Dephasing time and magnetoresistance of two-dimensional electron gas in spatially modulated magnetic fields

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The effect of a spatially modulated magnetic field on the weak-localization phenomenon in two-dimensional electron gas is studied. Both the dephasing time τ_H and magnetoresistance are shown to reveal a nontrivial behavior as functions of the characteristics of magnetic field profiles. The magnetic field profiles with rather small spatial scales d and modulation amplitudes H_0 such that $H_0 d^2 \ll \hbar c/e$ are characterized by the dephasing rate $\tau_H^{-1} \propto H_0^2 d^2$. The increase in the flux value $H_0 d^2$ results in a crossover to a standard linear dependence $\tau_H^{-1} \propto H_0$. Applying an external homogeneous magnetic field H one can vary the local dephasing time in the system and affect the resulting average transport characteristics. We have investigated the dependence of the average resistance vs the field H for some generic systems and predicted a possibility to observe a positive magnetoresistance at not too large H values. The resulting dependence of the resistance vs H should reveal a peak at the field values $H \sim H_0$.

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I. INTRODUCTION

The possibility to govern the electronic transport by applying an inhomogeneous magnetic field has recently attracted considerable interest. In particular, this problem is intensively studied in the hybrid ferromagnet/superconductor structures where the inhomogeneous magnetic field induced by the domain structure in the ferromagnet or a magnetic dot array is used to control the superconducting order parameter structure and the transport of Cooper pairs (see, e.g., Refs. 1 and 2 for review). It is important to note that the typical values of the fields used in such experiments can be relatively small: $H \sim 10-10^3$ Oe. Nevertheless in the vicinity of the superconducting transition even these field values allow to destroy the Cooper pairs and, thus, strongly affect the electronic transport.

Another possibility to change the conductance applying relatively weak magnetic fields can be realized even in the normal (i.e., nonsuperconducting) structures provided we consider the systems with measurable quantum interference effects, e.g., disordered two-dimensional electron gas (2DEG) at low temperatures T. In the latter case the electron conductance is known to be affected by the weak-localization effects, which are caused by the quantum interference between the electronic waves propagating along different time-reversed quasiclassical trajectories.[3] The weak-localization correction Δg to the Drude conductance g_D in the diffusive limit can be written in the form

$$\Delta g(\mathbf{r}) = -\frac{2e^2}{\pi\hbar} D \int_{\tau}^{\infty} W(\mathbf{r},t_0)dt_0. \tag{1}$$

Here $W(\mathbf{r},t_0)dt_0$ is the probability of electron return to the point $\mathbf{r}$ during the time interval $t_0 < t < t_0 + dt_0$, τ is the elastic scattering time, and D is the diffusion constant.

In the presence of an external magnetic field the probability of return is determined by the Green's function $C(\mathbf{r}_f,t_f,\mathbf{r}_i,t_i)$ satisfying the so-called Cooperon equation,

$$W(\mathbf{r},t_0) = C(\mathbf{r},t_i + t_0,\mathbf{r},t_i), \tag{2}$$

$$\left[\frac{\partial}{\partial t_f} + D\left(-i\frac{\partial}{\partial \mathbf{r}_f} - \frac{2e}{\hbar c}\mathbf{A}(\mathbf{r}_f)\right)^2 + \frac{1}{\tau_\varphi}\right] C = \delta(t_f - t_i)\,\delta(\mathbf{r}_f - \mathbf{r}_i), \tag{3}$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential and τ_φ is the characteristic dephasing time. In the limit of zero magnetic field the expression for the weak-localization correction to the conductance obtained from Eq. (3) takes the form

$$\Delta g(H=0) = -\frac{e^2}{2\pi^2\hbar}\ln\frac{\tau_\varphi}{\tau}. \tag{4}$$

