

# Spin generation in a Rashba-type diffusive electron system by nonuniform driving field

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We show that the Rashba spin-orbit interaction contributes to edge spin accumulation  $S_z$  in a diffusive regime when the driving field is nonuniform. Specifically, we solve the case of nonuniform driving field in the vicinity of a circular void locating in a two-dimensional electron system and we identify the key physical process leading to the edge spin accumulation. The void has radius  $R_0$  in the range of spin-relaxation length  $l_{so}$  and is far from both source and drain electrodes. The key physical process we find is originated from the nonuniform in-plane spin polarizations. Their subsequent diffusive contribution to spin current provides the impetus for the edge spin accumulation  $S_z$  at the void boundary. The edge spin accumulation is proportional to the Rashba coupling constant  $\alpha$  and is in a spin-dipole form oriented transversely to the driving field. We expect similar spin accumulation to occur if the void is at the sample edge.

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## I. INTRODUCTION

A major goal for the semiconductor spintronics is to generate and to manipulate spin polarization by mere electrical means. Spin-orbit interaction (SOI) provides the key leverage and spin-Hall effect (SHE) (Refs. 1–15) provides the key paradigm, where it is possible for a uniform driving electric field to induce bulk spin polarization and spin current and, in turn, out-of-plane spin accumulations  $S_z$  at lateral edges. The Rashba SOI (RSOI) (Ref. 16) is of particular interest because of its gate-tuning capability. However, background scatterers lead to a complete quenching of the RSOI's contribution to the edge spin accumulation  $S_z$ , a direct consequence of its linear dependence on the electron momentum  $k$ .<sup>17,18</sup> It is legitimate then to find ways to restore the RSOI's contribution to the edge spin accumulation. Our interest here is in the diffusive regime, when the spin-relaxation length  $l_{so} \gg l_e$ , the mean-free path. Even though the spin accumulation is finite in the mesoscopic ballistic regime ( $l_{so} < l_e$ , and  $L < l_\phi$ ),<sup>19–21</sup> with  $L$  the sample size and  $l_\phi$  the phase coherent length, it is still important to see whether the RSOI alone can contribute to SHE in the impurity-dominated regime.

Indeed, RSOI was found by Mishchenko *et al.*<sup>18</sup> to give rise to edge spin accumulation  $S_z$  near electrodes even though its contribution to bulk spin current vanishes. The edge spin accumulation is concentrated at the two ends of an electrode-sample interface, covering a region of size  $l_{so}$ . This finding was identified to arise from a nonzero spin current  $I_y^z$  flowing along the sample-electrode interface, in direction  $\hat{y}$ .<sup>18</sup> This nonzero spin current was understood from the way the spin current vanishes in the bulk, when an exact cancellation occurs between two terms, one related to the spin polarization and the other related to the driving field.<sup>18</sup> This exact cancellation no longer holds at the sample-electrode interface, when the driving field has reached its bulk value but the spin polarization has not. Similar result was also obtained by

Raimondi *et al.*,<sup>22</sup> where spin-density spatial profiles at the sample corners were obtained. Yet, it would be more desirable that we can find schemes and identify physical processes for the restoring of the RSOI-induced edge spin accumulation at locations other than the sample-electrode interfaces and according to our specification.

In this work, we turn to nonuniform driving field for the restoration of the RSOI-induced spin accumulation. The effect of nonuniform driving field on spin accumulation is also interesting in its own right. Earlier study considered nonuniform driving field in systems in the presence of “extrinsic” SOI, that is, SOI due to SOI impurities.<sup>23</sup> Here, instead, we consider nonuniform driving field in the vicinity of a circular void located in a diffusive RSOI-type two-dimensional electron gas (2DEG). We obtain spin accumulation in the vicinity of the void. This problem allows us to identify the key physical process for the spin accumulation and also sheds light on the case if the void were to form at a lateral edge. The radius  $R_0$  of the void is of the order of  $l_{so}$ .

Most important is our finding that the main physical process is in marked contrast to the conventional one. While the conventional one is associated with the nonvanishing of the out-of-plane spin current  $I_n^z$ ,<sup>18</sup> the key process we find is associated with the in-plane spin currents,  $I_n^x$  or  $I_n^y$ , and with the way they vanishes at the void boundary. Here,  $\hat{n}$  denotes the flow direction normal to the void boundary. The in-plane spin currents consist of two terms, a diffusive term and a term related to the spin accumulation  $S_z$ , which are given by

$$I_\rho^i = -2D \frac{\partial}{\partial \rho} S_i - R^{izi} S_z \hat{\rho} \cdot \hat{i}, \quad (1)$$

where  $\rho$  is the position vector measured from the center of the void,  $i \in \{x, y\}$ , and  $D$  is the diffusion constant.<sup>24</sup>  $R^{ijk}$  denotes the precession of  $S_j$  into  $S_i$  when it flows along  $\hat{k}$ . The factor  $\hat{\rho} \cdot \hat{i}$  denotes the projection of the flow. The RSOI governs the symmetry of  $R^{ijk}$  such that  $R^{izj} = 0$  for  $i \neq j$ .

