

Shot noise reduction of space charge limited electron injection through a Schottky contact for a GaN diode

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We present a one-dimensional model of shot noise reduction of space charge limited electron injection through a Schottky contact for a GaN diode assuming the length of the diode is much larger than the electron inelastic scattering mean-free path. The shot noise reduction due to both Coulomb repulsion and quantum partitioning is determined consistently, where the former is due to the space charge electrostatic field created by the injected electrons, and the latter is due to the electron tunneling through the self-consistent potential profile near to the contact. The shot noise reduction is calculated in the form of Fano factor over a wide range of applied voltage for various values of Schottky barrier height (0–0.5 eV), temperature (100–500 K), and length of the diode (0.1–10 μm). At high voltage, and high current regime, the shot noise suppression increases with large applied voltage, small diode length, low temperature, and small barrier height. Our model also indicates that the distributed traps in the solid almost has no effect to the shot noise reduction as compared to trap-free solid.

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I. INTRODUCTION

Shot noise is the fluctuation in the electrical signal due to the discreteness of electron charges. It was first studied by Schottky for the electron thermionic emission in a vacuum tube,¹ and the spectral power density of the shot noise is $S = 2qI$, where I is the mean value of the transmitted current and q is the electron charge. This equation is usually known as the uncorrelated (or full) shot noise if the electron emission is described by a random Poisson process. The deviation of the uncorrelated shot noise is normally given by the Fano factor $\gamma = S/2qI$, where $\gamma < 1$ indicates the suppression of shot noise due to the correlations among the electrons such as the Coulomb correlation, and the effects of quantum partitioning due to Pauli exclusion principle.^{2,3} The study of quantum shot noise is also important to understand the particle-wave duality of electrons in many mesoscopic systems.⁴

Recently, many novel devices are found to operate in space charge limited (SCL) regime, such as GaN nanorod,⁵ organic device,^{6,7} polymer transistor,⁸ nanowire,⁹ magnetoresistance,^{10,11} and nanocrystallites embedded silicon Schottky junction.¹² For a trap-free solid, SCL conduction is also known as Mott-Gurney (MG) law,^{13,14} which is

$$J_{\text{MG}} = \frac{9}{8} \mu_n \epsilon_r \epsilon_0 \frac{V_G^2}{L^3}. \quad (1)$$

Here, μ_n is the electron mobility, ϵ_r is the relative (dielectric) permittivity, ϵ_0 is vacuum permittivity, V_G is the applied voltage, and L is the length of the solid. For a solid with exponentially distributed traps, the scaling becomes $J \sim V_G^{l+1}/L^{2l+1}$, where l is the ratio of the distribution of the traps to free carriers. The equivalent of MG law for a vacuum gap is known as Child-Langmuir (CL) law.^{15,16} Both MG and CL law have been revised intensively recently including analytical multidimensional models, contact properties, quantum tunneling and ultrafast time scale.^{17–24}

It was first shown in the earlier works that shot noise can be suppressed by Coulomb repulsion of CL law in vacuum tube.²⁵ Similar suppression of shot noise was also later formulated for the MG law in solids.²⁶ More recently, the focus of shot noise reduction at SCL conduction was extended to diffusive conductors^{27–29} and ballistic conductors.^{30–32} An universal of 1/3 shot noise suppression was found in the SCL conduction in the three-dimensional system of nondegenerate diffusive conductor by including only the elastic scattering^{27,28} and the model was later revised to include the diffusive current and other types of scattering.²⁹

On the other hand, in the absence of scattering, shot noise is pretty much determined by the injecting contact determined by the potential barrier at the interface. In this case, the correlation comes from the quantum partitioning effect between the incoming charge and the transmitted charge through the potential barrier and its effect on shot noise reduction has been shown in many mesoscopic systems.³ Such effects of quantum partitioning on the shot noise reduction have also been suggested for electron field emission at metal-vacuum interface with a simplified potential profile,³³ and it has been extended to include more realistic potential profiles,^{34,35} and space charge effects in a quantum model.³⁶

In this paper, we are interested to calculate the shot noise reduction of SCL charge injection through a metal-semiconductor interface (or Schottky contact) of a diode, including the correlations due to the space charge effects of MG law and quantum partitioning of the electron tunneling through the Schottky barrier. Such a system is relevant to SCL charge injection required in various nanoelectronics or organic-based devices, which always have a non-Ohmic contact at the interface.