The maximal size of closed trajectories contributing to this value is defined by the characteristic dephasing length $L_\varphi = \sqrt{D\tau_\varphi}$. Applying an external magnetic field perpendicular to the plane of 2DEG system one destroys the coherence for closed trajectories which enclose the magnetic flux larger than the flux quantum $\Phi_0 = \pi\hbar c/|e|$. The resulting dephasing time τ_H becomes field dependent and can be obtained by comparing the flux through the contour of the size $\sqrt{D\tau_H}$ with Φ_0: $\tau_H^{-1} \sim DH/\Phi_0$. As a consequence, the 2DEG system has a negative magnetoresistance (see Ref. 3 and references therein) and the conductance takes the form

$$\Delta g = -\frac{e^2}{2\pi^2\hbar}\left\{\psi\left(\frac{1}{2} + \frac{\hbar c}{4eHD\tau}\right) - \psi\left(\frac{1}{2} + \frac{\hbar c}{4eHD\tau_\varphi}\right)\right\}, \tag{5}$$

where ψ is the digamma function. In the low field limit ($\tau_H \gg \tau_\varphi$) expression (5) transforms into the expansion

$$\Delta g = -\frac{e^2}{2\pi^2\hbar}\left\{\ln\frac{\tau_\varphi}{\tau} - \frac{2}{3}\left(\frac{eHD\tau_\varphi}{\hbar c}\right)^2 + \cdots\right\}. \tag{6}$$

Considering the magnetic fields which are modulated on microscopic length scales one should modify the above expressions taking into account the changes in the magnetic flux enclosed by the interfering trajectories passing through the regions with a rapidly changing magnetic field. The hybrid structures containing the 2DEG systems and certain sources of the spatially modulated magnetic fields attracted recently both the experimental and theoretical interests.[4–16] In part, these investigations have been stimulated by the possible potential of such systems for making detailed studies of the inhomogeneous magnetic field distributions. The magnetic field profiles with microscopic spatial scales can be induced, e.g., by a vortex lattice in a superconducting film,[4–8] as well as by a ferromagnetic film domain structure or a magnetic dot array positioned in the vicinity of the 2DEG system. Note also that the problem of the 2DEG conductance in a modulated magnetic field appears to be equivalent to the one of a rough 2DEG layer placed in a parallel magnetic field.[10,11] For the particular case of vortices trapped in a superconducting film the magnetic field takes the form of flux tubes. An appropriate theoretical description of the weak-localization phenomenon for different flux tube radii as compared to the L_φ length has been developed in Ref. 15. The corresponding contribution to the magnetoconductance at low average fields H appeared to be proportional to the vortex concentration, i.e., to the $|H|$ value, in contrast to the H^2 behavior in a uniform magnetic field. The numerical analysis of the conductance corrections for the case of a lattice of magnetic flux tubes for arbitrary relations between the tube radius and L_φ was performed in Ref. 8. Experimentally these predictions have been confirmed in Refs. 5 and 6 for GaAs/AlGaAs heterostructures.

One can expect that standard expressions (5) and (6) for local conductance should hold even for the spatially modulated magnetic fields provided the characteristic spatial scale d of such modulation is much larger than the size of the closed trajectories contributing to the conductance corrections. For a rather strong value of the z component of the local field $B(\mathbf{r})$ the latter size can be defined as a minimum of two lengths: (i) the dephasing length in the absence of the field $L_\varphi=\sqrt{D\tau_\varphi}$ and (ii) the length $L_B=\sqrt{D\tau_B}=\sqrt{\hbar c/eB(\mathbf{r})}$ which formally coincides with the textbook definition of the magnetic length. Here we denote the magnetic field component along the direction perpendicular to the plane of a 2DEG system as $B(\mathbf{r})=H+\delta H(\mathbf{r})$, where H is the average field value. Thus, considering rather strong fields and/or not very low temperatures one can use the above expressions for local conductance substituting the function $B(\mathbf{r})$ instead of the homogeneous field H. This adiabatic picture obviously breaks down when the closed interfering trajectories pass through the regions with rapidly changing magnetic field which happens either near the zeros of magnetic field or in the limit $d \lesssim \min[L_\varphi, L_B]$. The dephasing length and time in this case are no longer determined by the local field value and their dependence on the field modulation amplitude H_0 can become rather unusual. In particular, for the magnetic fields with zero spatial average the dephasing time is proportional to the square of the field amplitude ($\tau_B^{-1}\propto H_0^2$) which is in sharp contrast to the linear in H behavior of the dephasing rate for homogeneous fields. For some model, one-dimensional field profiles, such unusual field dependence of the dephasing rate has been previously predicted in Ref. 9. Experimentally this behavior $\tau_B^{-1}\propto H_0^2$ has been observed in Ref. 10 for random magnetic field profiles.

