Spin filtering and scaling of spin-dependent potentials in quasi-one-dimensional electron liquids with Rashba spin-orbit interaction

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We investigate theoretically the spin-filtering effect in a quasi-one-dimensional (Q1D) electron liquid with spin-orbit interaction. The Q1D system considered is formed from a two-dimensional electron-gas (2DEG) subject to both a lateral confining potential and an interface potential perpendicular to the 2DEG. Spin and charge degrees of freedom in the system are mixed by the interface potential through the Rashba mechanism of spin-orbit interaction [A. V. Moroz and C. H. W. Barnes, Phys. Rev. B **60**, 14272 (1999)] and we show that when a spin-dependent δ potential is further introduced into the system, for example, via implantation of magnetic/ferromagnetic impurities, the mixing leads to the spin-filtering effect which favors electrons with a certain spin orientation to transport through the δ potential. In particular, we calculate the scaling dimension of electron scattering both by spin-flip and by spin-independent δ potentials when the temperature is varied and show that, in the spin-flip case, the scaling of electron scattering with temperature varies with spin orientation. Conductance is calculated for both spin and charge transport, and the spin-filtering effect is discussed quantitatively in terms of the conductance.

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I. INTRODUCTION

Spintronics deals with the processing of polarized spin instead of charge signals and offers an innovative approach to technological applications including quantum computation.[1–4] Various spintronic devices have been proposed based on Fermi-liquid (FL) systems in two or three dimensions.[4] But with the advanced technology in fabrication of quantum wires, spintronic devices have also been suggested which are specifically based on the unique properties of quasi-one-dimensional (Q1D) systems.[5–11]

One distinct class of states in 1D considered for spintronic applications is known as the Tomonaga-Luttinger or simply Luttinger liquid (LL) and can be investigated within the framework of Tomonaga-Luttinger theory.[12–22] It has a unique low-energy excitation spectrum drastically different from that of FL. Specifically, the spectrum is linear, of bosonic nature, and consists of two branches describing separately excitations in association with spin or charge fluctuations. As a result of the unique spectrum, LL shows peculiar electronic properties, e.g., electronic density of states with power-law energy dependence. In the case of a spinless LL system, for example, the density of states $D(E) \propto E^{(K_\rho+1/K_\rho-2)/2}$, where K_ρ is the so-called Luttinger parameter specifying the electron-electron interaction, with $K_\rho<1$ for repulsive interaction, $K_\rho=1$ for a noninteracting system and $K_\rho>1$ for attractive interaction (we focus on the repulsive case throughout the work). In particular, $D(E)$ vanishes at the Fermi point as opposed to that of FL which is constant near the Fermi surface. This power-law characteristic of LL also shows up in electrical transport phenomena, via the dependence of transport on the density of states.[18,19] For example, in the case where a strong impurity potential is present, it leads to the ohmic tunneling conductance $G \propto T^{2/K_\rho-2}$, which scales with T (temperature) and, in particular, vanishes at $T=0$. Similarly, in the case where the impurity strength is weak, renormalization-group analysis shows that the effective impurity strength V_{eff} is a function of temperature which also scales according to the power law

$$V_0(\Lambda/k_BT)^{1-K_\rho}, \tag{1}$$

where Λ is the upper energy cutoff of the linear region of excitation spectrum. The ohmic conductance in this case is

$$G \approx G_0 - O(1)(V_0/\Lambda)^2(\Lambda/k_BT)^{2-2K_\rho}, \tag{2}$$

where $G_0=K_\rho e^2/2\pi$ ($\hbar=1$) is the conductance of LL free of impurities.

We are primarily interested, for spintronic applications, in the possibility of spin filtering which utilizes the above scaling property of conductance/impurity potential to produce spin polarization. It has been proposed that spin filtering can be achieved with the LL implanted with a nonmagnetic impurity.[7–10] It is shown that in the presence of a magnetic field, spin and charge degrees of freedom are mixed by the associated Zeeman effect and the mixing causes the exponent in Eqs. (1) and (2) for charge transport to become spin dependent, e.g., with new parameters $K_\uparrow$ and $K_\downarrow$ replacing K_ρ for up and down electrons, respectively. Generally, $K_\uparrow \neq K_\downarrow$ and, according to the equations, electrons of opposite spins thus see different impurity strength, which leads naturally to different conductance for up- and down-electron currents and gives rise to the effect of spin polarization.

In the forgoing proposal of spin-filtering devices, the magnetic field is indispensable and the presence of it satisfies two conditions important for the realization of spin filtering. First, it mixes spin and charge through the Zeeman effect. Second, it also breaks the time-reversal symmetry of the system in order to produce spin polarization. An interesting question arises as to whether it is possible to replace the magnetic field with one of electrical nature. A clue to the foregoing question has indeed been given in the important work of Moroz and Barnes.[23] They have shown that when a Q1D system is formed from a two-dimensional electron-gas

(2DEG) subject to both a lateral confining potential and an interface potential perpendicular to the 2DEG, the Rashba mechanism of spin-orbit interaction due to the interface potential leads to an asymmetric single-particle dispersion and as a result of the asymmetry, it gives rises to the required spin-charge mixing. Thus, the first condition, i.e., spin-charge mixing, for spin filtering is satisfied there without requiring the presence of any magnetic field. From the viewpoint of device implementation, it simplifies the design of spin filters. A further advantage with their system is that a gate voltage may be applied to adjust the interface potential and the required spin-charge mixing. It offers an electrical means of controlling spin filtering as opposed to the magnetic means in the original proposal. On the other hand, it is important to note that the Moroz-Barnes mechanism of mixing alone does not produce spin polarization since the time-reversal symmetry remains unbroken in the Rashba spin-orbit interaction-induced mixing. In order to break the symmetry and satisfy the second condition for spin filtering, a magnetic/ferromagnetic impurity is therefore introduced, in our work, into the system.

In summary, the foregoing reasoning motivates us to study the Q1D system similar to that of Moroz and Barnes but with the addition of a magnetic/ferromagnetic impurity into the system. In particular, we shall calculate the scaling dimension of a weak, spin-dependent impurity potential and show that the potential scales with temperature with different exponents for electrons of opposite spins. It thus provides a mechanism for spin filtering.

Before we present the calculation, we mention that Q1D systems showing strong Rashba spin-orbit interaction have also been proposed as asymmetric spin-polarization filters but the devices are based on a different principle.[11] It requires a magnetic field to be applied along the wire creating a Zeeman splitting of energy bands and opening a gap for spin filtering. We also note that, in addition to spin-charge mixing, spin-orbit coupling can renormalize Luttinger parameters or even affect the phase diagram of a Q1D system.[24,25] In the present work, we shall assume that the system remains in the LL phase and the Luttinger parameters used in our calculation are already renormalized.

The paper is organized as follows. In Sec. II, we present the calculation of scaling dimension of a spin-flip δ potential which models the spin-flip part of a magnetic/ferromagnetic impurity. The nonspin-flip part is modeled by a spin-independent δ potential and the calculation of corresponding scaling dimension is presented in Sec. III. In Sec. IV, we derive the conductance for both spin and charge transport and discuss the spin-filtering effect quantitatively in terms of the conductance. In Sec. V, we summarize the study.

II. SCALING DIMENSION OF A SPIN-FLIP δ potential

In Sec. II A, we discuss bosonization, the bosonized form of a spin-flip δ potential, and the path-integral formalism for our calculation. In Sec. II B, we derive the scaling dimension of the potential.

A. Bosonization, spin-flip δ potential, and path-integral representation of correlators

1. Bosonization

For a Q1D system of electrons, the low-energy spectrum of the system is linear (up to an energy Λ) and consists of only (bosonic) density fluctuations. Following the standard procedure of bosonization in the low-energy sector (see Ref. 22, for example), we express the electron field operator in terms of the fluctuations as follows. We write

$$\Psi_S(y) = e^{ik_F y}R_S(y) + e^{-ik_F y}L_S(y), \quad s = \uparrow \text{or} \downarrow,$$

$$R_S(y) = \frac{1}{\sqrt{2\pi a}}e^{i\varphi_{R,S}(y)} \quad L_S(y) = \frac{1}{\sqrt{2\pi a}}e^{-i\varphi_{L,S}(y)},$$

where we take the Q1D wire to be in the y direction, k_F is the Fermi wave vector, $R_S(y)$ and $L_S(y)$ are the right and left movers, and a is a length parameter determined by the upper wave vector cutoff of the linear, low-energy spectrum. The phase fields $\varphi_{R,S}(y)$ and $\varphi_{L,S}(y)$ are basically the accumulated fluctuating right- and left-moving charges with spin s, respectively,

$$\varphi_{R(L),S}(y) = 2\pi \int^y \delta\rho_{R(L),S}(y')dy'.$$

The above gives a transformation from the fermion field $\Psi_S(y)$ to the boson fields $\{\varphi_{R,S}(y), \varphi_{L,S}(y)\}$. In order to describe separately charge- and spin-fluctuation sectors (denoted below by ρ and σ, respectively), linear combinations of $\varphi_{R,S}(y)$ and $\varphi_{L,S}(y)$ are formed, with

$$\varphi_\rho(y) \equiv \frac{1}{2\sqrt{2}}[\varphi_{R,\uparrow}(y) + \varphi_{L,\uparrow}(y) + \varphi_{R,\downarrow}(y) + \varphi_{L,\downarrow}(y)],$$

$$\theta_\rho(y) \equiv \frac{1}{2\sqrt{2}}[\varphi_{R,\uparrow}(y) - \varphi_{L,\uparrow}(y) + \varphi_{R,\downarrow}(y) - \varphi_{L,\downarrow}(y)],$$

$$\varphi_\sigma(y) \equiv \frac{1}{2\sqrt{2}}[\varphi_{R,\uparrow}(y) + \varphi_{L,\uparrow}(y) - \varphi_{R,\downarrow}(y) - \varphi_{L,\downarrow}(y)],$$

$$\theta_\sigma(y) \equiv \frac{1}{2\sqrt{2}}[\varphi_{R,\uparrow}(y) - \varphi_{L,\uparrow}(y) - \varphi_{R,\downarrow}(y) + \varphi_{L,\downarrow}(y)],$$

$$\Pi_\rho(y) \equiv \frac{1}{\pi}\partial_y\theta_\rho(y), \quad \Pi_\sigma(y) \equiv \frac{1}{\pi}\partial_y\theta_\sigma(y).$$

These fields have the following interpretation. Within a multiplicative constant, $\partial\varphi_\rho(y)/\partial y$ and $\Pi_\rho(y)$ (or $\partial_y\theta_\rho$) are the fluctuating charge and charge current densities, respectively. Similarly, $\partial\varphi_\sigma(y)/\partial y$ and $\Pi_\sigma(y)$ (or $\partial_y\theta_\sigma$) are basically the spin and spin current densities, respectively. With the above boson fields, one can transform the Hamiltonian density of a Q1D system written in terms of fermion fields to one in terms of boson fields, e.g.,

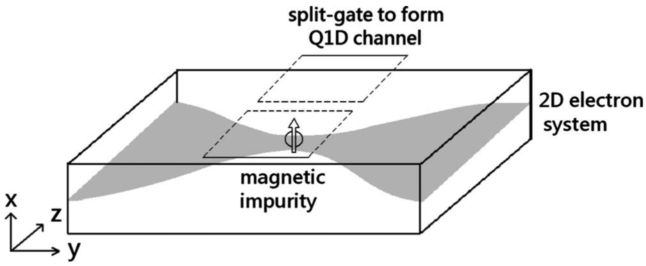

FIG. 1. The Q1D system is formed from a 2DEG on the y-z plane with the electron-gas subject to both a lateral confining potential along the z axis and an interface potential perpendicular to the 2DEG.

