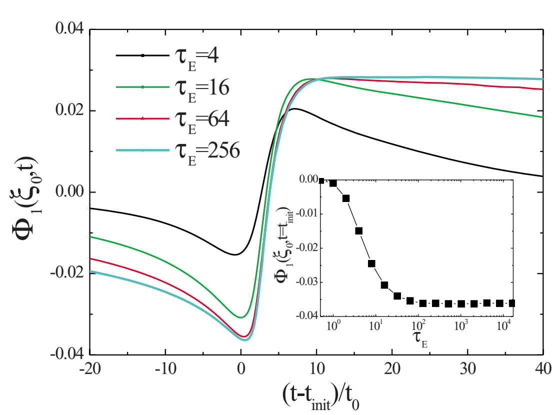

FIG. 3. (Color online) Time variation of the nonequilibrium potential Φ_1 in the phase slip center close to the moment of the phase slip ($t=t_{\mathrm{init}}$) for different values of τ_E. In the inset, we present the dependence of $\Phi_1(0)$ at the phase slip moment ($t=t_{\mathrm{init}}$) on τ_E.

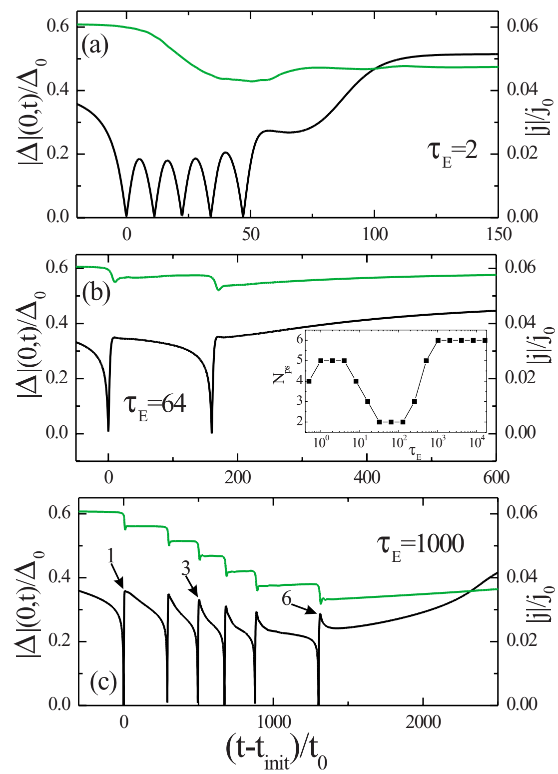

FIG. 4. (Color online) Time dependence of the order parameter $|\Delta|$ in the phase slip center (black curves) and the current density (green curves) for different values of τ_E [(a) $\tau_E=2$, (b) $\tau_E=64$, and (c) $\tau_E=1000$] for a ring with a large circumference $S=360\xi_0$ at $T=0.9T_c$. In the inset to Fig. 4(b) we show the number of phase slips as function of τ_E and the numbers in Fig. 4(c) correspond to the number of phase slips.

order parameter growth due to nonequilibrium effects (in the time interval $t_{\mathrm{init}} < t \lesssim t_{\mathrm{init}} + \tau_{GL}$) considerably influences the dynamics of the order parameter in rings of large radius where several subsequent phase slips can occur.

IV. RING WITH LARGE RADIUS

In Fig. 4, we present the time dependence of the order parameter for a ring with $S=360\xi_0 (R\sim 57\xi_0)$ at $T=0.9T_c$ and three values of τ_E after turning on the overcritical magnetic field $H_{\mathrm{appl}}=1.018H_c$. The initial part of the decay of the order parameter is not shown (it is almost the same as for a small radius ring). The main feature observed in our calculations—the nonmonotonic dependence of the number of phase slips on τ_E [see insert in Fig. 4(b)]. N_{ps} increases for small $\tau_E < \tau_{GL}$, than rapidly decreases (when $\tau_E \gtrsim \tau_{GL}$) and than increases again with a tendency to saturation for $\tau_E \gg \tau_{GL}$.