Thus in this context, several interesting questions arise: (a) is the correlation due to quantum partitioning important in high voltage regime where space charge effect is dominant due to the classical Coulomb correlation? (b) At the same applied electric field, is the quantum shot noise importance for different voltages and diode spacing? (c) What is the

dependence of shot noise on temperature, applied voltage, length of the diode, and Schottky barrier height? (d) How is the presence of traps in varying the shot noise suppression?

Before showing the exact calculations in answering the above questions (see figures below), we would like to present some qualitative picture here. First, at low applied field where the electron transport is based on Ohm's law, we expect no correlations, so zero shot noise suppression ($\gamma=1$). At high field where the space charge effects become important, the classical Coulomb correlation will provide a smoothing effect that suppress the shot noise. In this regime, the combined effects of Schottky barrier and space charge electrostatic field will serve as a potential barrier for electrons to be injected from metal into the solid through tunneling process, which provides the quantum partitioning correlation that will suppress the full shot noise ($\gamma<1$). The degree of suppression will depend on temperature, applied voltage, length of the diode, and Schottky barrier height. With larger Schottky barrier, the tunneling probability is lower and thus less shot noise suppression as compared to zero-Schottky barrier case (see Fig. 2). At fixed applied field with smaller diode length scale, we will have more shot noise suppression due to the relative significance of electron tunneling at the interface (see Fig. 3). At fixed diode length, shot noise suppression increases with higher voltages due to the relative importance of Coulomb correlation due to space charge effects (see Fig. 4).

II. MODEL

For simplicity, we first consider a trap-free solid (or dielectric) of length L (extended from $x=0$ to L) is sandwiched between two metallic electrodes with a biased voltage V_G at $x=L$. Our model is a one-dimensional (1D) model that has only spatial dependence on x , where the contact at $x=0$ and $x=L$ is, respectively, a Schottky contact with Schottky barrier height of ϕ_b , and an Ohmic contact. In the model, we only focus on the injection of electrons through the Schottky contact, and the hole injection is ignored completely. After tunneling through the contact, the transport of electrons is described by drift-diffusion equation on the assumption than L is much larger than the inelastic scattering mean-free path λ , which is determined by the mobility of the electrons.

Following the Landauer-Büttiker framework, the Fano factor of the shot noise reduction of SCL charge injection through the Schottky barrier may be expressed as³³⁻³⁵

$$\gamma = 1 - \frac{\int_{-\infty}^{\infty} C_T^2(E_x) g[-\beta E_x] dE_x}{\int_{-\infty}^{\infty} C_T(E_x) h[-\beta E_x] dE_x}, \quad (2)$$

where $\beta=1/kT$, k is Boltzmann's constant, T is the temperature, E_x is the conduction-band energy in the solid with respect to Fermi level E_F , C_T is the transmission coefficient through the energy barrier, $h(z)$ is defined as $h(z)=\ln(1+e^z)$, and $g(z)$ is defined as $g(z)=h(z)-dh(z)/dz$. Note both $g(z)$ and $h(z)$ are strictly positive functions. The transmission co-

efficient $C_T(E_x)$ is calculated by using the WKB approximation,³⁷

$$C_T(E_x) = \exp \left[-\frac{2}{\hbar} \int_{x_1}^{x_2} \sqrt{2m^*(\psi_C(x)q - E_x)} dx \right], \quad (3)$$

where \hbar is reduced Plank's constant, m^* is the effective electron mass, ψ_C is the conduction-band potential (to be calculated), q is electronic charge, x_1 and x_2 are roots of $\psi_C(x)q - E_x=0$.