$$h_0(\varphi_\rho,\varphi_\sigma,\Pi_\rho,\Pi_\sigma) = \text{Hamiltonian density}$$

$$= \frac{1}{2\pi}\{v_\rho K_\rho(\pi\Pi_\rho)^2 + v_\rho/K_\rho(\partial_y\varphi_\rho)^2 + v_\sigma K_\sigma(\pi\Pi_\sigma)^2 + v_\sigma/K_\sigma(\partial_y\varphi_\sigma)^2\},$$

where K_ρ and K_σ are the Luttinger parameters, and v_ρ and v_σ are the velocities of charge and spin excitations, respectively. These parameters depend on the electron-electron coupling constants $g_{1,\parallel}$, $g_{2,\parallel}$, $g_{2,\perp}$, $g_{4,\parallel}$, and $g_{4,\perp}$, with

$$v_\rho = v_0\sqrt{\left(1+\frac{g_{4,\parallel}+g_{4,\perp}}{2\pi v_0}\right)^2 - \left(\frac{g_{2,\parallel}+g_{2,\perp}-g_{1,\parallel}}{2\pi v_0}\right)^2},$$

$$K_\rho = \sqrt{\frac{2\pi v_0 + g_{4,\parallel}+g_{4,\perp}-g_{2,\parallel}-g_{2,\perp}+g_{1,\parallel}}{2\pi v_0 + g_{4,\parallel}+g_{4,\perp}+g_{2,\parallel}+g_{2,\perp}-g_{1,\parallel}}},$$

$$v_\sigma = v_0\sqrt{\left(1+\frac{g_{4,\parallel}-g_{4,\perp}}{2\pi v_0}\right)^2 - \left(\frac{g_{2,\parallel}-g_{2,\perp}-g_{1,\parallel}}{2\pi v_0}\right)^2},$$

$$K_\sigma = \sqrt{\frac{2\pi v_0 + g_{4,\parallel}-g_{4,\perp}-g_{2,\parallel}+g_{2,\perp}+g_{1,\parallel}}{2\pi v_0 + g_{4,\parallel}-g_{4,\perp}+g_{2,\parallel}-g_{2,\perp}-g_{1,\parallel}}}.$$

With the bosonized Hamiltonian density, one can study the dynamics of the system in terms of that of a boson gas, valid up to the energy Λ, the upper cutoff of linear region in the low-energy spectrum of the system.

In our case, we consider the Q1D system of length L_0, as shown in Fig. 1, formed from a 2DEG subject to both a lateral confining potential and an interface potential perpendicular to the 2DEG. The 2DEG is taken to lie on the y-z plane, with the x direction being normal to the 2DEG and the y direction being along the Q1D wire. The single-particle dispersion of the system is asymmetric due to the interface potential-induced Rashba spin-orbit interaction. Correspondingly, this modifies the bosonized Hamiltonian density. Let v_0 be the average and δv be the difference between the Fermi velocities of left and right branches of the dispersion. We have the following bosonized Hamiltonian density:[23]

$$h(\varphi_\rho,\varphi_\sigma,\Pi_\rho,\Pi_\sigma) = h_0(\varphi_\rho,\varphi_\sigma,\Pi_\rho,\Pi_\sigma) + \frac{\delta v}{2\pi}\{(\partial_y\varphi_\rho)(\pi\Pi_\sigma) + (\partial_y\varphi_\sigma)(\pi\Pi_\rho)\}. \tag{3}$$

When δv is finite, the system has the unique feature that spin and charge are mixed, as shown in Eq. (3) above. Throughout the paper, we take $\hbar=1$ and $\Lambda = v_0/a = k_B T_F$ for the model described here, where T_F is the Fermi temperature.

Next, we shall consider the addition of a spin-dependent potential to the Hamiltonian, when a magnetic/ferromagnetic impurity is implanted in the system. This additional term shall also be bosonized.

2. Spin-flip δ potential

We assume the impurity magnetic-moment points to the x direction (normal to the 2DEG) and write the impurity potential as $V_C\delta(y)+V_S\sigma_x\delta(y)$, a sum of nonspin-flip and spin-flip parts. We express the spin-flip part of potential energy, $\int dy \Psi_\uparrow^+(y)[V_S\delta(y)]\Psi_\downarrow(y)+$ Hermitian conjugate, in terms of the boson fields introduced earlier, and it yields

$$\frac{V_S}{\pi a}\{\cos[\sqrt{2}(\varphi_\rho+\theta_\sigma)|_{y=0}] + \cos[\sqrt{2}(\varphi_\rho-\theta_\sigma)|_{y=0}]\}. \tag{4}$$

The first term represents the backscattering process $R\uparrow \Leftrightarrow L\downarrow$ and the second term describes the process $R\downarrow \Leftrightarrow L\uparrow$.

We note that the two terms in Eq. (4) show different dependences on the boson fields and it leads naturally to the expectation of a possible difference in the strength of scattering. In the following, we shall study how the strength of each scattering scales with energy (or temperature). We shall show that the two types of scattering indeed scale differently and, moreover, the difference in the scattering strength increases with decreasing energy (or temperature).

3. Path-integral representation of correlators

In the case of LL, one can utilize the power-law scaling property of correlators to determine the scaling dimension of a term in the Hamiltonian.[26] According to Eq. (4), we shall consider specifically the binary correlator

$$\langle\cos[\sqrt{2}(\varphi_\rho+\theta_\sigma)|_{y=0,\tau_1}]\cos[\sqrt{2}(\varphi_\rho+\theta_\sigma)|_{y=0,\tau_2}]\rangle$$

for the $R\uparrow \Leftrightarrow L\downarrow$ backscattering process and

$$\langle\cos[\sqrt{2}(\varphi_\rho-\theta_\sigma)|_{y=0,\tau_1}]\cos[\sqrt{2}(\varphi_\rho-\theta_\sigma)|_{y=0,\tau_2}]\rangle$$

for the $R\downarrow \Leftrightarrow L\uparrow$ process, respectively. Once the scaling exponent of the correlator is obtained, we can split it evenly and obtain the exponent for each factor, i.e., $\cos[\sqrt{2}(\varphi_\rho \mp \theta_\sigma)|_{y=0,\tau}]$. The correlator shall be calculated in the path-integral formalism described in the following.

In our approach, the path integrals of correlators are written as

$$\langle\cos[\sqrt{2}(\varphi_\rho\mp\theta_\sigma)|_{y=0,\tau_1}]\cos[\sqrt{2}(\varphi_\rho\mp\theta_\sigma)|_{y=0,\tau_2}]\rangle \equiv \frac{1}{Z_0}\int D\varphi_\rho D\theta_\sigma D\Pi_\rho D\Pi'_\sigma \cos[\sqrt{2}(\varphi_\rho\mp\theta_\sigma)|_{y=0,\tau_1}] \times\cos[\sqrt{2}(\varphi_\rho\mp\theta_\sigma)|_{y=0,\tau_2}]e^{S_E[\varphi_\rho,\theta_\sigma,\Pi_\rho,\Pi'_\sigma]}, \tag{5}$$

where Z_0 is the partition function, (Π_ρ,Π'_σ) are the canonical conjugates of $(\varphi_\rho,\theta_\sigma)$, and S_E is the action in the Matsubara formalism given below

$$S_E[\varphi_\rho,\theta_\sigma,\Pi_\rho,\Pi'_\sigma]=\int_0^\beta d\tau\int dy l(\varphi_\rho,\theta_\sigma,\Pi_\rho,\Pi'_\sigma)|_{it\to\tau}$$

with $\beta=1/k_BT$ being basically the inverse temperature. Note that, in Eq. (5), $(\varphi_\rho,\theta_\sigma,\Pi_\rho,\Pi'_\sigma)$ are chosen as integration variables instead of $(\varphi_\rho,\varphi_\sigma,\Pi_\rho,\Pi_\sigma)$. In fact, it follows from the "charge-current" duality[17] that the path-integral representation of LL can be described in terms of the "charge" fields $(\varphi_\rho,\varphi_\sigma)$, the "current" fields $(\theta_\rho,\theta_\sigma)$, or the hybrid sets $(\varphi_\rho,\theta_\sigma)$ or $(\theta_\rho,\varphi_\sigma)$, and their corresponding canonical conjugates. The reason we choose the hybrid $(\varphi_\rho,\theta_\sigma)$ is that the correlators involve the fields $(\varphi_\rho,\theta_\sigma)$ instead of $(\varphi_\rho,\varphi_\sigma)$ and may hence be calculated more conveniently with $(\varphi_\rho,\theta_\sigma,\Pi_\rho,\Pi'_\sigma)$ as the integration variables. For this reason we shall now derive the Lagrangian density $l(\varphi_\rho,\theta_\sigma,\Pi_\rho,\Pi'_\sigma)$ which appears in the integral of Eq. (5).

We shall first convert the Hamiltonian density $h(\varphi_\rho,\varphi_\sigma,\Pi_\rho,\Pi_\sigma)$ in Eq. (3) to $h(\varphi_\rho,\theta_\sigma,\Pi_\rho,\Pi'_\sigma)$. We make the transformation from $(\varphi_\sigma,\Pi_\sigma)$ to the dual representation $(\theta_\sigma,\Pi'_\sigma)$ (with the substitution $\partial_y\varphi_\sigma\to\pi\Pi'_\sigma$ and $\pi\Pi_\sigma\to\partial_y\theta_\sigma$) in the spin part of h, and h becomes

$$h(\varphi_\rho,\theta_\sigma,\Pi_\rho,\Pi'_\sigma)=\frac{1}{2\pi}\left\{v_\rho K_\rho(\pi\Pi_\rho)^2+\frac{v_\rho}{K_\rho}(\partial_y\varphi_\rho)^2+v_\sigma K_\sigma(\partial_y\theta_\sigma)^2+\frac{v_\sigma}{K_\sigma}(\pi\Pi'_\sigma)^2\right\}+\frac{\delta v}{2\pi}\{(\partial_y\varphi_\rho)\times(\partial_y\theta_\sigma)+(\pi\Pi'_\sigma)(\pi\Pi_\rho)\}.$$

It yields the following Lagrangian density:

$$\begin{aligned}l(\varphi_\rho,\theta_\sigma,\Pi_\rho,\Pi'_\sigma)&=\Pi_\rho\partial_t\varphi_\rho+\Pi'_\sigma\partial_t\theta_\sigma-h(\varphi_\rho,\theta_\sigma,\Pi_\rho,\Pi'_\sigma)\\&=\Pi_\rho\partial_t\varphi_\rho+\Pi'_\sigma\partial_t\theta_\sigma-\frac{1}{2\pi}\left\{v_\rho K_\rho(\pi\Pi_\rho)^2+\frac{v_\rho}{K_\rho}(\partial_y\varphi_\rho)^2+v_\sigma K_\sigma(\partial_y\theta_\sigma)^2+\frac{v_\sigma}{K_\sigma}(\pi\Pi'_\sigma)^2\right\}\\&\quad-\frac{\delta v}{2\pi}\{(\partial_y\varphi_\rho)(\partial_y\theta_\sigma)+(\pi\Pi'_\sigma)(\pi\Pi_\rho)\}.\end{aligned}$$