In order to understand this $N_{ps}(\tau_E)$ dependence let us first discuss why subsequent phase slips can occur in a ring. After the first phase slip the order parameter $|\Delta|$ increases together with the supervelocity $Q\sim\nabla\phi$ but their rate of increase is described by different equations [Eqs. (3a) and (3b)] and they have different times of growth ($\tau_{|\Delta|}$ and $\tau_{\nabla\phi}$, respectively). Therefore, there are two situations possible: (i) $\tau_{|\Delta|} > \tau_{\nabla\phi}$ and (ii) $\tau_{|\Delta|} < \tau_{\nabla\phi}$. In the first case the term $-Q^2|\Delta|$

may become strong enough to make the RHS of Eq. (3a) negative before the order parameter reaches the stationary value (corresponding to the decreased value of the current density in the ring). From the physical point of view it corresponds to the situation when the superconducting current $j_s \sim |\Delta|^2 Q$ locally (in the phase slip core) becomes large enough to make the superconducting state with dynamically suppressed order parameter unfavorable and it leads to a decrease in the order parameter and the appearance of the next phase slip. In the case (ii) the supervelocity grows slower than the other terms in the RHS of Eq. (3a) and the next phase slip does not occur.

It is essential that the time scale over which Q (or $\nabla\phi$) varies is inversely proportional to the current density in the ring $\tau_{\nabla\phi} \sim 1/|j|$ (see also Ref. 32). Indeed the first term in the RHS of Eq. (3b) could be estimated as $\partial j_s/\partial x = -\partial j_n/\partial x \sim -j_n(0)/L$ where L is a characteristic length scale for the decay of the normal current near the phase slip core ($x=0$) and we used the condition $j = j_s(x,t) + j_n(x,t) = \mathrm{const}(t)$. The potential Φ_2 is roughly proportional to the electric potential (compare equations for Φ_2 and φ) and hence its gradient is proportional to the electric field $\mathbf{E}$ and the normal current density via Ohm's law $j_n = \mathbf{E}/\rho_n$. Because j_n cannot be larger than the total current $j(t)$ we obtain the relation $\tau_{\nabla\phi} \sim 1/|j|$ which has a simple physical meaning—the larger the electric field in the ring ($\mathbf{E} \sim j_n \lesssim j$) the faster will change the velocity of the superconducting electrons. Using the condition for the subsequent phase slip as $\tau_{|\Delta|} \gtrsim \tau_{\nabla\phi}$ we obtain an estimate for the lower critical current density when dynamic phase slip process can occur $|j| \gtrsim j_{c1} \sim 1/\tau_{|\Delta|}$.

In the ring, each phase slip decreases the current by $\delta j \simeq j_{\mathrm{dep}} \xi(T)/R$ and the maximal number of subsequent phase slips (assuming that all of them occur in the same point along the ring) is $N_{ps} = (j_{\mathrm{dep}} - j_{c1})/\delta j$. Our results for small rings show, that $\tau_{|\Delta|} \sim \delta t_{\mathrm{up}}$ is a nonmonotonic function of the inelastic time τ_E (see inset in Fig. 2 for δt_{up}) and it provides the related changes in $j_{c1} \sim 1/\tau_{|\Delta|} \sim 1/t\delta_{up}$ and N_{ps} for not very large τ_E [compare inset in Fig. 4(b) with inset in Fig. 2].

At relatively large $\tau_E \gg \tau_{GL}$ the effective heating of the electronic subsystem by the growth of the order parameter increases N_{ps} again. Due to the diffusion of the quasiparticles (with energy larger than $|\Delta|_{\max}$—maximal value of the order parameter near the phase slip center) Φ_1 in the phase slip center initially decays very fast after the phase slip (see positive peaks in Fig. 5), but than its decay time becomes proportional to τ_E (quasiparticles with energy less than $|\Delta|_{\max}$ cannot diffuse from the phase slip core—they relax to equilibrium via inelastic processes). Therefore, during some finite time the effective temperature $T_e \simeq T_{\mathrm{bath}} + \Phi_1 T_c$ near the phase slip center is larger than the bath temperature T_{bath} and the current density in the ring locally can be larger than $j_{c1}(T_e)$ or even $j_{\mathrm{dep}}(T_e)$. It provides the condition for the next phase slip. This effect becomes strong when the time between subsequent phase slips is smaller than τ_E and there is an essential accumulation of the "heat" in the phase slip core (see Fig. 5). It also explains the increase of $\Phi_1(0)$ with increasing τ_E after the second and further phase slips - compare the two curves in Fig. 5 [note, that after the first phase slip $\Phi_1(0)$ is practically the same for both values of τ_E—see discussion of Fig. 3]. But the difference is not large and this