Our scheme of  $S_z$  generation is made possible by Eq. (1), the boundary condition  $I_\rho^i=0$ , at  $\rho=R_0$ , and the presence of a radially nonuniform in-plane spin polarization  $S_i$ . The spin polarization  $S_\parallel=S_\parallel^{Ed}+\Delta S_\parallel$  we obtain in this work has

$$S_\parallel^{Ed} = -N_0\alpha\tau e/\hbar\hat{z} \times \mathbf{E}(\boldsymbol{\rho}), \quad (2)$$

which radial dependence is acquired from the driving field  $\mathbf{E}$ . Outside the void,  $\mathbf{E}(\boldsymbol{\rho})=E_0\hat{x}-E_0(R_0/\rho)^2(\cos 2\phi\hat{x}+\sin 2\phi\hat{y})$  where  $E_0\hat{x}$  is the uniform field far away from the void and  $\boldsymbol{\rho}\cdot\hat{x}=\cos\phi$ . The driving electric field  $\mathbf{E}=-\nabla\varphi(\boldsymbol{\rho})=\sigma_0\mathbf{j}$  has to satisfy the steady state condition  $\nabla\cdot\mathbf{j}=0$  and the boundary condition  $j_\rho=0$ . Here  $\sigma_0$  is the electric conductivity,  $\mathbf{j}$  is the electric current density,  $N_0$  is the energy density per spin,  $\alpha$  is the RSOI coupling constant,  $e>0$ , and  $\tau$  is the mean-free time. The term  $S_\parallel^{Ed}$  is an Edelstein-like spin polarization<sup>3</sup> which we have obtained for the case of nonuniform driving fields. That this term of  $S_\parallel$  alone fails to satisfy the boundary condition Eq. (1), because of its radial dependence, has prompted the generation of  $\Delta S_\parallel$  and  $S_z$ .

The aforementioned key physical process associated with the spin current is meant to establish a boundary condition for the spin-diffusion equation. We have used the conventional form of the spin current operator  $J_i^j=(1/4)(V_i\sigma_j+\sigma_jV_i)$ , where spin unit of  $\hbar$  is implied, and the kinetic velocity operator  $V_i=(1/i\hbar)[\hat{x}_i,H]$ . This is appropriate for hard wall boundary.<sup>24-26</sup> As the boundary condition is applied to a region much shorter in distance than  $l_{so}$  from the boundary, the effect of spin torque<sup>27,28</sup> here should be of secondary importance. In Sec. II we present the spin-diffusion equation for nonuniform driving fields, and the analytical solutions for spin densities around a circular void. In Sec. III we present our numerical results and discussion. Finally, in Sec. IV, we will present our conclusion.

## II. THEORY

The derivation of the spin-diffusion equation (SDE) by the Keldysh nonequilibrium Green's function method<sup>13,24</sup> is extended to the case when the driving field is nonuniform. With the RSOI Hamiltonian  $\mathbf{H}_{so}=\mathbf{h}_k\cdot\boldsymbol{\sigma}$  and  $\mathbf{h}_k=-\alpha\hat{z}\times\mathbf{k}$ , where  $\boldsymbol{\sigma}$ , and  $\mathbf{h}_k$  are, respectively, the Pauli's matrix vector and the SOI-effective magnetic field, the SDE is given by

$$\begin{aligned} D\nabla^2 S_x - \frac{\Gamma^{xx}}{\hbar^2} S_x + \frac{R^{vzx}}{\hbar} \frac{\partial}{\partial x} S_z - \frac{M^{x0}\cdot\nabla}{2\hbar^3} D_0^0 &= 0, \\ D\nabla^2 S_y - \frac{\Gamma^{yy}}{\hbar^2} S_y + \frac{R^{vzy}}{\hbar} \frac{\partial}{\partial y} S_z - \frac{M^{y0}\cdot\nabla}{2\hbar^3} D_0^0 &= 0, \\ D\nabla^2 S_z - \frac{\Gamma^{zz}}{\hbar^2} S_z + \frac{R^{zxx}}{\hbar} \frac{\partial}{\partial x} S_x + \frac{R^{zyy}}{\hbar} \frac{\partial}{\partial y} S_y &= 0, \end{aligned} \quad (3)$$

where the spin density  $S_i$  is in units of  $\hbar$ , and  $D=v_F^2\tau/2$ .