This is the Lagrangian density which enters the action S_E in Eq. (5). However, we can further simplify it as follows. We note that the field variables Π_ρ and Π'_σ in Eq. (5) can immediately be integrated out and it reduces to

$$\frac{1}{Z_0}\int D\varphi_\rho D\theta_\sigma\cos[\sqrt{2}(\varphi_\rho\mp\theta_\sigma)|_{y=0,\tau_1}]\times\cos[\sqrt{2}(\varphi_\rho\mp\theta_\sigma)|_{y=0,\tau_2}]e^{\int_0^\beta d\tau\int dy l(\varphi_\rho,\theta_\sigma)|_{it\to\tau}}$$

now with the Lagrangian density dependent only on $(\varphi_\rho,\theta_\sigma)$. $l(\varphi_\rho,\theta_\sigma)$ is given in terms of matrices

$$l(\varphi_\rho,\theta_\sigma)|_{it\to\tau}=\frac{1}{2\pi}\int dy'd\tau'(\varphi_\rho(y,\tau)\theta_\sigma(y,\tau))g_S^{-1}\times(y-y',\tau-\tau')\begin{pmatrix}\varphi_\rho(y',\tau')\\\theta_\sigma(y',\tau')\end{pmatrix},$$

where the matrix elements of g_S^{-1} are listed below

$$\begin{aligned}(g_S^{-1})_{11}&=\delta(y-y')\delta(\tau-\tau')[a_S(\partial_\tau)^2+b_S(\partial_y)^2],\\(g_S^{-1})_{22}&=\delta(y-y')\delta(\tau-\tau')[e_S(\partial_\tau)^2+f_S(\partial_y)^2],\\(g_S^{-1})_{12}=(g_S^{-1})_{21}&=\delta(y-y')\delta(\tau-\tau')[-c_S(\partial_\tau)^2+d_S(\partial_y)^2]\end{aligned}\tag{6}$$

with

$$a_S=\frac{v_\sigma}{K_\sigma\gamma_S},\quad b_S=\frac{v_\rho}{K_\rho},\quad c_S=\frac{\delta v}{2\gamma_S},\quad d_S=\frac{\delta v}{2},$$
$$e_S=\frac{v_\rho K_\rho}{\gamma_S},\quad f_S=v_\sigma K_\sigma,\quad \gamma_S=\frac{v_\rho K_\rho v_\sigma}{K_\sigma}-\frac{(\delta v)^2}{4}.$$

B. Scaling dimension

We determine the scaling dimension of the spin-flip potential as follows. We write the binary correlators

$$\langle\cos[\sqrt{2}(\varphi_\rho\mp\theta_\sigma)|_{y=0,\tau_1}]\cos[\sqrt{2}(\varphi_\rho\mp\theta_\sigma)|_{y=0,\tau_2}]\rangle=\sum_{n,m=1,2}C^{\mp}_{n.m},\tag{7}$$

where

$$C^{\mp}_{n.m}\equiv\frac{1}{4Z_0}\int D\varphi_\rho D\theta_\sigma e^{\int_0^\beta d\tau\int dy l^{\mp}_{n,m}(\varphi_\rho,\theta_\sigma)|_{it\to\tau}}$$

with

$$l^{\mp}_{n,m}=\frac{1}{2\pi}\int dy'd\tau'\Phi^t(y,\tau)g_S^{-1}(y-y',\tau-\tau')\Phi(y',\tau')+\frac{1}{2\pi}\{\Phi^t(y,\tau)J^{\mp}_{n,m}(y,\tau)+[J^{\mp}_{n,m}(y,\tau)]^t\Phi(y,\tau)\},$$

$$J^{\mp}_{n,m}(y,\tau)=j_{n,m}(y,\tau)\begin{pmatrix}1\\\mp1\end{pmatrix},\quad \Phi(y,\tau)=\begin{pmatrix}\varphi_\rho(y,\tau)\\\theta_\sigma(y,\tau)\end{pmatrix},$$

$$j_{n,m}(y,\tau)=i\sqrt{2}\delta(y)[(-1)^{n-1}\delta(\tau-\tau_1)+(-1)^{m-1}\delta(\tau-\tau_2)].$$

With the shift $\Phi(y,\tau)\to\Phi(y,\tau)+\int dy'd\tau'g_S(y-y',\tau-\tau')J^{\mp}_{n,m}(y',\tau')$, the path integral of $C^{\mp}_{n.m}$ is calculated and we obtain (with unimportant prefactors ignored)

$$C^{\mp}_{n.m}\propto\exp\left[\frac{1}{2\pi}\int dydy'd\tau d\tau' j_{n,m}(y,\tau)g_S^{(\mp)}\times(y-y',\tau-\tau')j_{n,m}(y',\tau')\right],\tag{8}$$

where $g_S^{(\mp)}=(g_S)_{11}\mp2(g_S)_{12}+(g_S)_{22}$. In order to study its scaling behavior, we adopt the following approach. We transform $g_S^{(\mp)}$ from (y,τ) space to (q,w) space and obtain

$$g_S^{(\mp)}(q,w) = N^{\mp}(q^2,w^2)/D(q^2,w^2),$$

where the numerator,

$$N^{\mp}(q^2,w^2) = -[v_\rho K_\rho + v_\sigma K_\sigma^{-1} \pm \delta v]w^2 - [v_\sigma K_\sigma + v_\rho K_\rho^{-1} \mp \delta v][v_\rho v_\sigma K_\rho K_\sigma^{-1} - (\delta v)^2/4]q^2$$

and the denominator,

$$D(q^2,w^2) = w^4 + [v_\rho^2 + v_\sigma^2 + (\delta v)^2/2]w^2 q^2 + [v_\rho v_\sigma K_\sigma K_\rho^{-1} - (\delta v)^2/4][v_\rho v_\sigma K_\rho K_\sigma^{-1} - (\delta v)^2/4]q^4.$$

Being a quadratic function of w^2, $D(q^2,w^2)$ can be factorized as

$$D(q^2,w^2) = (w^2 + v_+^2 q^2)(w^2 + v_-^2 q^2),$$

with

$$v_\pm^2 = \frac{1}{2}\{[v_\rho^2 + v_\sigma^2 + (\delta v)^2/2] \pm \sqrt{[v_\rho^2 + v_\sigma^2 + (\delta v)^2/2]^2 - 4[v_\rho v_\sigma K_\sigma K_\rho^{-1} - (\delta v)^2/4][v_\rho v_\sigma K_\rho K_\sigma^{-1} - (\delta v)^2/4]}\}.$$

Now, with the denominator in the factorized form, we arrange $g_S^{(\mp)}(q,w)$ as the following sum of fractions:

$$g_S^{(\mp)}(q,w) = -K_+^{\mp} v_+/(w^2 + v_+^2 q^2) - K_-^{\mp} v_-/(w^2 + v_-^2 q^2). \tag{9}$$

Here,

$$K_+^{\mp} + K_-^{\mp} = \frac{[v_\rho K_\rho + v_\sigma K_\sigma^{-1} \pm \delta v] + [v_\sigma K_\sigma + v_\rho K_\rho^{-1} \mp \delta v][v_\rho v_\sigma K_\rho K_\sigma^{-1} - (\delta v)^2/4]/v_+ v_-}{v_+ + v_-}. \tag{10}$$

Now, $g_S^{(\mp)}(q,w)$ as expressed in Eq. (9) shows a simple structure of poles and, with this form, we can easily transform $g_S^{(\mp)}$ from (q,w) space back to (y,τ) space by contour integration in the complex q plane and frequency summation. Again, with unimportant prefactors being ignored, it yields

$$g_S^{(\mp)}(y,\tau) \propto \frac{K_+^{\mp}}{4\pi}\ln|y^2 + v_+^2\tau^2| + \frac{K_-^{\mp}}{4\pi}\ln|y^2 + v_-^2\tau^2|$$

or, in the special case where $y=0$,

$$g_S^{(\mp)}(y=0,\tau) \propto \ln v_+ v_- |\tau|^{(K_+^{\mp}+K_-^{\mp})/2\pi}. \tag{11}$$

Putting together Eqs. (7), (8), and (11), we obtain

$$\langle \cos[\sqrt{2}(\varphi_\rho \mp \theta_\sigma)|_{y=0,\tau_1}]\cos[\sqrt{2}(\varphi_\rho \mp \theta_\sigma)|_{y=0,\tau_2}]\rangle \propto |\tau_1 - \tau_2|^{-K_+^{\mp}-K_-^{\mp}}.$$

It shows that the correlator has the scaling exponent $(-K_+^{\mp} - K_-^{\mp})$, previously given in Eq. (10) in terms of the parameters K_ρ, K_σ, v_ρ, v_σ, and δv of the present model. We further split it and obtain the exponent $-(K_+^{\mp}+K_-^{\mp})/2$ for each factor, i.e., $\cos[\sqrt{2}(\varphi_\rho \mp \theta_\sigma)|_{y=0,\tau}]$, in the correlator.

Finally, we can calculate the scaling dimension of the spin-flip terms, namely, $\frac{V_S}{\pi a}\cos[\sqrt{2}(\varphi_\rho + \theta_\sigma)|_{y=0}]$ (for the process $R\uparrow \leftrightarrow L\downarrow$) and $\frac{V_S}{\pi a}\cos[\sqrt{2}(\varphi_\rho - \theta_\sigma)|_{y=0}]$ (for the process $R\downarrow \leftrightarrow L\uparrow$), in Eq. (4) of the δ potential. We note that both terms have the prefactor $\frac{V_S}{\pi a}$ which carries the dimension of energy and scales with a unity exponent. Taking it into account, we obtain the total scaling dimension

$$\Delta_{R\uparrow\leftrightarrow L\downarrow} = 1 - (K_+^{+} + K_-^{+})/2,$$

$$\Delta_{R\downarrow\leftrightarrow L\uparrow} = 1 - (K_+^{-} + K_-^{-})/2. \tag{12}$$

Equation (12) shows that the two spin-flip scattering scale differently and implies the possibility of spin polarization. For example, one can apply a bias voltage and inject unpolarized electron current into the LL system from the left side. The incoming electrons will be scattered by the impurity potential which scales according to Eq. (12). As a result of the difference in scaling, electrons of one spin orientation will be backscattered more than those of the opposite orientation and the electrons collected on the right side will therefore be spin polarized. Quantitative spin polarization shall further be discussed in Sec. IV, where we calculate the conductance for spin current derived from the scaling asymmetry between the two spin orientations when spin-flip scattering is present.

