Dissipation and traversal time in Josephson junctions

Ilaria Cacciari,[1] Paolo Moretti,[2] and Anedio Ranfagni[1]
[1]*Istituto di Fisica Applicata "Nello Carrara" del Consiglio Nazionale delle Ricerche, via Madonna del Piano 10, 50019 Sesto Fiorentino, Firenze, Italy*
[2]*Istituto dei Sistemi Complessi del Consiglio Nazionale delle Ricerche, Sezione di Firenze, via Madonna del Piano 10, 50019 Sesto Fiorentino, Firenze, Italy*
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The various ways of evaluating dissipative effects in macroscopic quantum tunneling are re-examined. The results obtained by using functional integration, while confirming those of previously given treatments, enable a comparison with available experimental results relative to Josephson junctions. A criterion based on the shortening of the semiclassical traversal time τ of the barrier with regard to dissipation can be established, according to which $\Delta\tau/\tau \gtrsim N/Q$, where Q is the quality factor of the junction and N is a numerical constant of order unity. The best agreement with the experiments is obtained for $N=1.11$, as it results from a semiempirical analysis based on an increase in the potential barrier caused by dissipative effects.

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I. INTRODUCTION

The theme of macroscopic quantum tunneling (MQT) has been the subject of several investigations, due to its interest for fundamental physic, as well as for possible applications. Within this context, Josephson devices have been shown to be the most suitable ones for observing MQT in current biased junctions and macroscopic quantum coherence (MQC) in superconducting quantum interference devices.[1]

In spite of numerous efforts[2–8] certain aspects still need to be clarified, from both theoretical and experimental points of view. The present work deals with an analysis of these aspects. In particular, we will reconsider the effect of dissipation, first from the point of view of theory, namely, an evaluation of the so-called *effective action*. This quantity determines the transition probability, or decay rate, of the superconducting metastable state and, consequently, the traversal time of the barrier. Lastly, a comparison with the experimental results is made.

II. EFFECTIVE ACTION

We consider a Josephson junction connected to a single or a pair of open transmission lines, the total length of which is L. Let us denote the capacitance and the inductance per unit length by C_0 and L_0, respectively. Therefore, the characteristic impedance of the line (assuming the electrical losses to be negligible) is $Z_0=(L_0/C_0)^{1/2}$ and the wave velocity is $c=(L_0C_0)^{-1/2}$. The voltage $V(x,\tau)$ in terms of the magnetic flux $\phi(x,\tau)$ is given by $V(x,\tau)=\dot{\phi}(x,\tau)$, where x is the coordinate along the line and τ is the Euclidean time running from $(-T)$ to (T), $T\to\infty$. The Euclidean action of a single transmission line of length L can be expressed as

$$S_T[\phi]=\int_{-\infty}^{\infty}d\tau\int_0^L dx\left\{\frac{1}{2}C_0\dot{\phi}^2(x,\tau)+\frac{1}{2L_0}\left[\frac{\partial\phi(x,\tau)}{\partial x}\right]^2\right\}. \quad (1)$$

The action of the junction alone will be denoted by S_J and is given by

$$S_J[\varphi]=\int_{-\infty}^{\infty}d\tau\mathcal{L}[\varphi(\tau)], \quad (2)$$

where $\varphi(\tau)$ is the Cooper-pair phase difference across the junction. Dissipative effects are attributed only to the presence of the transmission line. The coupling between the line and the junction implies that

$$\phi(x=0,\tau)=(\Phi_0/2\pi)\varphi(\tau)=\lambda\varphi(\tau), \quad (3)$$

where $\Phi_0=h/2e$ is the flux quantum. Having referred to the literature for more physical details,[2,3] we are able to define an effective action for this system (from which the tunneling decay rate is determined) by performing a mean over all the possible field configurations $\phi(x,\tau)$. Defining $S_{\text{eff}}=S_J+S_{\text{int}}$, we can write

$$\exp\left\{-\frac{S_{\text{eff}}[\varphi(\tau)]}{\hbar}\right\}=\int\mathcal{D}\phi(x,\tau)\delta[\phi(0,\tau)-\lambda\varphi(\tau)]\times\exp\left(-\frac{S_T+S_J}{\hbar}\right). \quad (4)$$