To obtain the potential profile ψ_C , we solve the Poisson equation

$$\frac{d^2\psi_C}{dx^2} = \frac{q(n - n_i)}{\epsilon_r \epsilon_0}, \quad (4)$$

where n_i is the intrinsic electron concentration. The electron density (n) at the conduction band related to the conduction-band potential (ψ_C) is given by

$$n = N_C \exp \left[\frac{\psi_C - \phi_n}{kT/q} \right], \quad (5)$$

where N_C is the effective density of states of electrons at the conduction band and ϕ_n is quasi-Fermi electron potential. The ϕ_n term is solved by using the steady-state current continuity equation,

$$\frac{dj_n}{dx} = qG_n \quad (6)$$

and

$$j_n = -qn\mu_n \frac{d\psi_C}{dx} + qD_n \frac{dn}{dx} = -qn\mu_n \frac{d\phi_n}{dx}. \quad (7)$$

Here, j_n is the drift-diffusion electron current density, D_n is the electron diffusion coefficient, and μ_n is the electron mobility, G_n is the net generation rate of electrons due to the external charge injection at the contact through the process of electron tunneling and thermionic emission (or over barrier injection).

In our model, the Schottky contact (at $x=0$) is characterized by a barrier height of $q\phi_b = q\phi_m - q\chi$ (in electron volt), and the surface recombination current $j_{sn} = qv_{sn}(n_s - n_{eq})$, where ϕ_m is the work function of electrode, χ is the electron affinity of the solid, $v_{sn} = A^*T^2/(qN_C)$ is the surface recombination velocity, A^* is the effective Richardson constant, n_s is the surface electron density at $x=0$, and n_{eq} is the equilibrium electron density at $\phi_n=0$.^{38,39}

The boundary conditions for solving Eqs. (5)–(7) numerically are $\psi_C(x=0) = -\phi_b$, $\psi_C(x=L) = V_G - E_G/2q$, $\phi_n(x=0) = \psi_C(x=0) + (E_G/2q) + (kT/2q)\ln(N_C/N_V) - (kT/q)\ln(n_s/n_i)$, and $\phi_n(x=L) = V_G$. Here, E_G is the band gap, N_V is the effective density of states of hole in the valence band, and V_G is the applied voltage. The details of the numerical calculation can be found in our recent paper¹⁸ where we have studied the MG law with finite Schottky contact. To include the effect of electron temperature T , we take into account of N_C and E_G as a function of T (Ref. 38) with $N_C(T) = N_C^{300} \left[\frac{T}{300} \right]^{1.5}$ and

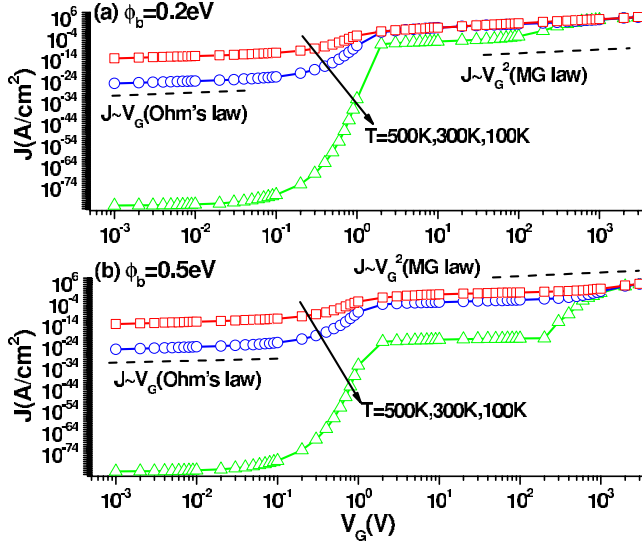


FIG. 1. (Color online) The calculated electron current density J as a function of V_G in log-log scale for a GaN diode of $L = 10 \mu\text{m}$ and $\mu_n = 900 \text{ cm}^2/\text{Vs}$ at (a) $q\phi_b = 0.2 \text{ eV}$ and (b) $q\phi_b = 0.5 \text{ eV}$ for different $T = 500, 300$, and 100 K as indicated by the arrow from top to bottom and are shown in different color lines, respectively, by red (square), blue (circle), and green (triangle).

$E_G(T) = E_G^{300} + 4.73 \times 10^{-4} \left[\frac{300^2}{300+636} - \frac{T^2}{T+636} \right]$, where N_C^{300} and E_G^{300} are the effective density of states in the conduction band and band gap at 300 K , respectively.