III. SCALING DIMENSION OF A SPIN-INDEPENDENT δ potential

Now, we consider the scaling of a nonspin-flip δ-potential $V_C\delta(y)$, which is also part of a magnetic/ferromagnetic impurity potential. We express the potential energy, $\int dy \Psi^+[V_C\delta(y)]\Psi$, in terms of the boson fields, which yields

$$\frac{V_C}{\pi a}\{\cos[\sqrt{2}(\varphi_\rho + \varphi_\sigma)|_{y=0}] + \cos[\sqrt{2}(\varphi_\rho - \varphi_\sigma)|_{y=0}]\}. \tag{13}$$

In order to study the scaling behavior of the above expression, we carry out the calculation of correlators

$$\langle\cos[\sqrt{2}(\varphi_\rho \mp \varphi_\sigma)|_{y=0,\tau_1}]\cos[\sqrt{2}(\varphi_\rho \mp \varphi_\sigma)|_{y=0,\tau_2}]\rangle \equiv \frac{1}{Z_0}\int D\varphi_\rho D\varphi_\sigma D\Pi_\rho D\Pi_\sigma \cos[\sqrt{2}(\varphi_\rho \mp \varphi_\sigma)|_{y=0,\tau_1}] \times\cos[\sqrt{2}(\varphi_\rho \mp \varphi_\sigma)|_{y=0,\tau_2}]e^{S_E[\varphi_\rho,\varphi_\sigma,\Pi_\rho,\Pi_\sigma]}. \quad (14)$$

Here we have

$$S_E[\varphi_\rho,\varphi_\sigma,\Pi_\rho,\Pi_\sigma] = \int_0^\beta d\tau \int dy l(\varphi_\rho,\varphi_\sigma,\Pi_\rho,\Pi_\sigma)|_{it\to\tau}.$$

Starting from the Hamiltonian density $h(\varphi_\rho,\varphi_\sigma,\Pi_\rho,\Pi_\sigma)$ in Eq. (3), the Lagrangian density is

$$\begin{aligned} l(\varphi_\rho,\varphi_\sigma,\Pi_\rho,\Pi_\sigma) &= \Pi_\rho\partial_t\varphi_\rho + \Pi_\sigma\partial_t\varphi_\sigma - h(\varphi_\rho,\varphi_\sigma,\Pi_\rho,\Pi_\sigma) \\ &= \Pi_\rho\partial_t\varphi_\rho + \Pi_\sigma\partial_t\varphi_\sigma - \frac{1}{2\pi}\Big\{ v_\rho K_\rho(\pi\Pi_\rho)^2 \\ &\quad + \frac{v_\rho}{K_\rho}(\partial_y\varphi_\rho)^2 + v_\sigma K_\sigma(\pi\Pi_\sigma)^2 + \frac{v_\sigma}{K_\sigma}(\partial_y\varphi_\sigma)^2\Big\} \\ &\quad - \frac{\delta v}{2\pi}\{(\partial_y\varphi_\rho)(\pi\Pi_\sigma) + (\partial_y\varphi_\sigma)(\pi\Pi_\rho)\}. \end{aligned}$$

Integrating out Π_ρ and Π_σ in Eq. (14), we obtain

$$\langle\cos[\sqrt{2}(\varphi_\rho \mp \varphi_\sigma)|_{y=0,\tau_1}]\cos[\sqrt{2}(\varphi_\rho \mp \varphi_\sigma)|_{y=0,\tau_2}]\rangle = \frac{1}{Z_0}\int D\varphi_\rho D\varphi_\sigma \cos[\sqrt{2}(\varphi_\rho \mp \varphi_\sigma)|_{y=0,\tau_1}] \times\cos[\sqrt{2}(\varphi_\rho \mp \varphi_\sigma)|_{y=0,\tau_2}]e^{\int_0^\beta d\tau \int dy l(\varphi_\rho,\varphi_\sigma)|_{it\to\tau}},$$

where

$$l(\varphi_\rho,\varphi_\sigma)|_{it\to\tau} = \frac{1}{2\pi}\int dy' d\tau' [\varphi_\rho(y,\tau)\varphi_\sigma(y,\tau)]g_C^{-1} \times(y-y',\tau-\tau')\begin{bmatrix}\varphi_\rho(y',\tau')\\ \varphi_\sigma(y',\tau')\end{bmatrix}$$

and

$$\begin{aligned} (g_C^{-1})_{11} &= \delta(y-y')\delta(\tau-\tau')[a_C(\partial_\tau)^2 + b_C(\partial_y)^2], \\ (g_C^{-1})_{22} &= \delta(y-y')\delta(\tau-\tau')[e_C(\partial_\tau)^2 + f_C(\partial_y)^2], \\ (g_C^{-1})_{12} &= (g_C^{-1})_{21} = \delta(y-y')\delta(\tau-\tau')[d_C\partial_\tau\partial_y] \end{aligned} \quad (15)$$

with

$$a_C = \frac{1}{K_\rho v_\rho}, \quad b_C = \frac{v_\rho}{K_\rho} - \frac{(\delta v)^2}{4K_\sigma v_\sigma},$$

$$d_C = i\frac{\delta v}{2}\left(\frac{1}{K_\rho v_\rho} + \frac{1}{K_\sigma v_\sigma}\right), \quad e_C = \frac{1}{K_\sigma v_\sigma},$$

$$f_C = \left[\frac{v_\sigma}{K_\sigma} - \frac{(\delta v)^2}{4K_\rho v_\rho}\right].$$

We then evaluate g_C, the correlator in Eq. (14), and the scaling dimension, in the way similar to the procedure described in Sec. II B for the calculation of the corresponding terms in the case of spin-flip scattering. It yields the following expression of scaling dimension:

$$\Delta_C = 1 - K/2 \quad (16)$$

with

$$K = \frac{v_\rho K_\rho + v_\sigma K_\sigma + [v_\rho K_\rho v_\sigma^2 + v_\sigma K_\sigma v_\rho^2 - (v_\rho K_\rho + v_\sigma K_\sigma)(\delta v)^2/4]/v_+ v_-}{v_+ + v_-}$$

for the nonspin-flip scattering. We note that the scaling dimension of a spin-independent δ potential has previously been calculated by Moroz *et al.*[23] However, the result in Eq. (16) is expressed in the form which treats K_ρ and K_σ as independent parameters of the model while that of Moroz *et al.* is specifically written for the case where the Luttinger parameters K_ρ and K_σ are related, e.g., $K_\rho^2+K_\sigma^2=2$. Such a relation follows the assumption in their LL model that electron-electron interactions are of pointlike density-density type only.

As Eqs. (12) and (16) show, for numerical calculation of the scaling dimension (and spin polarization in this work), numerical values of the following parameters are required, namely, K_ρ, K_σ, $u_\rho \equiv \frac{v_\rho}{v_0}$, $u_\sigma \equiv \frac{v_\sigma}{v_0}$, and the velocity asymmetry $\varepsilon \equiv \frac{\delta v}{v_0}$, where v_0 is the average of Fermi velocities of left and right branches of the energy bands. The numerical value of K_ρ is discussed in Ref. 27, where it is shown that $K_\rho > 0.5$ for a wide range of semiconductor quantum wire width and $K_\rho < 0.5$ for systems with strong electron-electron repulsion. We consider the range of $K_\rho \geq 0.6$ in the calculation to show the trend of scaling and spin polarization. As for the value of K_σ, both $K_\sigma > 1$ and $K_\sigma < 1$ have been mentioned in the literature.[24] We shall therefore consider both of the possibilities in the calculation. u_ρ and u_σ are taken to be $1/K_\rho$ and $1/K_\sigma$, respectively, or, equivalently, with $v_\rho K_\rho = v_\sigma K_\sigma = v_0$, which holds in the special case of pointlike density-density-type interactions.[23] Last, the velocity asymmetry ε is estimated to be about 0.1–0.2 according to Ref. 23. Numerical results of scaling dimension based on Eqs. (12) and (16) are listed below for two distinct cases, one with $K_\sigma > 1$ and the other with $K_\sigma < 1$.

(a) $(K_\rho, K_\sigma, u_\rho, u_\sigma) = (0.6, 1.1, \frac{1}{0.6}, \frac{1}{1.1})$. For $\boldsymbol{\varepsilon} = 0.1$, scaling dimension 0.2547 ($R\uparrow \leftrightarrow L\downarrow$); 0.237 ($R\downarrow \leftrightarrow L\uparrow$); and

0.1496 (nonflip). For $\varepsilon=0.2$, scaling dimension 0.2647 ($R\uparrow\leftrightarrow L\downarrow$); 0.2292 ($R\downarrow\leftrightarrow L\uparrow$); and 0.1487 (nonflip).

(b) $(K_\rho,K_\sigma,u_\rho,u_\sigma)=(0.6,0.9,\frac{1}{0.6},\frac{1}{0.9})$. For $\varepsilon=0.1$, scaling dimension 0.1507 ($R\uparrow\leftrightarrow L\downarrow$); 0.1387 ($R\downarrow\leftrightarrow L\uparrow$); and 0.2499 (nonflip). For $\varepsilon=0.2$, scaling dimension 0.1574 ($R\uparrow\leftrightarrow L\downarrow$); 0.1333 ($R\downarrow\leftrightarrow L\uparrow$); and 0.2496 (nonflip).

The above results show clearly that the scaling depends critically on the value of K_σ. When $K_\sigma>1$, the spin-flip scattering dominates at low energy while if $K_\sigma<1$, the nonspin-flip scattering dominates. Moreover, the difference in scaling between the two processes, $R\uparrow\leftrightarrow L\downarrow$ and $R\downarrow\leftrightarrow L\uparrow$, increases with the velocity asymmetry ε.

IV. CONDUCTANCE AND SPIN FILTERING

Next, we use the Kubo formula to calculate both the charge and spin conductance, as given below

spin conductance

$$=G_\sigma=\lim_{\omega\to 0}\frac{-e^2}{\omega}\times\left[\frac{1}{L_0}\int_{-L_0/2}^{L_0/2}dy_1\langle j_\sigma(y=0,-w)j_\rho(y_1,w)\rangle|_{iw\to\omega+i\delta}\right],$$

charge conductance

$$=G_\rho=\lim_{\omega\to 0}\frac{-e^2}{\omega}\times\left[\frac{1}{L_0}\int_{-L_0/2}^{L_0/2}dy_1\langle j_\rho(y=0,-w)j_\rho(y_1,w)\rangle|_{iw\to\omega+i\delta}\right], \tag{17}$$

where j_ρ and j_σ are charge and spin current densities, respectively, and, in terms of the boson fields, can be represented as

$$j_\rho=\frac{\sqrt{2}}{\pi}\partial_t\varphi_\rho,$$

$$j_\sigma=\frac{\sqrt{2}}{\pi}\partial_t\varphi_\sigma \quad \text{or} \quad \frac{\sqrt{2}}{\pi}v_\sigma K_\sigma\partial_y\theta_\sigma.$$

With them, the spin conductance in Eq. (17) is written as

$$G_\sigma=\lim_{\omega\to 0}\frac{2e^2}{\pi^2}v_\sigma K_\sigma\left[\frac{1}{L_0}\int_{-L_0/2}^{L_0/2}dy_1\langle\partial_y\theta_\sigma\times(y,-w)|_{y=0}\varphi_\rho(y_1,w)\rangle|_{iw\to\omega+i\delta}\right]. \tag{18}$$

The calculation of spin conductance therefore reduces to the evaluation of the following path integral:

$$\langle\partial_y\theta_\sigma\varphi_\rho\rangle=\frac{1}{Z}\int D\varphi_\rho D\theta_\sigma\partial_y\theta_\sigma\varphi_\rho e^{\int_0^\beta d\tau\int dy' l(\varphi_\rho,\theta_\sigma)|_{it\to\tau}+S_V},$$

$$Z=\int D\varphi_\rho D\varphi_\sigma e^{\int_0^\beta d\tau\int dy' l(\varphi_\rho,\theta_\sigma)|_{it\to\tau}+S_V}.$$