Even though the form of the SDE in Eq. (3) is essentially the same as that for the uniform driving field,<sup>24</sup> the spin-charge coupling term, through  $\nabla D_0^0$ , becomes position dependent. To get at this Eq. (3), we have performed a systematic scrutiny on possible additional terms in it that are up to appropriate orders, as will be detailed in the following. The spin-charge coupling terms, given by  $-M^{i0}\cdot\nabla D_0^0$ , have  $M^{i0} = 4\tau^2 h_k^3 \frac{\partial n_k^i}{\partial k} = -2\tau^2 h_F^2 \alpha (\hat{i} \times \hat{z})$  where  $D_0^0 = 2N_0 e \varphi(\boldsymbol{\rho})$  is the effective local equilibrium density. The overline denotes angular average over the Fermi surface,  $\varphi(\boldsymbol{\rho}) = -E_0(\rho + R_0^2/\rho) \cos\phi$  for  $\rho \geq R_0$  and  $\mathbf{n}_k = \mathbf{h}_k/h_k$ . The Edelstein-like spin polarization  $S_\parallel^{Ed}$  [Eq. (2)] is solved directly from Eq. (3).

The D'yakonov-Perel' (DP) spin-relaxation rates, given by  $\Gamma^{il} = 4\pi\hbar_k^2 (\delta^{il} - n_k^i n_k^l)$ ,<sup>30</sup> have  $\Gamma^{xx} = \Gamma^{yy} = \Gamma^{zz} / 2 = 2h_F^2 \tau$  for RSOI. Spin precession arising from diffusive flow is characterized by  $R^{ilm} = 4\tau \sum_n \epsilon^{ilm} h_k^n v_k^m$ , where  $\epsilon^{ilm}$  is the Levi-Civita symbol, and we have  $R^{zii} = -R^{izi} = -2h_F v_F \tau$  for RSOI and for  $i=(x,y)$ .<sup>24</sup> Since  $k_F l_e \gg 1$ , with  $l_e$  the mean-free path, the charge neutrality is maintained by the condition of zero charge density throughout due to screening effect. Within the linear response to the driving electric field, the effect of the screening potential on the spin accumulation can be neglected.

A brief note on the systematic scrutiny of the possible additional terms in Eq. (3) is in order here. The spin-charge coupling term in Eq. (3) is resulted from  $\Psi^{i0} D_0^0$ ,<sup>29</sup> which lowest order in RSOI and first order in spatial gradient is given by the expansion of  $\Psi^{i0}$  to the order  $h_F^3 q$ . This is appropriate for uniform driving field because  $\nabla\varphi$  would become position independent. We take caution here, for the case of nonuniform driving fields, to check for additional terms of higher order in  $q$  that could have arisen from  $\Psi^{i0} D_0^0$ . Here,<sup>29</sup>

$$\Psi^{il} = \frac{\Gamma}{2\pi N_0} \sum_{\mathbf{p}'} \text{Tr}[\tau^j G^{r(0)}(\mathbf{p}', \omega + \omega') \tau^l G^{a(0)}(\mathbf{p}' - \mathbf{q}, \omega')], \quad (4)$$

where  $\Gamma = 1/2\tau$ ,  $G^{r/a(0)}$  are retarded (advanced) Green's functions averaged over impurity configuration,  $\tau^{i=0} = 1$ , and  $\tau^{j=x,y,z} = \sigma_{x,y,z}$ . To identify additional expansion terms in  $\Psi^{i0}$  for nonuniform driving fields, we note first of all that  $S_\parallel^{Ed}$  is of order  $h_F q \varphi$ . If  $S_\parallel^{Ed}$  is to satisfy the SDE, all the terms in Eq. (3) involving  $S_i$  will have to be replaced by  $S_i - S_\parallel^{Ed}$ . This implies, according to Eq. (3), that terms of order  $h_F q^3 \varphi$  and  $h_F^2 q^2 \varphi$  will be needed, and thus we should look for terms of the same order in the expansion of the spin-charge coupling  $\Psi^{i0} D_0^0$ . The above two orders can also be identified based on symmetry argument, that the combined power in  $h_F$  and  $q$  must be even and that they are the lowest RSOI contributions to the respective  $q$  orders. Starting from

$$\Psi^{i0}(\omega=0, \omega', \mathbf{q}) = \frac{\Gamma}{2\pi N_0} \sum_p \text{Tr} \left[ \tau^j \frac{\left( \omega' - \varepsilon_p - \mathbf{q} \cdot \frac{\partial \varepsilon_p}{\partial \mathbf{p}} + i\Gamma \right) + \left( h_p^i + \mathbf{q} \cdot \frac{\partial h_p^i}{\partial \mathbf{p}} \right) \sigma^j}{\left( \omega' - \varepsilon_p - \mathbf{q} \cdot \frac{\partial \varepsilon_p}{\partial \mathbf{p}} + i\Gamma \right)^2} \left( \frac{1}{\omega' - \varepsilon_p - i\Gamma} + \frac{h_p^i \sigma^j}{(\omega' - \varepsilon_p - i\Gamma)^2} \right) \right], \quad (5)$$