In this paper, we will use gallium nitride (GaN) for our study since it has a relative higher breakdown electric field, and it was reported to be operated in the SCL current regime.⁵ The parameters used in the calculation are⁴⁰ $L = 10, 1$, and $0.1 \mu\text{m}$, $\epsilon_r = 9$, $\mu_n = 900 \text{ cm}^2/(\text{Vs})$, $\phi_b = 0 - 0.5 \text{ eV}$, and $T = 100, 300$, and 500 K . At this setting, the electron mean-free path for the inelastic scattering is about 24 nm for GaN, which is much smaller than the length of the solid $L \geq 100 \text{ nm}$. Note that maximum applied electric field used in the simulation is less than the breakdown field of GaN which is about 3.3 MV/cm .

It is important to note that the derivation of Eq. (2) is based on the assumption of using a fixed effective mass in the integration of the transverse momentum. In spite of this assumption, our model is however benchmarked by comparing our calculated current-voltage (IV) characteristic with an industry-standard TCAD tool⁴¹ as shown in Fig. 1. The comparison show good agreement which implies that the calculation of both transmission coefficient and Fano factor is reliable on the basis of device simulation. The effect of varying the effective mass during the injection process through the interface is beyond the scope of this paper.

III. RESULTS

In Fig. 1, the current density J as a function of V_G for GaN is plotted in a log-log plot at (a) $\phi_b = 0.2 \text{ eV}$ and (b) 0.5 eV for $T = 100, 300$, and 500 K , and $L = 10 \mu\text{m}$. At low V_G , the transport obeys Ohm's law ($J \sim V_G$) since the injected electron density is much lower than the intrinsic electron

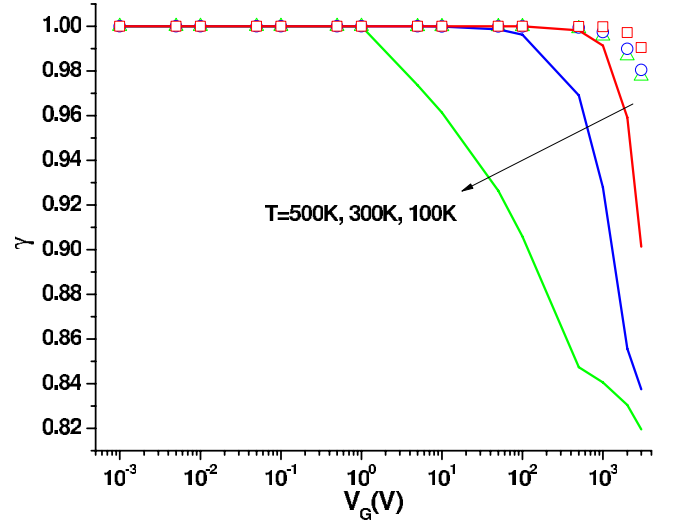


FIG. 2. (Color online) The calculated Fano factor γ as a function of V_G for GaN in linear-log scale at $L = 10 \mu\text{m}$ and $\mu_n = 900 \text{ cm}^2/\text{Vs}$ for $q\phi_b = 0$ (in solid lines) and 0.2 eV (in symbols) for different $T = 500, 300$, and 100 K as indicated by the arrow from top to bottom. The respective labeling are $(T, \text{color line, symbols}) = (500 \text{ K, red line, square}), (300 \text{ K, blue line, circle})$ and $(100 \text{ K, green line, triangle})$

concentration n_i of GaN. In this regime, electron tunneling is negligible at low V_G and J will only depend on the amount of available electrons injected at the surface $[n(x=0)]$ described in Eq. (5). The transition from Ohm's law to MG law is about 2 V . Above the transition voltage, the current density will become the SCL conduction following MG law ($J \sim V_G^2$). For a given value of ϕ_b , SCL conduction can only be reached if the electron tunneling is included self-consistently.¹⁸ Please note the results shown in Fig. 1 have been verified (in symbols) by using a two-dimensional device simulator,⁴¹ for which we have assigned the width of the system to be $W = 40 \mu\text{m} (\gg L)$, so that our 1D model is valid.