Here, S_V is the action contributed by the impurity potential scattering, which includes both the spin-flip and nonspin-flip ones as given in Eqs. (4) and (13). In order to simplify the analysis and get semiquantitative insight, we assume we are in the so-called high-temperature/long-system regime where the characteristic energy $k_BT\gg v_0/L_0$, which permits us to neglect the effect of electrode on the transport properties of LL here[18,20,21] and calculate the integral perturbatively treating the impurity potential strength as the small parameter. Detailed calculations are presented in the Appendix. Obviously, the zeroth-order conductance $G_\sigma^{(0)}$ (i.e., the conductance without the presence of any impurity scattering) is zero. It is found that the first nonzero correction $\delta G_\sigma^{(S)}$ derives from the spin-flip δ-potential $\frac{V_S}{\pi a}$ and is of $O(V_S^2)$,

$$\delta G_\sigma^{(S)}=\frac{-\pi^{3/2}e^2}{4}\left(\frac{V_S}{a\Lambda}\right)^2\left\{\left[v_\sigma K_\sigma\frac{\gamma_S}{v_+^2v_-^2}\left(-v_\rho K_\rho^{-1}-\frac{\delta v}{2}\right)\right]\frac{1}{v_++v_-}\left[\left(v_\rho K_\rho-\frac{\delta v}{2}\right)+\frac{\gamma_S}{v_+v_-}\left(v_\sigma K_\sigma+\frac{\delta v}{2}\right)\right]\right.$$
$$\times\frac{\Gamma\left(\frac{K_+^-+K_-^-}{2}\right)}{\Gamma\left(\frac{1+K_+^-+K_-^-}{2}\right)}\left(\frac{\pi}{\beta\Lambda}\right)^{K_+^-+K_-^--2}+\left[v_\sigma K_\sigma\frac{\gamma_S}{v_+^2v_-^2}\left(v_\rho K_\rho^{-1}-\frac{\delta v}{2}\right)\right]\frac{1}{v_++v_-}$$
$$\left.\times\left[\left(v_\rho K_\rho+\frac{\delta v}{2}\right)+\frac{\gamma_S}{v_+v_-}\left(v_\sigma K_\sigma-\frac{\delta v}{2}\right)\right]\frac{\Gamma\left(\frac{K_+^++K_-^+}{2}\right)}{\Gamma\left(\frac{1+K_+^++K_-^+}{2}\right)}\left(\frac{\pi}{\beta\Lambda}\right)^{K_+^++K_-^+-2}\right\}. \tag{19}$$

Similarly, the charge conductance in Eq. (17) can be written as

$$G_\rho = \lim_{\omega\to 0}\frac{-2e^2}{\pi^2} i(\omega+i\delta) \times\left[\frac{1}{L_0}\int_{-L_0/2}^{L_0/2} dy_1 \langle\varphi_\rho(y,-w)|_{y=0}\varphi_\rho(y_1,w)\rangle|_{iw\to\omega+i\delta}\right]. \quad (20)$$

The calculation of charge conductance therefore reduces to the evaluation of path integral $\langle\varphi_\rho\varphi_\rho\rangle$, which is done perturbatively in the Appendix, too. The zeroth-order G_ρ (i.e., the conductance without the presence of impurity scattering) has previously been obtained and is given as[18,19]

$$G_\rho^{(0)} = \frac{K_\rho e^2}{\pi}.$$

It is found that the correction to the conductance due to the nonspin-flip potential $\frac{V_C}{\pi a}$ [to $O(V_C^2)$] is

$$\delta G_\rho^{(C)} = \frac{-e^2}{2\sqrt{\pi}}\left(\frac{V_C}{a\Lambda}\right)^2\frac{1}{(v_+ + v_-)^2} \times\left(v_\rho K_\rho + \frac{\gamma_S}{v_+v_-}v_\sigma K_\sigma\right)^2\frac{\Gamma\left(\frac{K}{2}\right)}{\Gamma\left(\frac{1+K}{2}\right)}\left(\frac{\pi}{\beta\Lambda}\right)^{K-2} \quad (21a)$$

and the correction due to the spin-flip potential $\frac{V_S}{\pi a}$ [to $O(V_S^2)$] is

$$\delta G_\rho^{(S)} = \frac{-e^2}{4\sqrt{\pi}}\left(\frac{V_S}{a\Lambda}\right)^2\frac{1}{(v_+ + v_-)^2} \times\Bigg\{\left[\left(v_\rho K_\rho + \frac{\delta v}{2}\right) + \frac{\gamma_S}{v_+v_-}\left(v_\sigma K_\sigma - \frac{\delta v}{2}\right)\right]^2 \times\frac{\Gamma\left(\frac{K_+^+ + K_-^+}{2}\right)}{\Gamma\left(\frac{1+K_-^+ + K_-^+}{2}\right)}\left(\frac{\pi}{\beta\Lambda}\right)^{K_+^+ + K_-^+ - 2} + \left[\left(v_\rho K_\rho - \frac{\delta v}{2}\right) + \frac{\gamma_S}{v_+v_-}\left(v_\sigma K_\sigma + \frac{\delta v}{2}\right)\right]^2 \times\frac{\Gamma\left(\frac{K_+^- + K_-^-}{2}\right)}{\Gamma\left(\frac{1+K_+^- + K_-^-}{2}\right)}\left(\frac{\pi}{\beta\Lambda}\right)^{K_+^- + K_-^- - 2}\Bigg\}. \quad (21b)$$

In order to measure the effect of spin filtering, we define the polarization

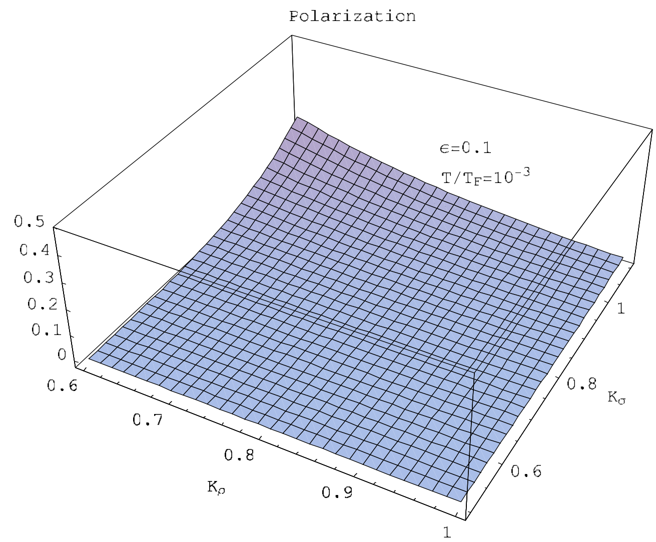

FIG. 2. (Color online) Polarization as a function of K_ρ and K_σ. K_ρ ranges from 0.6 to 1.0 and K_σ from 0.5 to 1.1. We set ε=0.1, $T=10^{-3}T_F$, and $\frac{V_S}{\Lambda a}$=0.3. Polarization increases with increasing K_σ and decreasing K_ρ.

$$P = \frac{G_\sigma}{G_\rho} \approx \frac{\delta G_\sigma^{(S)}}{G_\rho}.$$

The polarization as a function of K_ρ and K_σ is plotted in Fig. 2. In this numerical calculation, we take ε=0.1, V_C=0, $\frac{V_S}{\Lambda a}$ =0.3, and $\frac{k_BT}{\Lambda}=\frac{T}{T_F}=10^{-3}$. $v_\rho K_\rho = v_\sigma K_\sigma = v_0$ is also assumed in the calculation with Eqs. (19), (21a), and (21b), although the equations themselves are free from this assumption. The results with K_σ=0.9, 1.0, and 1.1 are singled out and plotted in Fig. 3. It shows that the polarization increases with increasing K_σ and with decreasing K_ρ, and the maximum polarization reaches 19%. In Fig. 4, we set ε=0.2 while keeping all the other parameters the same as those used in Fig. 2. The results with K_σ=0.9, 1.0, and 1.1 are again singled out and plotted in Fig. 5. It shows that the maximum polarization exceeds 40%. Comparing Figs. 2 and 3 with Figs. 4 and 5, we see that the spin polarization increases with the velocity

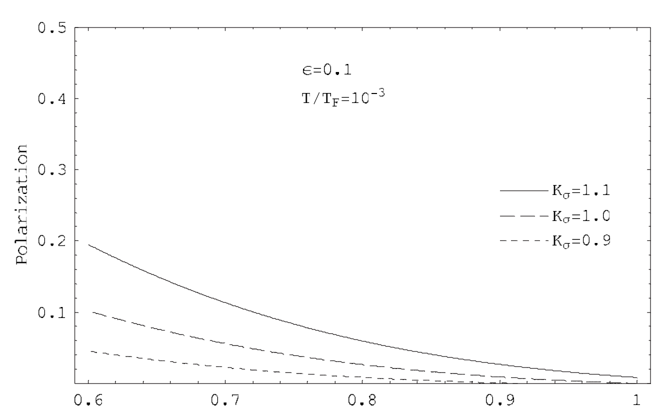

FIG. 3. Polarization vs K_ρ. The three curves plotted are taken from the result presented in Fig. 2 with K_σ=0.9, 1.0, and 1,1, respectively.

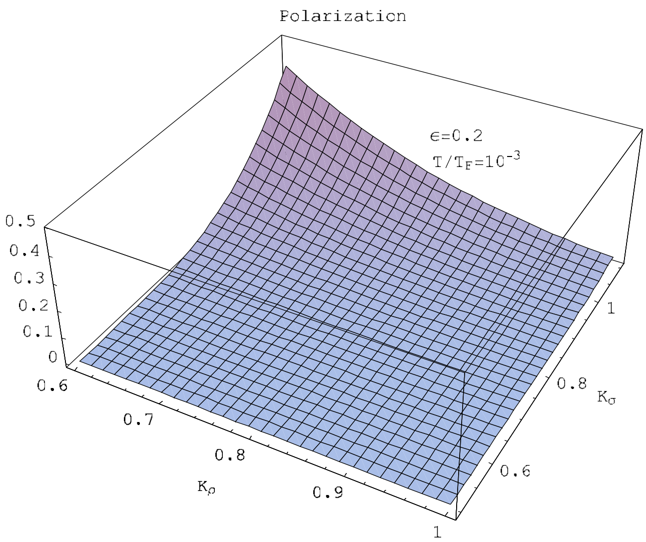

FIG. 4. (Color online) Polarization as a function of K_ρ and K_σ. We set $\varepsilon=0.2$ and keep the rest of parameters the same as those used in Fig. 2.

asymmetry ε. Equation (20) shows that the polarization depends on ε through both the scaling exponent of temperature and the prefactors of power function of temperature. In other words, a change in ε causes not only a corresponding variation in scaling dimensions but it also affects the polarization through the coefficients. In particular, when $\varepsilon=0$, $\delta G_\sigma^{(S)}=0$ and the polarization disappears, as Eq. (20) shows. Figure 6 shows the effect of temperature on polarization. Because spin-up and spin-down currents scale differently with negative powers of temperature [as shown in Eq. (12)], the polarization is enhanced when the temperature is lowered. Finally, Fig. 7 shows the variation in polarization with the impurity strength V_S and V_C. To keep the calculation within the access of perturbation theory, we take $\frac{V_S}{\Lambda a}\leq 0.3$ and $\frac{V_C}{\Lambda a}\leq 0.3$. The figure shows that the polarization grows with increasing V_S or V_C. The increase with V_S is easy to understand since the spin-flip scattering gives rise to spin polarization. On the other hand, when V_C increases, the charge conductance G_ρ decreases and, therefore, the polarization $P\approx\frac{\delta G_\sigma^{(S)}}{G_\rho}$ increases, too.