One of the goals of the present work is to suggest an analytical description of the weak-localization phenomenon in inhomogeneous magnetic field for a wide class of the field profiles. In Sec. II we consider different regimes of the weak localization which are realized in different regions of magnetic field parameters. Also in this section we demonstrate that in strong magnetic fields and/or at not very low temperatures the local approximation is applicable for calculation of the quantum correction to the conductance. In Sec. III we consider the regimes corresponding to the weak amplitude of magnetic field. In particular, in Sec. III A we present the calculations of a natural measurable quantity, i.e., the conductance averaged over the system area. As a next step, we proceed with the description of the dephasing rate behavior vs characteristics of the modulated magnetic field for a wide class of the periodic field profiles (see Sec. III B). In Sec. IV we consider the case of strong magnetic fields and show that the dependence of the magnetoresistance vs the average field value appears to reveal an unusual peak structure. An obvious reason for the nonmonotonous behavior of the resistance vs the average field is associated with partial field compensation effect which occurs in the regions where the z components of the average and local fields have the opposite signs. Thus, applying the external magnetic field to the system placed in a modulated field with zero average one can stimulate the interference effects in some regions of the sample. Depending on the particular shape of the field profile this effect can result in the negative or positive magnetoresistance of the sample. In other words, the 2DEG samples coupled with the subsystems inducing the inhomogeneous magnetic field can reveal a so-called "antilocalization" (see, e.g., Refs. 17 and 18) phenomenon when we apply an external magnetic field H. The results and suggestions for possible experiments are summarized in Sec. V.

Hereafter we focus on the case of classically weak magnetic fields $[eB(\mathbf{r})\tau/mc \ll 1]$. Such fields weakly affect the Drude conductance: the corresponding corrections are proportional to the factor $(\omega_c\tau)^2$ $[\omega_c=eB(\mathbf{r})/mc$ is the cyclotron frequency$]$. Indeed the diffusive approximation for the electron motion is applicable when $B\ll\Phi_0/l^2$. At such fields $\omega_c \ll (\hbar/\varepsilon_F\tau)1/\tau \ll 1/\tau$ (ε_F is the Fermi energy). Thus, these magnetic fields affect only the interference corrections to the transport characteristics, and we disregard the inhomogeneous field effect on the Drude-type contribution to the conductance which has been previously studied in Refs. 19–22.

Note that all results obtained in this paper are valid not only for ideal 2DEG with zero thickness but also for quasi-two-dimensional electron systems with finite thickness a, which has to satisfy the condition $a\ll L_\varphi$. In this case one can define the field range, in which longitudinal components do not affect the weak-localization correction to the conductance, while the transverse component does. Indeed, the effect of a weak longitudinal component of magnetic field $H_\parallel$ can be described by the renormalization of the characteristic dephasing time: the value τ_φ^{-1} has to be replaced by $\tau_\varphi^{-1}+\tau_{H_\parallel}^{-1}$ (see, e.g., Ref. 3), where

$$\frac{1}{\tau_{H_\parallel}}=\frac{1}{3}\left(\frac{eH_\parallel a}{\hbar c}\right)^2 D.$$

Thus, the influence of the longitudinal component becomes noticeable only for $H_\parallel \sim H_\parallel^* \sim \Phi_0/aL_\varphi$. This value is much