The evaluation of S_{int} is rather cumbersome and a detailed account of it will be reported elsewhere.[9] There, an important point is represented by a suitable choice of the "measure" in the functional integration. Here, we limit ourselves to report the final result. Denoting by χ the Fourier transform of φ, the final result can be expressed as

$$S_{\text{int}}=\frac{\lambda^2}{4\pi}\int_{-\infty}^{\infty}d\omega\frac{|\chi(\omega)|^2}{g(\omega)}, \quad (5)$$

where $g(\omega)$ is given, in the case of a single transmission line (case A) or of a pair (case S), by

$$g_{\text{A}}(\omega)=\frac{Z_0}{\omega}\coth\left(\frac{\omega L}{c}\right),$$

$$g_{\text{S}}(\omega)=\frac{1}{2}\frac{Z_0}{\omega}\coth\left(\frac{\omega L}{2c}\right). \quad (6)$$

The result of Eq. (5) is not a new one and can be considered as a confirmation of other, more or less, simplified treatments that have already been presented. In particular, in addition to the elegant work by Chakravarty and Schmid[2]—a somewhat hermetic and concise one—many others have been dedicated to this subject and a review paper has been available since 1987.[4] Subsequently, the argument of dissipation has been reexamined: for instance, in Ref. 5 for double-well potentials (case of MQC) and for the case of our interest (MQT) in Refs. 3 and 6 along the lines of Ref. 2. More recently,[7] an equivalent expression for Eq. (5) was also obtained by avoiding the delicate instrument of functional integration. This argument seems to be consolidated; see, however, in the following.

III. TRAVERSAL-TIME VARIATION IN JOSEPHSON JUNCTIONS

The variation in the bounce action due to the load of the transmission line ($\Delta S_B \equiv S_{\rm int}$), can be rewritten, in a more explicit form, as

$$\Delta S_B = \frac{\eta}{2}\int_{-\infty}^{+\infty} d\omega |\xi(\omega)|^2 \omega \tanh(\omega\tau_0), \tag{7}$$

where $\eta=(\Phi_0/2\pi)^2/Z_0$, $\xi(\omega)$ is the Fourier transform of the bounce trajectory $\varphi(\tau)=\varphi_B \,\mathrm{sech}^2(\Omega\tau/2)$, according to the transform definition in which $d\omega/2\pi \to d\omega$, and $\tau_0=kL/\omega$ is the delay time of the line of length L (case A). In the limit of $\Omega\tau_0 \gg 1$, or in the case of pure resistive load Z_0, Eq. (7) supplies the result $\Delta S_B \simeq 0.465\eta\varphi_B^2$, with φ_B being the bounce amplitude.[7] Our goal is now to evaluate the semiclassical traversal (or tunneling) time in a junction and, more precisely, the variation in this quantity due to the dissipative effects. In another paper[8] dedicated to the same problem, the variation in the action ΔS, due to the presence of a transmission line, was evaluated applying a different method. By using the artifice of halving the bounce time (an artifice also adopted in Ref. 6) and, in the case of an artificial line, a Laplace transform technique, the results obtained were in substantial agreement with the previous ones. In particular, in the case of a line with distributed constants for the action variation we obtained the quantity $\Delta S=0.436\eta\varphi_B^2$ as a result of the integration of the following Lagrangian

$$\mathcal{L}(\tau) = \eta\int_0^{\tau} d\tau' [\dot\varphi(\tau')]^2, \tag{8}$$

where $\dot\varphi(\tau)$ is the time derivative of the bounce trajectory.[10] In the presence of dissipative effects, the potential $V(\varphi)$ of the barrier is augmented by an amount

$$W(\varphi) = \eta\int_0^{\varphi} d\varphi' |\dot\varphi|, \tag{9}$$

which is the equivalent of Eq. (8). Consequently, the semiclassical traversal time[11] is shortened by an amount which, in the limit of $W(\varphi) \ll V(\varphi)-E$, with E being the energy of the level considered, is given by[8]

$$\Delta\tau/\tau \simeq \left\langle \frac{W(\varphi)}{2[V(\varphi)-E]} \right\rangle, \tag{10}$$

where $\langle \cdots \rangle$ means an average over the barrier extension. In Eq. (10), $W(\varphi)$, as given by Eq. (9), is found to be