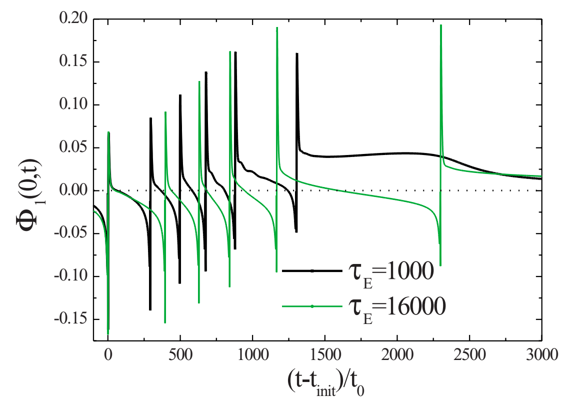

FIG. 5. (Color online) Time dependence of the nonequilibrium term $\Phi_1(0)$ in the Ginzburg-Landau equation [Eq. (3a)] for two values of τ_E in case of a large ring ($S=360\xi_0$).

can be the reason for the saturation of N_{ps} [see inset in Fig. 4(b)] with increasing τ_E for the ring with the chosen parameters. In our case $\delta j \simeq 0.05 j_{\mathrm{dep}}$ is relatively large and we cannot detect a decrease in j_{c1} for large τ_E.

Another effect which comes from the nonlocality of the nonequilibrium state is the two peaks in the distribution $|\Delta|(x)$ around the phase slip center (see Fig. 6) that appears during the transition period. At $t < t_{\mathrm{init}}$ the maximal deviation from equilibrium occurs at $x=0$ and due to the diffusion of the quasiparticles it spreads in space on a scale of about L_E. Because Φ_1 is negative at $t < t_{\mathrm{init}}$ it leads to an enhancement of the order parameter near the phase slip center. Besides the term $2N_2R_2Q\nabla f_T$ in Eq. (2a) is positive for almost all time and it acts as an additional source of nonequilibrium, which enhances superconductivity near the phase slip core when ∇f_T (which is proportional to electric field) is different from zero. During the transition period there are voltage pulses (corresponding to phase slips) and it keeps the presence of two peaks in the spatial dependence of the order parameter and even leads to their increase (see Fig. 6). Appearance of these peaks enhances the effective 'heating' (discussed in above paragraph) because the nonequilibrium quasiparticles

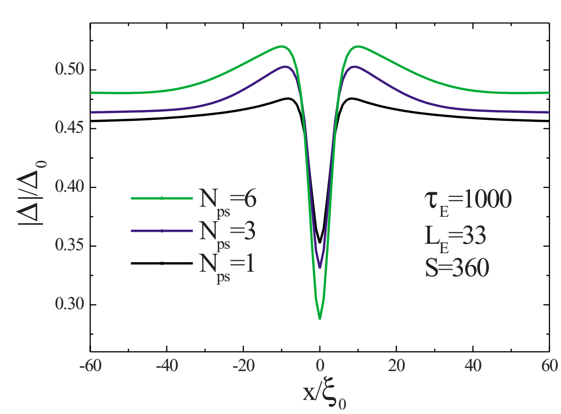

FIG. 6. (Color online) Order parameter distribution in a large radius ring after the first (black curve), the third (blue curve) and the sixth (green curve) phase slips at the times when the order parameter reaches its maximal value in the phase slip center [see Fig. 4(c)].

in a wider energy interval $0<E\lesssim|\Delta|_{max}$ cannot diffuse from the phase slip core and have to relax to equilibrium within a time $\sim\tau_E$. We should note that for relatively small $\tau_E\lesssim\tau_{GL}$ such an effect is very weak.

V. DISCUSSION

The above results were obtained for $T=0.9T_c$. We also performed calculations for higher temperatures $0.9<T/T_c<0.98$ and found qualitatively similar results. The dynamics of the order parameter is governed by τ_E: the initial growth time of $|\Delta|$ in the phase slip core after the phase slip is a nonmonotonic function of $\tilde{\tau}_E$ which has a maximum at $\tilde{\tau}_E\simeq\tilde{\tau}_{GL}=\pi\hbar/8k_B(T_c-T)$ and a tendency to saturate for large $\tilde{\tau}_E\gtrsim\tilde{\tau}_{GL}$. This time is much smaller than the decay time of $|\Delta|$, which depends on τ_E. We also find peaks in the order parameter distribution near the phase slip core during the transition period which are most pronounced when $\tau_E\gg\tau_{GL}$. But due to the increased coherence length $\xi(T)$ when approaching T_c (and hence an increase in the ratio $\delta j/j_{dep}$) already at $T=0.98T_c$ we find transitions only with $N_{ps}=2$ for a ring with $S=360\xi_0$ in the considered range of τ_E. This makes it difficult to study the effect of 'heating' on the phase slip process and the critical current density j_{c1} (already after two phase slips the current density in such a ring decreases by 40%).