we expand it, for instance, up to the order  $h_F q^3$ , and obtain

$$\Psi^{i0}(h_F q^3) = 2N_0 \int d\varepsilon \times \left[ \frac{\overline{(\mathbf{q} \cdot \mathbf{v}_p)^3 h_p^i}}{(\omega' - \varepsilon + i\Gamma)^4 (\omega' - \varepsilon + i\Gamma)^2} + \frac{3(\mathbf{q} \cdot \mathbf{v}_p)^2 \mathbf{q} \cdot \frac{\partial h_p^i}{\partial \mathbf{p}}}{(\omega' - \varepsilon + i\Gamma)^4 (\omega' - \varepsilon - i\Gamma)} \right], \quad (6)$$

where  $\mathbf{v}_p = \mathbf{p}/m^*$ . The angular averages in Eq. (6) over the Fermi surface give rise to  $\mathbf{q}$  dependences of the form  $q^2 q_{j=x,y}$ , which will not contribute to Eq. (3) because  $\nabla^2 \varphi = 0$ . Following similar procedure,  $\Psi^{i0}(h_F^2 q^2)$  is found to be identically zero. Thus Eq. (3) is the SDE for the case of nonuniform driving field.

As has been explained in the previous section,  $\mathbf{S}_{\parallel}^{Ed}$  alone cannot satisfy the boundary condition  $I_{\rho}^{i=x,y} = 0$ . Thus in the end we expect to have an additional  $\Delta \mathbf{S}$  so that  $\mathbf{S} = \mathbf{S}_{\parallel}^{Ed} + \Delta \mathbf{S}$ . On the other hand, the contribution from  $\mathbf{S}_{\parallel}^{Ed}$  to  $I_{\rho}^z$  is found to vanish already. The generation of  $\Delta \mathbf{S}_z$  thus does not fall into the conventional scheme that spin accumulation  $S_z$  is caused by  $I_n^z$  near the sample boundary. Our major task in the following is to calculate  $\Delta \mathbf{S}$ .

Putting the coordinates in units of  $l_{s0} = \sqrt{D\tau_{s0}}$ , with  $\tau_{s0} = 2\hbar^2/(h_F^2 \tau)$ , the SDE for  $\Delta \mathbf{S}$  is given by

$$\nabla^2 \Delta S_x - 4\Delta S_x + 4 \frac{\partial}{\partial x} \Delta S_z = 0,$$

$$\nabla^2 \Delta S_y - 4\Delta S_y + 4 \frac{\partial}{\partial y} \Delta S_z = 0,$$

$$\nabla^2 \Delta S_z - 8\Delta S_z - 4 \frac{\partial}{\partial x} \Delta S_x - 4 \frac{\partial}{\partial y} \Delta S_y = 0. \quad (7)$$

Modes of solution of Eq. (7) have the form  $\Delta S_j^{(q)} = \sum_m a_j^{(q)} e^{im(\delta+\phi)} H_m^{(1)}(\gamma q \rho)$ , where  $H_m^{(1)}(z)$  is the Hankel function of the first kind and the index  $q$  denotes the  $q$ -th mode. Substituting into Eq. (7) we obtain

$$\begin{bmatrix} (-\gamma^2 - 4) & 0 & 4i\gamma \sin \delta \\ 0 & (-\gamma^2 - 4) & 4i\gamma \cos \delta \\ -4i\gamma \sin \delta & -4i\gamma \cos \delta & (-\gamma^2 - 8) \end{bmatrix} \begin{bmatrix} a_x^{(q)} \\ a_y^{(q)} \\ a_z^{(q)} \end{bmatrix} = 0. \quad (8)$$

The asymptotic behavior required of  $\Delta S_j^{(q)}$  leads to  $\text{Im } \gamma > 0$ . Thus  $\gamma_1 = 2i$ ,  $\gamma_2 = \sqrt{2+2i\sqrt{7}}$ , and  $\gamma_3 = -\gamma_2^*$ . We

also have  $(a_x^{(1)}, a_y^{(1)}, a_z^{(1)}) = a_x^{(1)}(1, -\tan \delta, 0)$ ,  $(a_x^{(2)}, a_y^{(2)}, a_z^{(2)}) = a_z^{(2)}(2ig_2 \sin \delta, 2ig_2 \cos \delta, 1)$ , and  $(a_x^{(3)}, a_y^{(3)}, a_z^{(3)}) = a_z^{(3)}(2ig_2 \sin \delta, 2ig_2 \cos \delta, 1)^*$ , for  $q=1, 2$ , and  $3$ , respectively. Here  $g_2 = \gamma_2/(\gamma_2^2 + 4)$ . As  $\delta$  takes on continuous values, there are effective infinite solutions per  $q$ -mode. In terms of these modes  $\Delta S_j$  is expanded in the form

$$\Delta S_j = \int_0^{2\pi} d\delta \sum_{q=1}^3 \sum_m a_j^{(q)}(\delta) H_m^{(1)}(\gamma q \rho) e^{im(\delta+\phi)}. \quad (9)$$

The condition that  $\Delta \mathbf{S}$  is real requires  $a_x^{(1)}$  to be pure imaginary and  $a_z^{(2)} = -a_z^{(3)*}$ .