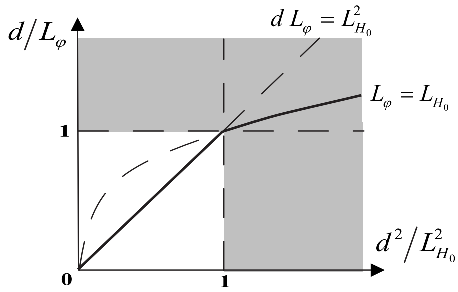

FIG. 1. The diagram of different weak-localization regimes in the plane of key parameters. In the gray region the scale of the magnetic field inhomogeneity d is large enough so that the dephasing is controlled by the local magnetic field. In the white square the magnetic field is weak but its inhomogeneity reveals in the renormalization of the effective electron dephasing time. Above the solid curve the dephasing occurs at the dephasing length L_φ and the influence of magnetic filed reveals in a small additional correction to the conductivity.

larger than the characteristic value $H_\perp^* \sim \Phi_0/L_\varphi^2$ of transverse component, which can strongly affect the weak-localization correction. Hereafter we assume $H_\parallel \ll H_\parallel^*$ and neglect the effect of longitudinal magnetic field components.

II. DIFFERENT REGIMES OF WEAK LOCALIZATION AND POSSIBLE APPROXIMATE APPROACHES

In this section we outline the approximate approaches which are used for describing dephasing regimes in different regions of the system's parameters.

In the presence of an inhomogeneous magnetic field the weak-localization correction to the conductance of 2DEG is determined by the interplay of three length scales: (i) dephasing length L_φ, which at low temperatures grows as $T^{-1/2}$ (see Ref. 24), (ii) the scale of the magnetic field inhomogeneity d, and (iii) the magnetic length $L_{H_0}=\sqrt{\Phi_0/H_0}$, where H_0 is the amplitude of the periodic magnetic field. The ratios of these lengths define the behavior of the weak-localization correction to the conductance. For the analysis of the behavior of quantum correction to the conductance it is convenient to use the diagram shown in Fig. 1. We choose the parameter d/L_φ to describe the temperature dependence of weak-localization correction to the conductance and the parameter $d^2/L_{H_0}^2$ to consider the influence of modulated magnetic field. Note that for a two-dimensional lattice with translational vectors $\mathbf{R}_n = n_1 d_1 \mathbf{a}_1 + n_2 d_2 \mathbf{a}_2$ ($\mathbf{a}_1$, $\mathbf{a}_2$ are unit vectors and n_1, n_2 are integers) the value d is the absolute value of the smallest vector $\mathbf{R}_n (d=\min[d_1,d_2])$.

Depending on the ratios d/L_{H_0} and d/L_φ there exist two qualitatively different mechanisms of the electron dephasing caused by the inhomogeneous field. Provided $d/L_{H_0}<1$ and simultaneously the d length scale is smaller than L_φ (white square in Fig. 1) the dephasing scenario in a modulated field with zero average can be explained by the following qualitative arguments. Let us consider a certain quasiclassical trajectory of the length $L=\sqrt{D\tau_B}$ which encloses many primitive cells of the periodic field profile. The magnetic flux coming from the cells which are positioned inside the contour appears to be averaged to zero. The only residual flux is associated with the cells which are crossed by the quasiclassical trajectory and give a flux contribution which strongly fluctuates with the increase in the area enclosed by the trajectory. The characteristic amplitude of these flux fluctuations can be estimated as the number of the elementary cells crossed by the trajectory (L/d) multiplied by the typical flux value $H_0 d^2$: $\delta\Phi \sim L d H_0$. Comparing this fluctuating flux with the flux quantum Φ_0 we find the length of the dephasing $L \sim \Phi_0/dH_0 \sim L_{H_0}^2/d$ and corresponding dephasing rate $\tau_B^{-1} \sim D d^2 H_0^2/\Phi_0^2$. These qualitative arguments are in beautiful agreement with the quantitative consideration in Sec. III B carried out on the basis of the "nearly free electron" approximation. In the opposite limit $d/L_{H_0}>1$ or $d/L_\varphi>1$ the dephasing is controlled by the local magnetic field value (this regime corresponds to the gray region in Fig. 1).