In Fig. 2, we compare the shot noise reduction or Fano factor γ [using Eq. (2)] at zero barrier $\phi_b = 0$ (solid lines) and $\phi_b = 0.2 \text{ eV}$ (symbols) as a function of V_G for $T = 100, 300$, and 500 K , and $L = 10 \mu\text{m}$. At low V_G (below the transition voltage of 2 V), the γ values are ≈ 1 independent of ϕ_b . At this low current regime, the current density follows Ohm's Law and the electron tunneling is negligible. Thus, the Fano factor $\gamma \approx 1$ as there are no correlation effects due to coulomb repulsion and quantum partitioning effects to reduce the shot noise.

At high voltage regimes, the tunneling probability is lower for $\phi_b = 0.2 \text{ eV}$, and thus less shot noise suppression as compared to zero-Schottky barrier case ($\phi_b = 0$). At this high current regime, the tunneling of electrons through the potential barrier resulted from the Schottky barrier and electrostatic space charge field will provide the correlations due to both Coulomb repulsion and quantum partitioning, which will reduce the Fano factor γ to be less than 1. The Coulomb repulsion has the effect of reducing γ slightly at lower T and the tunneling will further suppress shot noise at large V_G . In terms of temperature, the electron tunneling dominates the

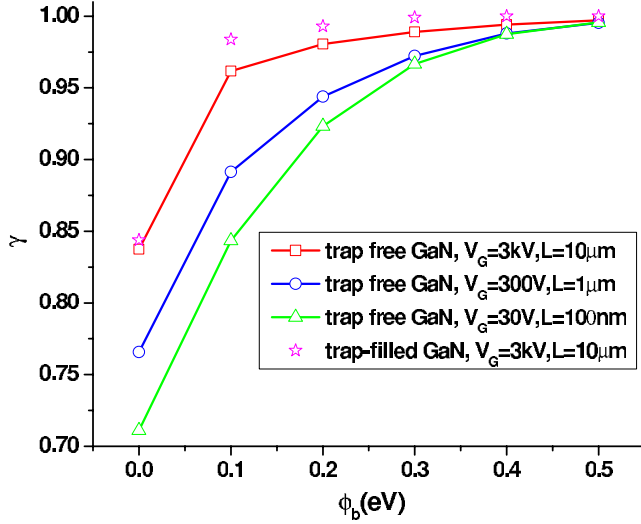


FIG. 3. (Color online) The calculated Fano factor γ as a function of $q\phi_b=0-0.5$ eV for GaN at $T=300$ K and fixed applied field $V_G/L=3$ MV/cm for different combinations of $V_G=3, 0.3$, and 0.03 kV, and $L=10, 1$, and 0.1 μm . The magenta star symbols are the calculations at trap-limited SCL conduction at $L=10$ μm and $V_G=3$ kV case.

current conduction at low temperature, and thus more shot noise suppression at low temperature. For example, at $V_G=3$ kV (or 3 MV/cm), and $q\phi_b=0$ eV, we have $\gamma=0.819, 0.834$, and 0.901 , respectively, at $T=100, 300$, and 500 K. At $q\phi_b=0.2$ eV, we have $\gamma=0.991-0.999$ in the range of $T=100-500$ K at same 3 MV/cm.

It is clear the probability of tunneling does not depend only on the applied field but also on the length of the GaN diode L . In Fig. 3, we show the dependence of shot noise reduction as a function of barrier height ($0-0.5$ eV) at room temperature $T=300$ K and fixed applied electric field of 3 MV/cm with different combination of $V_G=(3, 0.3, 0.03)$ kV and $L=(10, 1, 0.1)$ μm . For smaller L , we have more shot noise suppression (smaller γ) as the electron tunneling is more likely. For example, γ is decreased to about 0.71 and 0.93, respectively, at $\phi_b=0$ and 0.2 eV for the $L=100$ nm case as compared to $L=10$ μm case shown in Fig. 2. Thus, the tunneling is the dominant effect for shot noise suppression as the length of device material gets smaller. The relatively insignificance of the correlation due to Coulomb repulsion can be seen from the MG law, which varies as L^{-1} for fixed V_G/L .