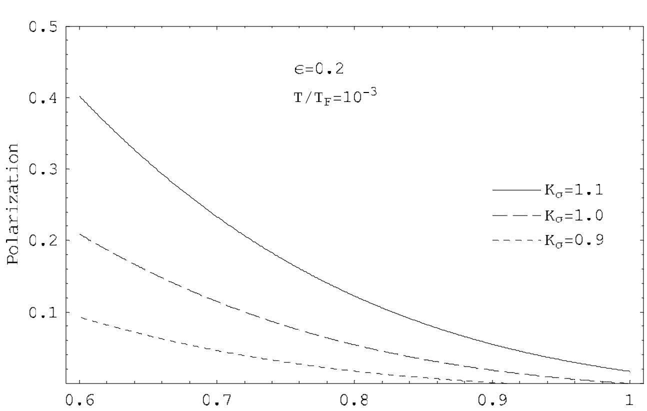

FIG. 5. Polarization vs K_ρ. The three curves plotted are taken from the result presented in Fig. 4 with $K_\sigma=0.9$, 1.0, and 1.1, respectively.

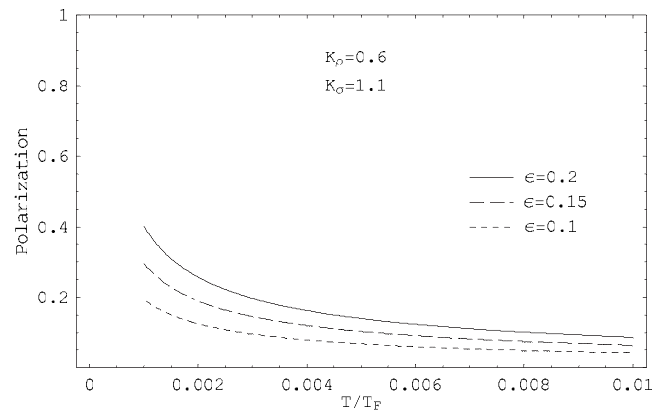

FIG. 6. Polarization as a function of T. Three curves are plotted for $\varepsilon=0.1$, 0.15, and 0.2, respectively, with T ranging from $T=10^{-3}T_F$ to $T=10^{-2}T_F$, $K_\rho=0.6$, $K_\sigma=1.1$, and $\frac{V_S}{\Lambda a}=0.3$.

V. SUMMARY AND CONCLUSION

We have presented a theoretical study of the spin-filtering effect in an interacting Q1D system, which is formed from a constrained 2DEG with the Rashba effect and a magnetic impurity implanted in it. It is shown that the magnetic impurity strength alone scales differently for spin-up and spin-down electrons, with the difference enhanced as the characteristic energy scales down. In contrast, other devices previously suggested, such as those using the Rashba effect to form asymmetric spin filters, or those using nonmagnetic impurities, all need the presence of magnetic fields to break time-reversal symmetry while this is not required in our device with a magnetic impurity. Moreover, our study shows that, with the variation in parameters $(v_\rho,K_\rho,v_\sigma,K_\sigma,\varepsilon)$ the scaling effect brings the system from the regime of nonspin-flip dominant scattering to one dominated by spin-flip scattering. Last, with the temperature lowering down to

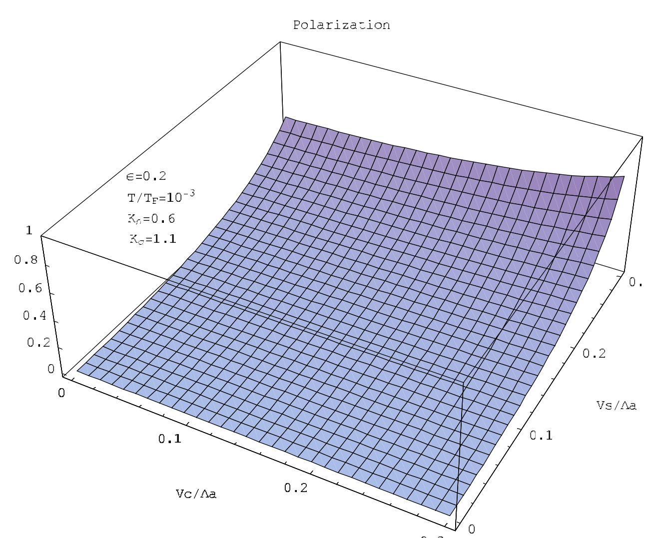

FIG. 7. (Color online) Polarization as a function of V_S and V_C, with $\frac{V_S}{\Lambda a}\leq 0.3$ and $\frac{V_S}{\Lambda a}\leq 0.3$. We set $\varepsilon=0.2$, $T=10^{-3}T_F$, $K_\rho=0.6$, and $K_\sigma=1.1$.

$T=10^{-3}T_F$ according to our conductance calculation, we expect the spin polarization to be in the range of 10% or more.

ACKNOWLEDGMENTS

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APPENDIX

In this appendix, we discuss the perturbative calculation of path integrals appearing in the conductance. We treat the impurity potential strength (including both V_c for the nonspin-flip scattering and V_s for the spin-flip scattering) as the small expansion parameter.

1. Calculation of $\langle \partial_y \theta_\sigma(y,-w)|_{y=0}\varphi_\rho(y_1,w)\rangle$

The calculation of $\langle \partial_y \theta_\sigma(y,-w)|_{y=0}\varphi_\rho(y_1,w)\rangle$ is essential for the spin conductance in Eq. (18) and the integral is expanded as follows:

$$\langle \partial_y \theta_\sigma(y,-w)|_{y=0}\varphi_\rho(y_1,w)\rangle = \int \frac{dq'}{2\pi}\frac{dq}{2\pi}e^{iqy_1}(iq')\frac{(N_0+N_1+N_2)}{(Z_0+Z_1+Z_2)} + O(V_S^3,V_C^3), \quad \text{(A1)}$$

where

$$N_0 = \int D\varphi_\rho D\theta_\sigma \theta_\sigma(q',-w)\varphi_\rho(q,w)e^{S_E},$$

$$N_1 = \int d\tau_1 \sum_{j=1}^{4} V_j n_j(\tau_1)$$

$$N_2 = \frac{1}{4}\int d\tau_1 d\tau_2 \sum_{m=\pm 1}\sum_{j=1}^{4} V_j^2 n_{jj}^{(m)}(\tau_1-\tau_2),$$

$$V_1 = V_2 = \frac{V_S}{\pi a} \quad V_3 = V_4 = \frac{V_C}{\pi a},$$

$$n_1(\tau_1) = \int D\varphi_\rho D\theta_\sigma \theta_\sigma(q',-w)\varphi_\rho(q,w)e^{i\sqrt{2}(\varphi_\rho+\theta_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$n_2(\tau_1) = \int D\varphi_\rho D\theta_\sigma \theta_\sigma(q',-w)\varphi_\rho(q,w)e^{i\sqrt{2}(\varphi_\rho-\theta_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$n_3(\tau_1) = \int D\varphi_\rho D\varphi_\sigma \theta_\sigma(q',-w)\varphi_\rho(q,w)e^{i\sqrt{2}(\varphi_\rho+\varphi_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$n_4(\tau_1) = \int D\varphi_\rho D\varphi_\sigma \theta_\sigma(q',-w)\varphi_\rho(q,w)e^{i\sqrt{2}(\varphi_\rho-\varphi_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$n_{11}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\theta_\sigma \theta_\sigma(q',-w)\varphi_\rho(q,w) \times e^{i\sqrt{2}[(\varphi_\rho+\theta_\sigma)|_{0,\tau_1}+m(\varphi_\rho+\theta_\sigma)|_{0,\tau 2}]}e^{S_E},$$

$$n_{22}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\theta_\sigma \theta_\sigma(q',-w)\varphi_\rho(q,w) \times e^{i\sqrt{2}[(\varphi_\rho-\theta_\sigma)|_{0,\tau_1}+m(\varphi_\rho-\theta_\sigma)|_{0,\tau 2}]}e^{S_E},$$

$$n_{33}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\varphi_\sigma \theta_\sigma(q',-w)\varphi_\rho(q,w) \times e^{i\sqrt{2}[(\varphi_\rho+\varphi_\sigma)|_{0,\tau_1}+m(\varphi_\rho+\varphi_\sigma)|_{0,\tau 2}]}e^{S_E},$$

$$n_{44}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\varphi_\sigma \theta_\sigma(q',-w)\varphi_\rho(q,w) \times e^{i\sqrt{2}[(\varphi_\rho-\varphi_\sigma)|_{0,\tau_1}+m(\varphi_\rho-\varphi_\sigma)|_{0,\tau 2}]}e^{S_E} \quad \text{(A2a)}$$

and

$$Z_0 = \int D\varphi_\rho D\theta_\sigma e^{S_E},$$

$$Z_1 = \int d\tau_1 \sum_{j=1}^{4} V_j z_j(\tau_1)$$

$$Z_2 = \frac{1}{4}\int d\tau_1 d\tau_2 \sum_{m=\pm 1}\sum_{j=1}^{4} V_j^2 z_{jj}^{(m)}(\tau_1-\tau_2),$$

$$z_1(\tau_1) = \int D\varphi_\rho D\theta_\sigma e^{i\sqrt{2}(\varphi_\rho+\theta_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$z_2(\tau_1) = \int D\varphi_\rho D\theta_\sigma e^{i\sqrt{2}(\varphi_\rho-\theta_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$z_3(\tau_1) = \int D\varphi_\rho D\varphi_\sigma e^{i\sqrt{2}(\varphi_\rho+\varphi_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$z_4(\tau_1) = \int D\varphi_\rho D\varphi_\sigma e^{i\sqrt{2}(\varphi_\rho-\varphi_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$z_{11}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\theta_\sigma e^{i\sqrt{2}[(\varphi_\rho+\theta_\sigma)|_{0,\tau_1}+m(\varphi_\rho+\theta_\sigma)|_{0,\tau 2}]}e^{S_E},$$

$$z_{22}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\theta_\sigma e^{i\sqrt{2}[(\varphi_\rho-\theta_\sigma)|_{0,\tau_1}+m(\varphi_\rho-\theta_\sigma)|_{0,\tau 2}]}e^{S_E},$$

$$z_{33}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\varphi_\sigma e^{i\sqrt{2}[(\varphi_\rho+\varphi_\sigma)|_{0,\tau_1}+m(\varphi_\rho+\varphi_\sigma)|_{0,\tau 2}]}e^{S_E},$$

$$z_{44}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\varphi_\sigma e^{i\sqrt{2}[(\varphi_\rho-\varphi_\sigma)|_{0,\tau_1}+m(\varphi_\rho-\varphi_\sigma)|_{0,\tau 2}]}e^{S_E}. \quad \text{(A2b)}$$

Note that the above path integrals n_j, $n_{jj}^{(m)}$, z_j, and $z_{jj}^{(m)}$ correspond to the contribution from the spin-flip scattering when $j=1$ or 2 while they correspond to the contribution from the nonspin-flip scattering when $j=3$ or 4. The action in the integrals is given as

$$S_E = \int_0^\beta d\tau \int dy l(\varphi_\rho,\theta_\sigma)|_{it\to\tau} \quad \text{(spin-flip scattering)},$$

$$S_E = \int_0^\beta d\tau \int dy l(\varphi_\rho,\varphi_\sigma)|_{it\to\tau} \quad \text{(nonspin-flip scattering)}.$$

The Lagrangian densities $l(\varphi_\rho,\theta_\sigma)$ and $l(\varphi_\rho,\varphi_\sigma)$ here are already discussed in Sec. II.