Equations (2a) and (2b) are valid if the relaxation of the quasiparticle distribution function to equilibrium occurs mainly via inelastic electron-phonon interactions (with characteristic time $\tilde{\tau}_E=\tau_{e-ph}$). Furthermore it was assumed that the phonon subsystem is in equilibrium and the electron-electron relaxation time $\tau_{e-e}\gg\tau_{e-ph}$.[24] If $\tau_{e-e}\lesssim\tau_{e-ph}$ (for example in Al and Zn they are of the same order already at $T\sim T_c$) in Eqs. (2a) and (2b) the collision integrals should be added, which are responsible for the change of f_L and f_T due to the electron-electron interaction. They lead to a nonequilibrium quasiparticle distribution function of Fermi-like type with an effective temperature after some relaxation time of the order τ_{e-e}. But the transition period (from the first phase slip up to the last one) is about $\tilde{\tau}_E$ for large $\tilde{\tau}_E\gg\tilde{\tau}_{GL}$ [see Fig. 4(c) and 5] and, therefore, we expect a small influence on our results when $\tau_{e-e}\sim\tau_{e-ph}$.

Our theoretical results could be verified with superconducting rings of different radius and made of two types of material—with low and strong electron-phonon coupling. The good candidates are Al($\tau_E\sim10^3$) and Nb($\tau_E\sim10^2$). We expect that the number of phase slips should be the same in rings of small radius $[R\lesssim3\xi(T)]$ and N_{ps} should be much larger for Al rings than for Nb when $R\gg\xi(T)$.

We believe that the nonequilibrium effects discussed in our work that are nonlocal in time and space should also affect the minimal critical current density j_{c1} at which the dynamical phase slip process can exist in a current carrying wire. We expect that it has a nonmonotonic dependence on τ_E: it should have a local minimum at $\tau_E\sim\tau_{GL}$, then a local maximum at $\tau_E>\tau_{GL}$ and for large $\tau_E\gg\tau_{GL}$ j_{c1} should monotonically decay with increasing τ_E.

Moreover, for a wire with length $\xi(T)\ll L<L_E$, the boundary conditions at its ends may influence the value of j_{c1}. If for example we have a "bad" contact (tunnel barrier with low transparency) the diffusion of the nonequilibrium quasiparticles to the leads is strongly suppressed and we should observe a decrease of j_{c1} in comparison with long wires with $L\gg L_E$ (effective heating increases in this case).

In the case of a "good" contact (no tunnel junctions) the diffusion of the nonequilibrium quasiparticles depends on the value of the order parameter in the lead $|\Delta|_{lead}$. Usually it is larger than $|\Delta|$ in the wire where it is suppressed by the transport current. Therefore, the diffusion is also suppressed (but not so strong as for the "bad" contact) and we may expect a small decrease of j_{c1} (in comparison with a long wire $L\gg L_E$). But if we suppress $|\Delta|_{lead}$ (for example by an external magnetic field) the diffusion of nonequilibrium quasiparticles becomes stronger, the heating is weaker and j_{c1} should increase. This mechanism could be responsible for the observed, in many experiments, magnetic field enhanced superconductivity seen in *short* superconducting nanowires[10–16] when the conductivity and/or the critical current of the nanowire increases in weak magnetic fields. In our opinion the decrease of the charge imbalance length $L_Q\sim L_E$ in weak magnetic fields (which also could influence the phase slip process[10,18,33]) is irrelevant to this effect at least for part of these experiments[12,15,16] because the critical current density was large (it was comparable with the depairing current density) and it has a stronger effect on L_Q (see for example Ref. 34) than relatively weak magnetic fields used in these works.

ACKNOWLEDGMENTS

This work was supported by the Flemish Science Foundation (FWO-Vl) and the Belgian Science Policy (IAP). D.Y.V. also acknowledges support from the Russian Foundation for Basic Research, Federal Target Programme "Scientific and scientific-pedagogical personnel of innovative Russia in 2009–2013" and Dynasty Foundation.

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