The boundary condition for the nonuniform driving field is established by applying to the spin current expression similar procedure that we have applied to Eq. (3). The spin-current expression is found to resemble the uniform driving field case,<sup>24</sup> albeit now that  $\nabla \varphi$  becomes position dependent. We have

$$I_j^i = -2D \nabla_j S_i - R^{ixj} S_x - R^{iyj} S_y - R^{izj} S_z + \sum_{l=x,y} 4\tau^2 \epsilon^{xyi} v_F^j \left( \mathbf{h}_p \times \frac{\partial \mathbf{h}_p}{\partial k_l} \right)_z e N_0 \nabla_l \varphi(\mathbf{r}), \quad (10)$$

where the last term is the explicit contribution from the driving field and is nonzero for  $I_j^i$  only. The boundary condition  $I_{\rho}^i(\rho=\rho_0) = 0$  becomes

$$-\nabla_{\rho} \Delta S_x - 2 \cos \phi \Delta S_z - 2\alpha \tilde{E}/\rho \sin 2\phi|_{\rho=\rho_0} = 0,$$

$$-\nabla_{\rho} \Delta S_y - 2 \sin \phi \Delta S_z + 2\alpha \tilde{E}/\rho \cos 2\phi|_{\rho=\rho_0} = 0,$$

$$-\nabla_{\rho} \Delta S_z + 2 \cos \phi \Delta S_x + 2 \sin \phi \Delta S_y|_{\rho=\rho_0} = 0, \quad (11)$$

where  $\nabla_{\rho} \equiv \partial/\partial \rho$ ,  $\rho_0 = R_0/l_{s0}$ , and  $\tilde{E} = eE_0 N_0 \tau/\hbar$ . We note that the  $\tilde{E}$  terms in Eq. (11) originate from the spin current due to  $\mathbf{S}_{\parallel}^{Ed}$ , which are the driving terms here. We solve Eqs. (9) and (11) for  $a_j^{(q)}(\delta)$  by a direct numerical approach and by an analytical approach. Excellent matching is obtained between the two approaches. The analytical approach is facilitated by the assumed forms  $a_x^{(1)} = it_x \sin 2\delta$  and  $a_z^{(2)} = t_z \cos \delta$ , where  $t_x$  is real and  $t_z$  is complex. The former is guided by the observation, from Eq. (11), that  $\Delta S_x$  depends on  $\phi$  as  $\sin 2\phi$ . Substituting these forms into Eqs. (9) and (11), and after some algebra, gives

$$\begin{aligned}
-\gamma_1 H_2^{(1)'}(z_1)t_x + i \operatorname{Im}[t_z X] &= i\alpha\tilde{E}/(\pi\rho_0), \\
-i\gamma_1 H_1^{(1)}(z_1)t_x + 2 \operatorname{Im}[t_z Y] &= 0, \\
\frac{4}{z_1} H_1^{(1)}(z_1)t_x + i \operatorname{Im}[t_z Z] &= 0,
\end{aligned} \tag{12}$$

where  $z_1 = \gamma_1 \rho_0$ ,  $f'(z) \equiv df/dz$ ,  $X = 2H_1^{(1)}(z_1) - 2g_2\gamma_2 H_2^{(1)'}(z_2)$ ,  $Y = (g_2\gamma_2 - 1)H_1^{(1)}(z_2)$ ,  $Z = 2(\gamma_2 - 4g_2)H_1^{(1)'}(z_2)$ , and  $z_2 = \gamma_2 \rho_0$ . Equation (12) allows us to solve for  $t_x$  and  $t_z$  analytically, which are proportional to  $\alpha\tilde{E}$ . Explicit expressions of  $t_x$  and  $t_z$  are

$$t_x = -\frac{4\alpha\tilde{E}}{\pi} \frac{\operatorname{Im}[YZ^*]}{8H_1^{(1)}(z_1)\operatorname{Im}[XY^*] + \gamma_1 z_1 H_1^{(1)}(z_1)\operatorname{Im}[ZX^*] + 2\gamma_1 z_1 H_2^{(1)'}(z_1)\operatorname{Im}[ZY^*]}, \tag{13}$$

and

$$t_z = \frac{\alpha\tilde{E}}{\pi\rho_0} \frac{H_1^{(1)}(z_1)\operatorname{Im}[8Y^* - \gamma_1 z_1 Z^*]}{8H_1^{(1)}(z_1)\operatorname{Im}[XY^*] + \gamma_1 z_1 H_1^{(1)}(z_1)\operatorname{Im}[ZX^*] + 2\gamma_1 z_1 H_2^{(1)'}(z_1)\operatorname{Im}[ZY^*]}.$$