$$W(\varphi) = \frac{2}{5}\eta\Omega\varphi_B^2 f(\varphi), \tag{11}$$

where $f(\varphi)$ is a numerical function given by[12]

$$f(\varphi) = \frac{2}{3} - \left(1-\frac{\varphi}{\varphi_B}\right)^{1/2}\left[1-\left(\frac{\varphi}{\varphi_B}\right)^2\right] + \frac{1}{3}\left(1-\frac{\varphi}{\varphi_B}\right)^{3/2}, \tag{12}$$

whose maximum value at $\varphi=\varphi_B$ holds 2/3. As for $V(\varphi)$, the unperturbed potential of the barrier in the absence of dissipation is given by $V(\varphi)=\epsilon\varphi^2(1-\varphi/\varphi_B)$, where $\epsilon = C(\Phi_0/2\pi)^2\Omega^2/2$, with C being the capacitance of the junction. Within the limit of $V_{\max}(\varphi)=\Omega S_0/3.6 \gg E$, where $S_0 = (4/15)C(\Phi_0/2\pi)^2\Omega\varphi_B^2$ is the half-bounce action in the absence of dissipation,[8] an approximate expression for Eq. (10) can be obtained by taking for the potential its maximum value $V_{\max}$ and for $f(\varphi)$ the value assumed at the coordinate of $V_{\max}$, $\varphi_{\max}=(2/3)\varphi_B$, namely, $\overline{f(\varphi)} \simeq 0.41$. In this way, the resulting expression can be considered as a lower limit for Eq. (10), i.e.,

$$\Delta\tau/\tau \gtrsim \frac{\frac{2}{5}\eta\varphi_B^2\overline{f(\varphi)}}{2\Omega S_0/3.6} = \frac{1.11}{Q} \simeq \frac{2}{3}\frac{\Delta S}{S_0}, \tag{13}$$

where $\Delta S=0.465\eta\varphi_B^2$ and the quality factor $Q=\Omega RC$ with $R(\equiv Z_0)$ is the shunt resistance of the junction.

We recall that a very similar expression to Eq. (13) was reported in Ref. 8, taking $\overline{f(\varphi)} \simeq 0.44$ and $\Delta S=0.436\eta\varphi_B^2$, and obtaining $\Delta\tau/\tau \simeq (1.15 \pm 0.25)/Q$, where the uncertainty in the numerical factor roughly accounts for the approximations involved. On the other hand, the expression reported in Ref. 7, namely, $\Delta\tau/\tau \simeq (2/3)\Delta S_B/S_B = 0.58/Q$, where $\Delta S_B = 0.465\eta\varphi_B^2$ as before but $S_B=2S_0$ is the action of the complete bounce trajectory, has to be considered wrong since $V_{\max}$ in entering the final expression (13) is derived from S_0 and not from S_B. Otherwise $V_{\max}$ should be given by $\Omega S_B/7.2$, which is exactly the same quantity. However, there are other reasons that would make the last expression of $\Delta\tau/\tau=0.58/Q$ acceptable. The above-mentioned arguments, which lead to Eq. (13), are based on the assumption that the action-variation $\Delta S=0.465(\text{or}0.436)\eta\varphi_B^2$ is due to the increase in $W(\varphi)$ potential, with ΔS being attributed to a half-bounce trajectory, i.e., the artifice of halving the bounce time adopted in Refs. 6 and 8; see also Ref. 10. If ΔS_B is considered as being due to the complete bounce, it automatically halves its effect and the final results of $\Delta\tau/\tau=0.58/Q$ can be confirmed.

In summary, we have considered two different approaches to the problem of evaluating the traversal-time variation, both based on the calculation of the action variation and on the increase in the potential barrier as a consequence of dissipation. Both approaches lead to the same (approximate) simple expression, namely, $\Delta\tau/\tau \gtrsim N/Q$, where the numerical constant N assumes the value 1.11 of Eq. (13) (1.15 in Ref. 8) or the value 0.58 in the analogous expression in Ref. 7. There is therefore a rough factor of 2 as a consequence of considering the half-bounce action S_0 (according to Refs. 6 and 8) or the complete bounce action $S_B=2S_0$ (according to Refs. 3 and 7). A choice between these two different approaches is a delicate point and ultimately rests on a comparison with the available experimental results.