Of course, the magnetic field provides a dominating dephasing mechanism only at low temperatures. For rather high temperatures when $L_\varphi < \max[L_{H_0}, L_{H_0}^2/d]$ (the region above the solid curve in Fig. 1) the dephasing occurs at the length L_φ and one can analyze the magnetic field effect on the weak-localization correction to the conductance perturbatively (see Sec. III A). Defining the range of parameters where the perturbative description is applicable one should compare the dephasing length L_φ with the scale L_{H_0} for $d > L_{H_0}$ and the scale $L_{H_0}^2/d$ for $d < L_{H_0}$.

Now let us focus on the gray region in Fig. 1 ($d \gg L_{H_0}$ or $d > L_\varphi$). In this region the weak-localization correction to the conductance can be obtained within the local approximation. This means that the conductivity at each point of the sample depends on the local magnetic field. The validity of the local approximation in this regime can be shown directly from Eq. (3). Let us introduce the vectors

$$\mathbf{R} = \frac{\mathbf{r}_f + \mathbf{r}_i}{2}, \quad \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i.$$

An electron is dephased at the length scale which is the minimum of the scales L_φ and L_{H_0}; i.e., only the region $|\mathbf{r}| < \min[L_\varphi, L_{H_0}]$ makes the contribution to the weak-localization correction to the conductance. Therefore in the limit $d \gg \min[L_\varphi, L_{H_0}]$ it is necessary to find the solution of Eq. (3) only in the case when $|\mathbf{r}| \ll d$. In this case we can expand the vector potential $\mathbf{A}(\mathbf{r}_f)$,

$$\mathbf{A}(\mathbf{r}_f) = \mathbf{A}\left(\mathbf{R} + \frac{\mathbf{r}}{2}\right) \approx \mathbf{A}(\mathbf{R}) + \frac{1}{2}\left(\mathbf{r}, \frac{\partial}{\partial \mathbf{R}}\right)\mathbf{A}(\mathbf{R}).$$

Then after introducing a modified Green's function

$$\tilde{C}(\mathbf{R},\mathbf{r}) = C(\mathbf{R},\mathbf{r}) \exp\left[-\frac{2ie}{\hbar c}[\mathbf{A}(\mathbf{R}),\mathbf{r}]\right]$$

one can obtain the following equation:

$$\left[\frac{\partial}{\partial t_f} + D\left(-i\frac{\partial}{\partial \mathbf{r}} - \frac{i}{2}\frac{\partial}{\partial \mathbf{R}} - \frac{2e}{\hbar c}\widetilde{\mathbf{A}}(\mathbf{R},\mathbf{r})\right)^2 + \frac{1}{\tau_\varphi}\right]\widetilde{C} = \exp\left[-\frac{2ie}{\hbar c}[\mathbf{A}(\mathbf{R}),\mathbf{r}]\right]\delta(\mathbf{r})\delta(t_f - t_i), \quad (7)$$

where

$$\widetilde{\mathbf{A}}(\mathbf{R},\mathbf{r}) = \frac{1}{2}[\widetilde{\mathbf{H}}(\mathbf{R}),\mathbf{r}], \quad \widetilde{\mathbf{H}}(\mathbf{R}) = \left[\frac{\partial}{\partial \mathbf{R}},\mathbf{A}(\mathbf{R})\right].$$

The right part of Eq. (7) contains $\delta(\mathbf{r})$, so we can set $\mathbf{r}=0$ in the exponential prefactor.