All the integrals in Eqs. (A2a) and (A2b) can be evaluated by the shifting

$$\Phi(y,\tau) \to \Phi(y,\tau) + \int dy' d\tau' g(y,\tau;y',\tau') J_{jj}^{\mp}(y',\tau'),$$

where $\Phi(y,\tau)=\binom{\varphi_\rho(y,\tau)}{\theta_\sigma(y,\tau)}$ for the spin-flip scattering and $\Phi(y,\tau)=\binom{\varphi_\rho(y,\tau)}{\varphi_\sigma(y,\tau)}$ for the nonspin-flip scattering. The results needed for the correction to the zeroth-order conductance are given below:

$$n_{jj}^{(-1)}(\tau_1-\tau_2) = z_{jj}^{(-1)}(\tau_1-\tau_2) \times\left[\frac{N_0}{Z_0} - (gJ_{jj}^-)_2(q',-w)(gJ_{jj}^-)_1(q,w)\right]$$

$$j=1-4 \tag{A3a}$$

and

$$z_{11}^{(-1)}(\tau_1-\tau_2) = Z_0\left(\frac{2\pi}{\beta\Lambda}\right)^{K_+^+ + K_-^+} \times\left\{2-2\cos\left[\frac{2\pi}{\beta}(\tau_1-\tau_2)\right]\right\}^{-(K_+^+ + K_-^+)/2},$$

$$z_{22}^{(-1)}(\tau_1-\tau_2) = Z_0\left(\frac{2\pi}{\beta\Lambda}\right)^{K_+^- + K_-^-}\left\{2-2\cos\left[\frac{2\pi}{\beta}(\tau_1-\tau_2)\right]\right\}^{-(K_+^- + K_-^-)/2},$$

$$z_{33}^{(-1)}(\tau_1-\tau_2) = z_{44}^{(-1)}(\tau_1-\tau_2) = Z_0\left(\frac{2\pi}{\beta\Lambda}\right)^{K}\left\{2-2\cos\left[\frac{2\pi}{\beta}(\tau_1-\tau_2)\right]\right\}^{-K/2},$$

$$J_{11}^- = J_{33}^- = \sqrt{2}\pi(e^{iw\tau_1}-e^{iw\tau_2})\begin{pmatrix}1\\1\end{pmatrix}$$

$$J_{22}^- = J_{44}^- = \sqrt{2}\pi(e^{iw\tau_1}-e^{iw\tau_2})\begin{pmatrix}1\\-1\end{pmatrix}. \tag{A3b}$$

In Eq. (A3a), (gJ_{jj}^-) is a two-component vector and its subscript indicates the component of vector to be taken in the expression there. In particular, g is the Green's function and when $j=1$ or 2, we have

$$g(q,w) = g_S(q,w) = \begin{pmatrix}(g_S)_{11} & (g_S)_{12}\\(g_S)_{21} & (g_S)_{22}\end{pmatrix} = \frac{\gamma_S}{D(q^2,w^2)}\begin{pmatrix}e_S w^2 + f_S q^2 & c_S w^2 - d_S q^2\\ c_S w^2 - d_S q^2 & a_S w^2 + b_S q^2\end{pmatrix} \tag{A4a}$$

and when $j=3$ or 4, we have

$$g(q,w) = g_C(q,w) = \begin{pmatrix}(g_C)_{11} & (g_C)_{12}\\(g_C)_{21} & (g_C)_{22}\end{pmatrix} = \frac{\gamma_C}{D(q^2,w^2)}\begin{pmatrix}e_C w^2 + f_C q^2 & -d_C wq\\ -d_C wq & a_C w^2 + b_C q^2\end{pmatrix}. \tag{A4b}$$

Here, $\gamma_C = v_\rho K_\rho v_\sigma K_\sigma$ and definitions of the symbols $a_S \sim f_S$ in $g_S(q,w)$, $a_C \sim f_C$ in $g_C(q,w)$, γ_C, and $D(q^2,w^2)$ appearing above are given in Eqs. (6) and (15) in Sec. II.

Then, in the order of $O(V_S^2)$, we have

$$\langle \partial_y \theta_\sigma(y,-w)|_{y=0}\varphi_\rho(y_1,w)\rangle$$

$$= -\frac{1}{Z_0}\int \frac{dq'}{2\pi}\frac{dq}{2\pi}e^{iqy_1}(iq')\frac{1}{4}\left(\frac{V_S}{\pi a}\right)^2\sum_{j=1}^{2}\int d\tau_1 d\tau_2 z_{jj}^{(-1)}(\tau_1-\tau_2) \times (g_S J_{jj}^-)_2(q',-w)(g_S J_{jj}^-)_1(q,w)$$

$$= \frac{-\pi^2}{Z_0}\beta\left(\frac{V_S}{\pi a}\right)^2\int\frac{dq'}{2\pi}(iq')[(g_S)_{21} + (g_S)_{22}]|_{(q',-w)}\int\frac{dq}{2\pi}e^{iqy_1}[(g_S)_{11}+(g_S)_{12}]|_{(q,w)}$$

$$\int d(\tau_1-\tau_2)\{1-\cos[w(\tau_1-\tau_2)]\}z_{11}^{(-1)}(\tau_1-\tau_2)$$

$$-\frac{\pi^2}{Z_0}\beta\left(\frac{V_S}{\pi a}\right)^2\int\frac{dq'}{2\pi}(iq')[(g_S)_{21}-(g_S)_{22}]|_{(q',-w)} \times\int\frac{dq}{2\pi}e^{iqy_1}[(g_S)_{11}-(g_S)_{12}]|_{(q,w)}$$

$$\int d(\tau_1-\tau_2)\{1-\cos[w(\tau_1-\tau_2)]\}z_{22}^{(-1)}(\tau_1-\tau_2). \tag{A5}$$

Moreover, substituting into Eq. (A5) the result below (obtained by contour integration in the complex q plane),

$$\int_{-\infty}^{\infty} dq e^{iqy_1}[(g_S)_{11} \pm (g_S)_{12}]|_{(q,w)}$$
$$= \frac{\pi\gamma_S}{(v_+^2 - v_-^2)|w|}\left\{\exp(-|wy_1|/v_+)\left[\frac{v_+}{\gamma_S}\left(v_\rho K_\rho \pm \frac{\delta v}{2}\right) - \frac{1}{v_+}\left(v_\sigma K_\sigma \mp \frac{\delta v}{2}\right)\right] - \exp(-|wy_1|/v_-) \times\left[\frac{v_-}{\gamma_S}\left(v_\rho K_\rho \pm \frac{\delta v}{2}\right) - \frac{1}{v_-}\left(v_\sigma K_\sigma \mp \frac{\delta v}{2}\right)\right]\right\} \quad \text{(A6a)}$$

and

$$\int_{-\infty}^{\infty} dq e^{iqy_1}[(g_S)_{21} \pm (g_S)_{22}]|_{(q,w)}$$
$$= \frac{\pi\gamma_S}{(v_+^2 - v_-^2)|w|}\left\{\exp(-|wy_1|/v_+) \times\left[\frac{v_+}{\gamma_S}\left(\pm v_\sigma K_\sigma^{-1} + \frac{\delta v}{2}\right) - \frac{1}{v_+}\left(\pm v_\rho K_\rho^{-1} - \frac{\delta v}{2}\right)\right] - \exp(-|wy_1|/v_-) \times\left[\frac{v_-}{\gamma_S}\left(\pm v_\sigma K_\sigma^{-1} + \frac{\delta v}{2}\right) - \frac{1}{v_-}\left(\pm v_\rho K_\rho^{-1} - \frac{\delta v}{2}\right)\right]\right\}, \quad \text{(A6b)}$$

we have, finally,

$$\langle \partial_y \theta_\sigma(y,-w)|_{y=0}\varphi_\rho(y_1,w)\rangle = \frac{-\sqrt{\pi}\pi^2}{2}\left(\frac{V_S}{\pi a}\right)^2\Bigg\{\left[v_\sigma K_\sigma \frac{\gamma_S}{v_+^2 v_-^2}\left(v_\rho K_\rho^{-1} - \frac{\delta v}{2}\right)\right]\frac{\gamma_S}{(v_+^2 - v_-^2)}\left\{\exp(-|wy_1|/v_+)\left[\frac{v_+}{\gamma_S}\left(v_\rho K_\rho + \frac{\delta v}{2}\right) - \frac{1}{v_+}\left(v_\sigma K_\sigma - \frac{\delta v}{2}\right)\right] - \exp(-|wy_1|/v_-)\left[\frac{v_-}{\gamma_S}\left(v_\rho K_\rho + \frac{\delta v}{2}\right) - \frac{1}{v_-}\left(v_\sigma K_\sigma - \frac{\delta v}{2}\right)\right]\right\}$$
$$\times \frac{\Gamma\left(\frac{K_+^+ + K_-^+}{2}\right)}{\Gamma\left(\frac{1 + K_+^+ + K_-^+}{2}\right)}\left(\frac{\pi}{\beta\Lambda}\right)^{K_+^+ + K_-^+ - 2} + \left[v_\sigma K_\sigma \frac{\gamma_S}{v_+^2 v_-^2}\left(-v_\rho K_\rho^{-1} - \frac{\delta v}{2}\right)\right]\frac{\gamma_S}{(v_+^2 - v_-^2)}\left\{\exp(-|wy_1|/v_+) \times\left[\frac{v_+}{\gamma_S}\left(v_\rho K_\rho - \frac{\delta v}{2}\right) - \frac{1}{v_+}\left(v_\sigma K_\sigma + \frac{\delta v}{2}\right)\right] - \exp(-|wy_1|/v_-) \times\left[\frac{v_-}{\gamma_S}\left(v_\rho K_\rho - \frac{\delta v}{2}\right) - \frac{1}{v_-}\left(v_\sigma K_\sigma + \frac{\delta v}{2}\right)\right]\right\}\frac{\Gamma\left(\frac{K_+^- + K_-^-}{2}\right)}{\Gamma\left(\frac{1 + K_+^- + K_-^-}{2}\right)}\left(\frac{\pi}{\beta\Lambda}\right)^{K_+^- + K_-^- - 2}\Bigg\}. \quad \text{(A7)}$$