IV. ANALYSIS OF SOME EXPERIMENTS

Only a few experimental results suitable for application of the above obtained criterion are available in the literature (to which we refer for technical details), as reported here as follows. (a) In a work by Voss and Webb,[13] the results obtained with small junctions of 1 μm are reported. The results allow for a direct test of Eq. (13) since they explicitly supply the ratio $\Delta S/S_B$ in the form $8A/7.2Q$, where A is a numerical factor of order unity. In the case of a junction with critical current $I_c=1.62$ μA, capacitance $C=0.1$ pF, and shunting resistance $R=320$ Ω, the fitting of experimental data of transition rate versus the bias current, at temperatures below as ~ 5 mK, make it possible to determine $A\simeq 4.5$, which is considerably greater than unity. This, in turn, supplies the value $5/Q$ for $\Delta S/S_B$, or $\Delta S/2S_0$, which corresponds to $\Delta\tau/\tau \simeq 3.33/Q$, where the numerical factor appears to be a disproportionate amount with respect to the predicted one of 0.58. The situation improves by adopting the other criterion according to which $\Delta S/S_0=8A/3.6Q$ implies a halving of parameter $A\simeq 2.25$. Again, the results are $\Delta S/S_0=5/Q$ and $\Delta\tau/\tau \simeq 3.33/Q$, where the numerical factor is more comparable with 1.11 of Eq. (13). However, still in Ref. 13, lower values for A were also considered to be plausible, depending on a different choice of R, and hence of coefficient $Q=\Omega RC$, Ω being the plasma frequency. With $\Omega \simeq 10^{11}$ s^{-1}, $R=320$ Ω, and $C=0.1$ pF, we have $Q\simeq 3.2$ but a reduction in R up to about 1/3 was considered to be acceptable. Under this assumption, the numerical factor would be lowered down to 1.11, which is in excellent agreement with the prediction of Eq. (13) while it remains twice the value predicted by the other criterion.

(b) Another case is presented in the paper by Esteve *et al.*[14] where a junction of ~ 10 μm was tested at temperatures of 18 and 65 mK, with a critical current $I_c\simeq 7$ μA, capacitance $C=2.7$ pF, and a load consisting of a line of variable length with characteristic impedance $Z_0=72$ Ω, terminated with an impedance $Z_t\simeq 20$ Ω. By assuming the passage time obtained as tunneling time $\tau_p=78$ ps, this can be compared with the half period in harmonic approximation $\tau=\pi/\Omega\simeq 85$ ps, with $\Omega\simeq 3.7\times 10^{10}$ s^{-1}, which represents the semiclassical tunneling time when the temperature is comparable with the crossover temperature $T_{co}=47$ mK. In this situation, since the load is represented by $R(\equiv Z_0)=72$ Ω for a sufficiently long line, we have $Q=\Omega RC\simeq 7.2$. Therefore, since $\Delta\tau/\tau=7/85=8.2\%$, we obtain a value of 0.59 for the numerical factor in Eq. (13), a value which seems to be in good agreement with the other criterion giving $\Delta\tau/\tau=0.58/Q$. However, if we consider that the results are attributed to a temperature well below T_{co}, i.e., 18 mK, a better determination of the semiclassical tunneling time is given by $\tau_t=3.6/\Omega=97$ ps.[15] In this way we obtain $\Delta\tau/\tau=19/97=19.6\%$. This value corresponds to a numerical factor in Eq. (13) of 1.41, which is in quite good agreement with the predicted one (1.11).

(c) A similar result was obtained in Ref. 16 where a junction of 10×10 μm^2, with critical current of 46 μA that was reduced by applying a magnetic field to $I_c\simeq 5$ μA. The experiments were performed at a temperature of ≈ 60 mK while the temperature at the junction holder was of 20–30 mK. The traversal time τ_t was determined by measuring the lifetime τ_d of the zero-voltage state, as a function of the bias current, according to the well-known relation[16]

$$\tau_t = -\frac{\hbar}{2}\frac{\partial}{\partial V_B}\ln(\tau_d^{-1}), \tag{14}$$