In addition, we also study the effect of traps on the shot noise reduction for a trap-filled GaN at the SCL conduction regime. In doing so, we assume the traps are exponentially distributed in energy by equation: $n_t=N_t \exp[(\psi_C - \phi_n)/(kT/q)]$, where N_t is the trap density and $l=T_t/T$ with T_t is the characteristic distribution of the traps. At $l=1$ ($T_t=T$), the traps are independent of energy and it is equivalent to the trap-free case but with a reduced current density J (by a numerical constant as compared to the trap-free MG law). By using the device simulator,⁴¹ we extract the conduction-band potential profile $\psi_C(x)$ from the simulation

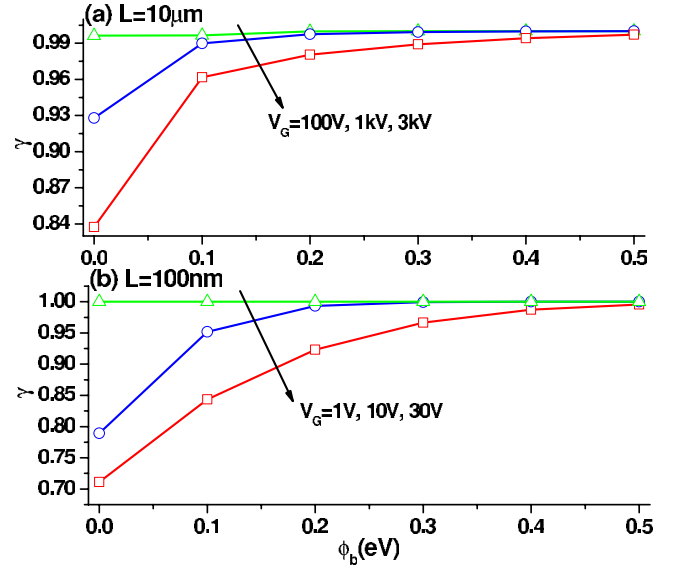


FIG. 4. (Color online) The calculated Fano factor γ as a function of $q\phi_b$ for GaN at $T=300$ K and fixed $L=(a)10$ μm for $V_G=100, 1$, and 3 kV, and (b) 100 nm for $V_G=1, 10$, and 30 V.

including the effects of traps, and calculate numerically the Fano factor using Eq. (2). The calculated results using the parameters of ($l=2, N_t=5 \times 10^{17}$ cm^{-3} , $V_G=3$ kV, and $L=10$ μm) is plotted (see symbols) in Fig. 3 for comparison. Our calculations show that shot noise reduction has little effect (within 2%) in the trap-limited SCL conduction as compared to trap-free case (red solid line).

For completeness, the dependence of shot noise suppression ($T=300$ K) is shown Fig. 4 as a function of barrier height ($0-0.5$ eV) at (a) $L=10$ μm and (b) $L=100$ nm for various V_G . The results prove that shot noise suppression increases with higher voltages at fixed L , due to the relative importance of Coulomb correlation due to increment of space charge effects at high voltages.

IV. SUMMARY

In conclusion, we have calculated shot noise reduction (or Fano factor) of SCL electron injection through a Schottky contact in a GaN diode, including the correlation effects of Coulomb repulsion and quantum partitioning due to electron tunneling through the self-consistent determined potential barrier near to the Schottky contact. The effects of the shot noise reduction on temperature, Schottky barrier height, applied voltage, length of the diode are investigated. It is found that shot noise suppression cannot be ignored for smaller barrier height (<0.2 eV) at the interface at high voltage regimes especially for smaller diode length scale. At low voltage and low current regime, the electron transport obeys Ohm's law and there is no suppression of shot noise. At high voltage, and high current regime, the electron transport is space charge limited by the MG law and the shot noise suppression increases with large applied voltage, small diode length, low temperature, and small barrier height. Our model also indicates that the distributed traps in the solid nearly has

no effect to the shot noise reduction as compared to trap-free solid. Finally, we have also varied the value of mobility, and it is found that the findings of the shot noise suppression reported here remain valid as long as the electron transport is at space charge limited conduction regime.

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