2. Calculations of $\langle \varphi_\rho(y=0,-w)\varphi_\rho(y_1,w)\rangle$

The calculation of $\langle \varphi_\rho(y=0,-w)\varphi_\rho(y_1,w)\rangle$ is essential for the charge conductance in Eq. (20). The integral is expanded in terms of the impurity potential as follows:

$$\langle \varphi_\rho(y=0,-w)\varphi_\rho(y_1,w)\rangle = \int \frac{dq'}{2\pi}\frac{dq}{2\pi}e^{iqy_1}\frac{(N_0 + N_1 + N_2)}{(Z_0 + Z_1 + Z_2)} + O(V_S^3, V_C^3), \quad \text{(A8)}$$

where in the numerator,

$$N_0 = \int D\varphi_\rho D\theta_\sigma \varphi_\rho(q',-w)\varphi_\rho(q,w)e^{S_E},$$

$$N_1 = \int d\tau_1 \sum_{j=1}^{4} V_j n_j(\tau_1)$$

$$N_2 = \frac{1}{4}\int d\tau_1 d\tau_2 \sum_{m=\pm 1}\sum_{j=1}^{4} V_j^2 n_{jj}^{(m)}(\tau_1 - \tau_2),$$

$$n_1(\tau_1) = \int D\varphi_\rho D\theta_\sigma \varphi_\rho(q',-w)\varphi_\rho(q,w)e^{i\sqrt{2}(\varphi_\rho + \theta_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$n_2(\tau_1) = \int D\varphi_\rho D\theta_\sigma \varphi_\rho(q',-w)\varphi_\rho(q,w)e^{i\sqrt{2}(\varphi_\rho - \theta_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$n_3(\tau_1) = \int D\varphi_\rho D\varphi_\sigma \varphi_\rho(q',-w)\varphi_\rho(q,w)e^{i\sqrt{2}(\varphi_\rho+\varphi_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$n_4(\tau_1) = \int D\varphi_\rho D\varphi_\sigma \varphi_\rho(q',-w)\varphi_\rho(q,w)e^{i\sqrt{2}(\varphi_\rho-\varphi_\sigma)|_{0,\tau_1}}e^{S_E},$$

$$n_{11}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\theta_\sigma \varphi_\rho(q',-w)\varphi_\rho(q,w) \times e^{i\sqrt{2}[(\varphi_\rho+\theta_\sigma)|_{0,\tau_1}+m(\varphi_\rho+\theta_\sigma)|_{0,\tau 2}]}e^{S_E},$$

$$n_{22}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\theta_\sigma \varphi_\rho(q',-w)\varphi_\rho(q,w) \times e^{i\sqrt{2}[(\varphi_\rho-\theta_\sigma)|_{0,\tau_1}+m(\varphi_\rho-\theta_\sigma)|_{0,\tau 2}]}e^{S_E},$$

$$n_{33}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\varphi_\sigma \varphi_\rho(q',-w)\varphi_\rho(q,w) \times e^{i\sqrt{2}[(\varphi_\rho+\varphi_\sigma)|_{0,\tau_1}+m(\varphi_\rho+\varphi_\sigma)|_{0,\tau 2}]}e^{S_E},$$

$$n_{44}^{(m)}(\tau_1-\tau_2) = \int D\varphi_\rho D\varphi_\sigma \varphi_\rho(q',-w)\varphi_\rho(q,w) \times e^{i\sqrt{2}[(\varphi_\rho-\varphi_\sigma)|_{0,\tau_1}+m(\varphi_\rho-\varphi_\sigma)|_{0,\tau 2}]}e^{S_E}. \quad \text{(A9)}$$

Definitions of V_j, Z_0, Z_1, and Z_2 are already given in Eqs. (A2a) and (A2b).

The path integrals in Eq. (A9) can again be evaluated by shifting the variable Φ and the results needed for the conductance correction are given below:

$$n_{jj}^{(-1)}(\tau_1-\tau_2) = z_{jj}^{(-1)}(\tau_1-\tau_2)\left[\frac{N_0}{Z_0} - (gJ_{jj}^-)_1(q',-w) \times (gJ_{jj}^-)_1(q,w)\right] \quad j=1-4.$$

Here, $z_{jj}^{(-1)}(\tau_1-\tau_2)$, J_{jj}^-, and $g(q,w)$ are already given in Eqs. (A3b), (A4a), and (A4b). Again, in the last expression of $n_{jj}^{(-1)}$, when j=1 or 2, it corresponds to the contribution from spin-flip potential with $g(q,w)=g_S(q,w)$, and when j=3 or 4, it corresponds to the contribution from nonspin-flip potential with $g(q,w)=g_C(q,w)$.

With these, we write the contribution from the spin-flip part (with $j=1$ or 2) [in the order of $O(V_S^2)$],

$$\langle\varphi_\rho(y=0,-w)\varphi_\rho(y_1,w)\rangle \quad \text{(spin-flip part only)}$$
$$= \frac{-\pi^2}{Z_0}\beta\left(\frac{V_S}{\pi a}\right)^2 \int \frac{dq'}{2\pi}[(g_S)_{11}+(g_S)_{12}]|_{(q',-w)} \times \int \frac{dq}{2\pi}e^{iqy_1}[(g_S)_{11}+(g_S)_{12}]|_{(q,w)}$$
$$\int d(\tau_1-\tau_2)\{1-\cos[w(\tau_1-\tau_2)]\}z_{11}^{(-1)}(\tau_1-\tau_2)$$
$$-\frac{\pi^2}{Z_0}\beta\left(\frac{V_S}{\pi a}\right)^2 \int \frac{dq'}{2\pi}[(g_S)_{11}-(g_S)_{12}]|_{(q',-w)} \times \int \frac{dq}{2\pi}e^{iqy_1}[(g_S)_{11}-(g_S)_{12}]|_{(q,w)}$$
$$\int d(\tau_1-\tau_2)\{1-\cos[w(\tau_1-\tau_2)]\}z_{22}^{(-1)}(\tau_1-\tau_2). \quad \text{(A10)}$$

Substituting Eq. (A6a) into Eq. (A10), we obtain

$$\langle\varphi_\rho(y=0,-w)\varphi_\rho(y_1,w)\rangle \quad \text{(spin-flip part only)}$$

$$= \frac{-\sqrt{\pi}\pi}{2}\left(\frac{V_S}{\Lambda a}\right)^2 \frac{1}{|w|}\Bigg\{ \frac{1}{v_++v_-}\left[\left(v_\rho K_\rho+\frac{\delta v}{2}\right)+\frac{\gamma_S}{v_+v_-}\left(v_\sigma K_\sigma-\frac{\delta v}{2}\right)\right]\frac{1}{(v_+^2-v_-^2)}$$
$$\times\left\{\exp(-|wy_1|/v_+)\left[v_+\left(v_\rho K_\rho+\frac{\delta v}{2}\right)-\frac{\gamma_S}{v_+}\left(v_\sigma K_\sigma-\frac{\delta v}{2}\right)\right]-\exp(-|wy_1|/v_-)\left[v_-\left(v_\rho K_\rho+\frac{\delta v}{2}\right)-\frac{\gamma_S}{v_-}\left(v_\sigma K_\sigma-\frac{\delta v}{2}\right)\right]\right\}$$
$$\times\frac{\Gamma\left(\frac{K_+^++K_-^+}{2}\right)}{\Gamma\left(\frac{1+K_+^++K_-^+}{2}\right)}\left(\frac{\pi}{\beta\Lambda}\right)^{K_+^++K_-^+-2} + \frac{1}{v_++v_-}\left[\left(v_\rho K_\rho-\frac{\delta v}{2}\right)+\frac{\gamma_S}{v_+v_-}\left(v_\sigma K_\sigma+\frac{\delta v}{2}\right)\right]\frac{1}{(v_+^2-v_-^2)}\left\{\exp(-|wy_1|/v_+)\left[v_+\left(v_\rho K_\rho-\frac{\delta v}{2}\right)\right.\right.$$
$$\left.\left.-\frac{\gamma_S}{v_+}\left(v_\sigma K_\sigma+\frac{\delta v}{2}\right)\right]-\exp(-|wy_1|/v_-)\left[v_-\left(v_\rho K_\rho-\frac{\delta v}{2}\right)-\frac{\gamma_S}{v_-}\left(v_\sigma K_\sigma+\frac{\delta v}{2}\right)\right]\right\}\frac{\Gamma\left(\frac{K_+^-+K_-^-}{2}\right)}{\Gamma\left(\frac{1+K_+^-+K_-^-}{2}\right)}\left(\frac{\pi}{\beta\Lambda}\right)^{K_+^-+K_-^--2}\Bigg\}. \quad \text{(A11)}$$

Similarly, we can write the contribution from the nonspin-flip part (with $j=3$ or 4) which is of $O(V_C^2)$,

$$\langle \varphi_\rho(y=0,-w)\varphi_\rho(y_1,w)\rangle \quad \text{(nonspin part only)}$$

$$= \frac{-\pi^2}{Z_0}\beta\left(\frac{V_C}{\pi a}\right)^2 \int \frac{dq'}{2\pi}[(g_C)_{11}+(g_C)_{12}]|_{(q',-w)} \times \int \frac{dq}{2\pi}e^{iqy_1}[(g_C)_{11}+(g_C)_{12}]|_{(q,w)}$$

$$\int d(\tau_1-\tau_2)\{1-\cos[w(\tau_1-\tau_2)]\}z_{33}^{(-1)}(\tau_1-\tau_2)$$

$$-\frac{\pi^2}{Z_0}\beta\left(\frac{V_C}{\pi a}\right)^2 \int \frac{dq'}{2\pi}[(g_C)_{11}-(g_C)_{12}]|_{(q',-w)} \times \int \frac{dq}{2\pi}e^{iqy_1}[(g_C)_{11}-(g_C)_{12}]|_{(q,w)}$$

$$\int d(\tau_1-\tau_2)\{1-\cos[w(\tau_1-\tau_2)]\}z_{44}^{(-1)}(\tau_1-\tau_2). \quad \text{(A12)}$$

Using the following result (obtained by contour integration in the complex q plane):

$$\int_{-\infty}^{\infty} dq e^{iqy_1}[(g_C)_{11}\pm(g_C)_{12}]|_{(q,w)} = \frac{v_\rho v_\sigma K_\rho K_\sigma}{(v_+^2-v_-^2)}\frac{\pi}{|w|}\Bigg(\exp(-|wy_1|/v_+) \times\left\{\frac{v_+}{v_\sigma K_\sigma}-\frac{1}{v_+}\left[\frac{v_\sigma}{K_\sigma}-\frac{(\delta v)^2}{4v_\rho K_\rho}\right]\right\}-\exp(-|wy_1|/v_-) \times\left\{\frac{v_-}{v_\sigma K_\sigma}-\frac{1}{v_-}\left[\frac{v_\sigma}{K_\sigma}-\frac{(\delta v)^2}{4v_\rho K_\rho}\right]\right\}\Bigg), \quad \text{(A13)}$$

we obtain

$$\langle \varphi_\rho(y=0,-w)\varphi_\rho(y_1,w)\rangle \quad \text{(nonspin part only)}$$

$$= -\sqrt{\pi}\pi\left(\frac{V_C}{\Lambda a}\right)^2 \frac{v_\rho v_\sigma K_\rho K_\sigma}{(v_++v_-)(v_+^2-v_-^2)}\frac{\pi}{|w|}\left(v_\rho K_\rho+\frac{\gamma_S}{v_+v_-}v_\sigma K_\sigma\right) \times\Bigg(e^{-|wy_1|/v_+}\left\{\frac{v_+}{v_\sigma K_\sigma}-\frac{1}{v_+}\left[\frac{v_\sigma}{K_\sigma}-\frac{(\delta v)^2}{4v_\rho K_\rho}\right]\right\} - e^{-|wy_1|/v_-}\left\{\frac{v_-}{v_\sigma K_\sigma}-\frac{1}{v_-}\left[\frac{v_\sigma}{K_\sigma}-\frac{(\delta v)^2}{4v_\rho K_\rho}\right]\right\}\Bigg) \times\frac{\Gamma\left(\frac{K}{2}\right)}{\Gamma\left(\frac{1+K}{2}\right)}\left(\frac{\pi}{\beta\Lambda}\right)^{K-2} \quad \text{(A14)}$$

Finally, with the above results, we can calculate the conductance as follows. Substituting Eq. (A7) into Eq. (18) yields $\delta G_\sigma^{(S)}$ in Eq. (19). Substituting Eq. (A11) into Eq. (20) yields $\delta G_\rho^{(S)}$ in Eq. (21b). Substituting Eq. (A14) into Eq. (20) yields $\delta G_\rho^{(C)}$ in Eq. (21a).

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