where V_B is the barrier height. The result was obtained of 91 ps. This time was shorter than the one predicted in harmonic approximation, which was preferred in this case since the temperature was not low enough but rather was greater than the crossover temperature $T_{co}\approx 25$ mK. By considering the contributions of the different levels, as well as the one due to the thermal overcoming of the barrier, the traversal time should be of 113 ps, considerably greater than the measured one (91 ps), by an amount given by $\Delta\tau/\tau=22/113=19.5\%$. By considering that the quality factor $Q=\Omega RC=11$, with $\Omega=2\times 10^{10}$ s^{-1}, $C=6.6$ pF, and $R=85$ Ω, the numerical factor in Eq. (13) should be of 2.1: a value which, although again compatible with relation (13), is considerably higher than the prediction (1.11). By considering the curves of potential barrier for different friction coefficient (that is, $\propto R^{-1}$), values relative to the junction in object,[15] we arrived at the conclusion, through Eq. (10), that $\Delta\tau/\tau$ should be approximately 10%, a value which corresponds to a numerical factor in Eq. (13) of ≈ 1, in agreement with the test prediction, thus demonstrating its consistency.

(d) A similar experiment to the previous one was performed with a junction of 9 μm^2, with critical current $I_c=7.1$ μA at a temperature ≈ 50 mK, comparable with the crossover temperature $T_{co}\approx 57$ mK.[17] By assuming that the capacitance $C\simeq 2$ pF [a value that was estimated by adding that of the wiring to the intrinsic capacitance of the junction (≈ 1 pF)], we find that the plasma frequency $\Omega=4.71\times 10^{10}$ s^{-1}. However, the shunting resistance R was unknown. According to the same procedure as point (c), an experimental value for τ_t was found to be ≈ 67 ps. This value might be compared with $\tau=3.6/\Omega=76.4$ ps and τ thermic as given by $\tau_{\text{ther}}=\frac{1}{2}\beta\hbar$, where $\beta=1/k_BT$, with k_B the Boltzmann constant, which was nearly the same: 76.1 ps. We could thus assume $\tau\simeq 76$ ps to be the value in the absence of dissipation. Therefore, $\Delta\tau/\tau\simeq 9/76=11.8\%$. By using Eq. (13) we can determine the quality factor $Q\simeq 1.11\tau/\Delta\tau=9.37$, and an estimate of R is given by $Q/\Omega C\simeq 100$ Ω, which is a plausible one. However, this case cannot be considered to be a true test of relation (13) and the experiment deserves to be repeated.

TABLE I. Summary of the data relative to the considered experiments. The data in parentheses refer to alternative determinations.

Case	I_c (μA)	T (mK)	C (pF)	Ω (s^{-1})	R (Ω)	Q	N	Ref.
(a)	1.62	5	0.1	$\sim 10^{11}$	320 (107)	3.2 ($\sim$1.1)	3.33 (1.11)	13
(b)	7	18	2.7	3.7×10^{10}	72	7.2	0.58 (1.41)	14
(c)	5	60	6.6	2×10^{10}	85	11	2.1 ($\sim$1)	16
(d)	7.1	50	$\sim$2	4.71×10^{10}	$\sim$100	9.37	1.11	17

The results are summarized in Table I. In all cases, except in one of the two determinations in (b), the criterion expressed by $\Delta\tau/\tau \gtrsim N/Q$ is better satisfied with $N=1.11$, as given in Eq. (13), rather than with $N=0.58$.

In the light of the cases considered, it would seem that the criterion expressed by Eq. (13) with $N=1.11$ is the most acceptable one for interpreting the experiments. It is worth recalling that, although approximated, relation (13) was obtained by adopting the value for the action variation as given by Eq. (5) or Eq. (7). However, in the following relations from Eqs. (10)–(13), this contribution, in accordance with Refs. 6 and 8, is compared with the half-bounce action S_0, rather than with the complete bounce action S_B.