The spin densities  $\Delta S_i$  are then obtained to give

$$\Delta S_x = 2\pi\{-it_x H_2^{(1)}(\gamma_1\rho) + 2 \operatorname{Im}[t_z g_2 H_2^{(1)}(\gamma_2\rho)]\}\sin 2\phi,$$

$$\Delta S_y = 2\pi\{-it_x H_0^{(1)}(\gamma_1\rho) - 2 \operatorname{Im}[t_z g_2 H_0^{(1)}(\gamma_2\rho)]\} - \Delta S_x \cot 2\phi,$$

$$\Delta S_z = -4\pi \operatorname{Im}[t_z H_1^{(1)}(\gamma_2\rho)]\sin \phi. \tag{14}$$

This and Eq. (2) together are our main results. In particular,  $\Delta S_z \neq 0$  confirms that RSOI's contribution to spin accumulation can be restored in a nonuniform driving field. The parity in  $\phi$  of  $\Delta S_i$  is consistent with that implied in Eq. (11), which is determined by the  $\tilde{E}$  terms. The spin accumulation, given in its entirety by  $\Delta S_z$ , is in a dipole distribution which orients transversely to the driving field  $E_0\hat{x}$ . Furthermore, Eq. (14) shows that  $\gamma_1$  and  $\gamma_2$  contribute to, respectively, decaying and oscillatory behavior in  $\Delta S_i$ .

### III. NUMERICAL RESULTS

Figure 1 presents the spin accumulation  $S_z$  in the vicinity of the circular void. We use for our numerical results material parameters that are consistent with GaAs: effective mass  $m^* = 0.067m_0$  with  $m_0$  the free-electron mass; electron density  $n_e = 1 \times 10^{12} \text{ cm}^{-2}$ ; electron mean free path  $l_e = 0.43 \text{ } \mu\text{m}$ ; radius of the circular hole  $R_0 = 0.5l_{so}$ ; and Rashba coupling constant  $\alpha = 0.3 \times 10^{-12} \text{ eV m}$ .<sup>31,32</sup> The spin-relaxation length is  $l_{so} = 3.76 \text{ } \mu\text{m}$  and the driving field is  $E_0 = 40 \text{ mV}/\mu\text{m}$ . As shown in Fig. 1, the core of the spin accumulation consists of two spin pockets of opposite spin and of largest spin density magnitude at  $\phi = \pm \pi/2$ . The spin pockets have radial thickness of about  $0.3l_{so} \sim 1.1 \text{ } \mu\text{m}$ . In the outer region, spin densities of opposite signs and of smaller magnitudes are dispersed to a wider spatial extent, in the form of two curved spin clouds. The spin cloud center is located about one  $l_{so}$  from the void boundary at  $\phi = \pm \pi/2$ .

Both the spin pocket thickness and the spin cloud distance from the void boundary are not sensitive to the void radius  $R_0$ .

This spin accumulation can be probed optically by Kerr rotation. To simulate the case of an optical probe scanning along the  $\phi = \pi/2$  direction, we calculate the net number of out-of-plane electron spin within the probe area which center is located at a distance  $d$  from the void center. For simplicity, we take the probe area to be the same as that of the void. The result is presented in Fig. 2, where we have included several  $R_0$  cases, from  $R_0 = 0.5l_{so}$  up to  $R_0 = 1.2l_{so}$ . Distinct contributions from the spin pocket and the spin cloud can be identified. The former are negative minima around  $d \approx 0.5l_{so}$  and

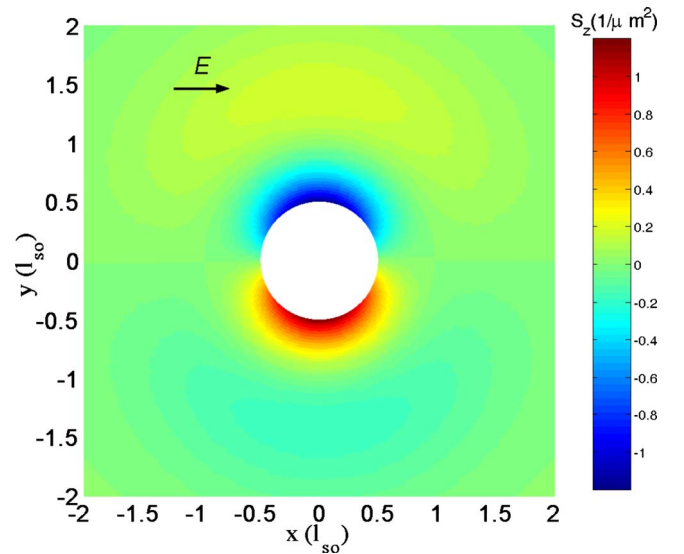


FIG. 1. (Color online) Spin accumulation  $S_z$  in the vicinity of a circular void (white circle).  $S_z$  is in unit of  $1/\mu\text{m}^2$ , void radius  $R_0 = 0.5l_{so}$ , and  $l_{so} = 3.76 \text{ } \mu\text{m}$ . Dark arrow indicates the driving field direction.

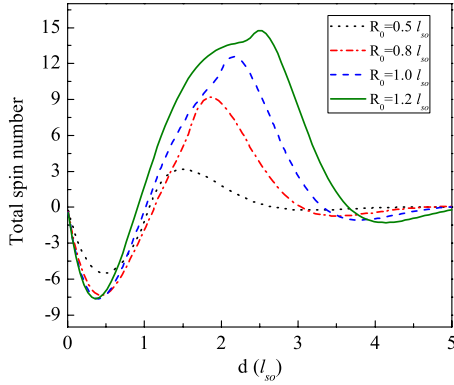


FIG. 2. (Color online) Net number of out-of-plane electron spin within a circular area the same size as the void. The center is shifted by a distance  $d$  from the void center along  $\phi = \pi/2$ .

the latter are positive peaks around  $d \approx R_0 + l_{so}$ . That the negative minima are essentially unshifted reflects the insensitivity of the core radial thickness to the void radius  $R_0$ . Additional peak for the  $R_0 = 1.2l_{so}$  curve at  $d \approx 2R_0$  corresponds to the situation when the probe area moves out of the spin pocket.

The spin accumulation in a spin-dipole form oriented transversely to the driving field is a generic feature signifying the redistribution of spin rather than the net transport of spin. It has been found in the vicinity of a non-SOI elastic scatterer in a RSOI 2DEG,<sup>33,34</sup> and in the vicinity of a mesoscopic cylindrical barrier in a 2DEG with the barrier profile providing the SOI.<sup>35</sup> Both objects are of sizes much less than  $l_e$ . Of course, the physical mechanisms leading to all the above spin-dipole forms are entirely different. Furthermore, here we demonstrate that such spin-dipole feature can exist in the neighborhood of a much larger object  $R_0 \approx l_{so}$ , is robust against background scatterers and is within reach of present measurement technology.

Finally, we note that the spin accumulation features we obtain above are relevant to the case when the circular void

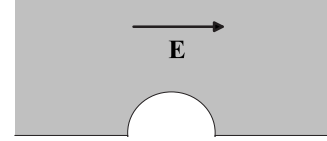


FIG. 3. Patterning the sample edge with a semicircular void

is located at a sample edge: an edge-semicircular void (ESV) as shown in Fig. 3. The nonuniform driving field  $E(\rho)$  for the circular void satisfies also the additional boundary condition  $j_y = 0$  imposed by the ESV case at the sample edge,  $\phi = (0, \pi)$ . Thus the same  $E(\rho)$  holds in the two cases. However, to satisfy the additional boundary condition for spin current at the sample edge, a further additional spin accumulation  $\Delta S_{ESV}$  is needed, leading to the total spin accumulation  $S = S_{\parallel}^{Ed} + \Delta S + S_{ESV}$ . The imposing of the spin current boundary condition in this case is much more complicated, particularly for the spin accumulation near the two corners of the ESV structure, but we find that the spin pocket and the spin cloud features in Fig. 1 remains essentially intact except for near corner regions of the ESV structure.<sup>36</sup>

#### IV. CONCLUSIONS

In conclusions, we have demonstrated that nonuniform driving field can give rise to spin accumulation in a diffusive Rashba-type 2DEG. The nonuniform driving field can be realized by patterning the sample such as with a circular void in the sample or with a semicircular void at the sample edge. The physical process is identified to be associated with spin current for the in-plane spin at the boundary. Our proposed scheme of restoring the RSOI contribution to gate-tunable spin accumulation is relatively simple, and we hope that this will draw experimental effort in the near future.

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<sup>1</sup>M. I. D'yakonov and V. I. Perel', Phys. Lett. **35A**, 459 (1971).  
<sup>2</sup>A. A. Bakun, B. P. Zakharchenya, A. A. Rogachev, M. N. Tkachuk, and V. G. Fleisher, JETP Lett. **40**, 1293 (1984).  
<sup>3</sup>V. M. Edelstein, Solid State Commun. **73**, 233 (1990).  
<sup>4</sup>A. G. Aronov, Yu. B. Lyanda-Geller, and G. E. Pikus, Sov. Phys. JETP **73**, 537 (1991).  
<sup>5</sup>J. E. Hirsch, Phys. Rev. Lett. **83**, 1834 (1999).  
<sup>6</sup>S. Zhang, Phys. Rev. Lett. **85**, 393 (2000).  
<sup>7</sup>S. Murakami, N. Nagaosa, and S. C. Zhang, Science **301**, 1348 (2003).  
<sup>8</sup>J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. **92**, 126603 (2004).  
<sup>9</sup>Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science **306**, 1910 (2004).  
<sup>10</sup>S. Q. Shen, Phys. Rev. B **70**, 081311(R) (2004).  
<sup>11</sup>J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. **94**, 047204 (2005).