At this stage, it is difficult to decide whether this discrepancy should be attributed to a residual ambiguity in the theoretical analysis or to the approximations involved in the experimental interpretation. In this respect, it is worth recalling that, besides the above considered motives of uncertainty [mainly relative to the determination of the R values in the cases (a) and (d)], more sophisticated effects, as those analyzed in Refs. 18 and 19, should be taken into account. In particular, we have not considered that when the mean traversal time of the barrier, as calculated from the switching current distribution employing current ramps, cases (a) and (b), the result can seriously depend on the repetition frequency of the ramp pulses.[19] Moreover, the usual Kramer's approximation of the escape time, determining the thermal overcoming of the barrier for temperatures not sufficiently low, cases (c) and (d), can give significant error in the case of small barriers.[18]

Further investigations, mainly (but not only) from the point of view of the experimental work, need to be made. Moreover, we wish to mention that the semiclassical traversal time discussed here, the variation in which is a consequence of dissipative effects, must be considered not properly representative of the true duration of the process. To be more precise, when the tunneling is analyzed within a complex-time framework, this quantity is found to be much shorter: typically by one order of magnitude.[20]

[1] For a survey on the matter see, for example, Proceedings of the International Workshop on Macroscopic Quantum Tunneling and Coherence, preface by: P. Silverstrini, B. Ruggiero, F. Petruccione, and B. Barone [J. Supercond. **12**, 681 (1999)].

[2] S. Chakravarty and A. Schmid, Phys. Rev. B **33**, 2000 (1986).

[3] P. Moretti, D. Mugnai, F. Pignatelli, and A. Ranfagni, Phys. Lett. A **271**, 139 (2000).

[4] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987); where, however, Ref. 2 is not quoted.

[5] K. Fujikawa, S. Iso, M. Sasaki, and H. Suzuki, Phys. Rev. Lett. **68**, 1093 (1992).

[6] A. Agresti, A. Ranfagni, R. Ruggeri, and P. Sandri, Phys. Rev. E **67**, 027602 (2003).

[7] A. Ranfagni, P. Moretti, I. Cacciari, and M. A. Vitali, Phys. Rev. E **77**, 057601 (2008).

[8] A. Agresti, M. Barilli, A. Ranfagni, R. Ruggeri, and C. Susini, Phys. Rev. E **65**, 066616 (2002).

[9] P. Moretti *et al.* (unpublished).

[10] It is worth to note that adopting the Lagrangian corresponding to Eq. (7), namely, Ref. 7. $\mathcal{L}(\omega,\tau,\tau_0)=\frac{1}{2}\frac{\eta}{\omega}[\dot{\varphi}(\tau)]^2|\tanh(\omega\tau_0)|$, the time integration from $-\infty$ to ∞ supplies for $\omega=\Omega$ and in the limit of $\Omega\tau_0 \gg 1$, a result ($0.267\eta\varphi_B^2$) which is sensibly smaller than the one given by Eq. (7) ($0.465\eta\varphi_B^2$). A result which is practically the same ($0.436\eta\varphi_B^2$) is indeed obtained when Eq. (8) is subsequently integrated from 0 to ∞ (or from $-\infty$ to 0) (Ref. 8).

[11] We recall that the semiclassical traversal time of a "particle" of mass m and energy E, through a potential barrier $V(x)$, is given by $\tau=(\frac{m}{2})^{1/2}\int_a^b \frac{dx}{[V(x)-E]^{1/2}}$, where a and b are the turning points at $V(x)=E$.

[12] See Eq. (17) in Ref. 3, for temperature $T\to 0$ and lower signs, where $\tanh(\Omega\tau/2)=(1-\varphi/\varphi_B)^{1/2}$.

[13] R. F. Voss and R. A. Webb, Phys. Rev. Lett. **47**, 265 (1981).

[14] D. Esteve, J. M. Martinis, C. Urbina, E. Turlot, and M. H. Devoret, Phys. Scr., T **T29**, 121 (1989).

[15] P. Fabeni, D. Mugnai, G. P. Pazzi, F. Pignatelli, A. Ranfagni, R. Cristiano, L. Frunzio, and L. S. Schulmann, J. Supercond. **12**, 829 (1999).

[16] A. Ranfagni, P. Fabeni, D. Mugnai, G. P. Pazzi, L. S. Schulman, G. Ventura, R. Cristiano, and L. Frunzio, Phys. Scr. **58**, 538 (1998).

[17] F. Pignatelli, PhD thesis, University of Florence, 1999.

[18] A. N. Malakhov and A. L. Pankratov, Physica C **269**, 46 (1996).

[19] A. L. Pankratov and M. Salerno, Phys. Lett. A **273**, 162 (2000).

[20] A. Ranfagni, I. Cacciari, P. Sandri, C. Ranfagni, R. Ruggeri, and A. Agresti, Phys. Lett. A **343**, 469 (2005).