<sup>12</sup>H. A. Engel, B. I. Halperin, and E. I. Rashba, Phys. Rev. Lett. **95**, 166605 (2005).  
<sup>13</sup>A. G. Mal'shukov and K. A. Chao, Phys. Rev. B **71**, 121308(R) (2005).  
<sup>14</sup>W. K. Tse and S. Das Sarma, Phys. Rev. Lett. **96**, 056601 (2006).  
<sup>15</sup>D. Culcer and R. Winkler, Phys. Rev. Lett. **99**, 226601 (2007).  
<sup>16</sup>E. I. Rashba, Sov. Phys. Solid State **2**, 1109 (1960); Yu. A. Bychkov and E. I. Rashba, JETP Lett. **39**, 78 (1984).  
<sup>17</sup>J. I. Inoue, G. E. W. Bauer, and L. W. Molenkamp, Phys. Rev. B **70**, 041303(R) (2004); A. A. Burkov, A. S. Núñez, and A. H. MacDonald, *ibid.* **70**, 155308 (2004); R. Raimondi and P. Schwab, *ibid.* **71**, 033311 (2005); O. V. Dimitrova, *ibid.* **71**, 245327 (2005).  
<sup>18</sup>E. G. Mishchenko, A. V. Shytov, and B. I. Halperin, Phys. Rev. Lett. **93**, 226602 (2004).  
<sup>19</sup>B. K. Nikolić, S. Souma, L. P. Zárbo, and J. Sinova, Phys. Rev.

- Lett. **95**, 046601 (2005).
- <sup>20</sup>J. Li and S. Q. Shen, Phys. Rev. B **76**, 153302 (2007).
- <sup>21</sup>P. G. Silvestrov, V. A. Zyuzin, and E. G. Mishchenko, Phys. Rev. Lett. **102**, 196802 (2009).
- <sup>22</sup>R. Raimondi, C. Gorini, P. Schwab, and M. Dzierzawa, Phys. Rev. B **74**, 035340 (2006).
- <sup>23</sup>I. G. Finkler, H. A. Engel, E. I. Rashba, and B. I. Halperin, Phys. Rev. B **75**, 241202(R) (2007); V. Sih, W. H. Lau, R. C. Myers, V. R. Horowitz, A. C. Gossard, and D. D. Awschalom, Phys. Rev. Lett. **97**, 096605 (2006).
- <sup>24</sup>A. G. Mal'shukov, L. Y. Wang, C. S. Chu, and K. A. Chao, Phys. Rev. Lett. **95**, 146601 (2005).
- <sup>25</sup>O. Bleibaum, Phys. Rev. B **74**, 113309 (2006).
- <sup>26</sup>Y. Tserkovnyak, B. I. Halperin, A. A. Kovalev, and A. Brataas, Phys. Rev. B **76**, 085319 (2007).
- <sup>27</sup>J. R. Shi, P. Zhang, D. Xiao, and Q. Niu, Phys. Rev. Lett. **96**, 076604 (2006).
- <sup>28</sup>P. Zhang, Z. G. Wang, J. R. Shi, D. Xiao, and Q. Niu, Phys. Rev. B **77**, 075304 (2008).
- <sup>29</sup>L. Y. Wang, C. S. Chu, and A. G. Mal'shukov, Phys. Rev. B **78**, 155302 (2008).
- <sup>30</sup>M. I. D'yakonov and V. I. Perel', Sov. Phys. JETP **33**, 1053 (1971) [Zh. Eksp. Teor. Fiz. **60**, 1954 (1971)].
- <sup>31</sup>J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. **78**, 1335 (1997).
- <sup>32</sup>L. Meier, G. Salis, I. Shorubalko, E. Gini, S. Schön, and K. Ensslin, Nat. Phys. **3**, 650 (2007).
- <sup>33</sup>A. G. Mal'shukov and C. S. Chu, Phys. Rev. Lett. **97**, 076601 (2006).
- <sup>34</sup>A. G. Mal'shukov, L. Y. Wang, and C. S. Chu, Phys. Rev. B **75**, 085315 (2007).
- <sup>35</sup>K. Y. Chen, C. S. Chu, and A. G. Mal'shukov, Phys. Rev. B **76**, 153304 (2007).
- <sup>36</sup>L. Y. Wang, C. S. Chu, and A. G. Mal'shukov (unpublished).