Note that in Eq. (7) one can neglect the term containing the derivative $\partial/\partial\mathbf{R}$. Indeed, it has the order d^{-1} whereas the value $\widetilde{\mathbf{A}}(\mathbf{R},\mathbf{r})$ and the term containing the derivative $\partial/\partial\mathbf{r}$ have the order $1/\min[L_\varphi, L_{H_0}]$, so the terms containing $\partial/\partial\mathbf{R}$ are negligible. In this case Eq. (7) takes the form

$$\left[\frac{\partial}{\partial t_f} + D\left(-i\frac{\partial}{\partial \mathbf{r}} - \frac{2e}{\hbar c}\widetilde{\mathbf{A}}(\mathbf{R},\mathbf{r})\right)^2 + \frac{1}{\tau_\varphi}\right]\widetilde{C} = \delta(t_f - t_i)\delta(\mathbf{r}). \quad (8)$$

Equation (8) formally coincides with Eq. (3) for the case of homogeneous magnetic field $\widetilde{\mathbf{H}}(\mathbf{R})$, which depends on the variable $\mathbf{R}$ as a parameter. Thus, in the gray region in Fig. 1 one can use the local approximation to calculate the weak-localization correction to the conductance. In what follows we show that in this case the spatially modulated magnetic field with zero average can cause the effect of positive magnetoresistance of 2DEG in the external homogeneous magnetic field.

Note that the local approximation breaks down at the points where the magnetic field is changing rapidly, i.e., near the zeros of the magnetic field. Nevertheless considering the spatially averaged conductance one can neglect the correction coming from these regions which appears to be small in the limit $L_\varphi/d \ll 1$.

III. WEAK MAGNETIC FIELDS: NEGATIVE MAGNETORESISTANCE

A. Magnetoresistance of 2DEG: Second-order perturbation theory

1. Quantum correction to the conductance in the field with arbitrary spatial configuration

Let us consider the case of magnetic field with arbitrary spatial configuration but with zero spatial average. In this section we find an analytical solution of Eq. (3) in the extreme case of low magnetic field. This means that the magnetic flux through any closed contour of the size $\min[L_\varphi, \sqrt{L_\varphi d}]$ is mush less than Φ_0. In this case the magnetic field weakly affects the weak-localization correction to the conductance, and Eq. (3) can be solved within the frames of the perturbation theory with a small parameter proportional to the value of magnetic field.

Let us introduce the Fourier transform of the magnetic field,

$$H_z(\mathbf{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H_{\mathbf{k}} e^{i\mathbf{kr}} d^2\mathbf{k}. \quad (9)$$

Here $\mathbf{r}$ is a vector in the plane of 2DEG. We assume all spatial harmonics $H_{\mathbf{k}}$ of magnetic field to be small.

The corresponding vector potential can be chosen in the form

$$\mathbf{A}(\mathbf{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{A}_{\mathbf{k}} e^{i\mathbf{kr}} d^2\mathbf{k}, \quad (10)$$

where

$$\mathbf{A}_{\mathbf{k}} = \frac{i[\mathbf{k},\mathbf{z}_0]}{k^2} H_{\mathbf{k}}. \quad (11)$$

In the zero order of the small parameter we considered the Green's function as the one without magnetic field,

$$C_0 = \frac{1}{4\pi D t_0}\exp\left(-\frac{r_0^2}{4Dt_0} - \frac{t_0}{\tau_\varphi}\right).$$

Here $\mathbf{r}_0 = \mathbf{r}_f - \mathbf{r}_i$, $t_0 = t_f - t_i$. Further we represent the operator in the left part of Eq. (3) as a sum of operators $\hat{F} = \hat{F}_0 + \hat{F}_1 + \hat{F}_2$ where

$$\hat{F}_0 = \left[\frac{\partial}{\partial t_f} - D\frac{\partial^2}{\partial \mathbf{r}_f^2} + \frac{1}{\tau_\varphi}\right],$$

$$\hat{F}_1 = \frac{4eD}{\hbar c}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{k} H_{\mathbf{k}} e^{i\mathbf{kr}_f}\left[\mathbf{k}, \frac{\partial}{\partial \mathbf{r}_f}\right],$$

$$\hat{F}_2 = \frac{4e^2 D}{\hbar^2 c^2}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{k}\frac{i[\mathbf{k},\mathbf{z}_0]}{k^2} H_{\mathbf{k}} e^{i\mathbf{kr}}\right]^2.$$

In the first order of perturbation theory the correction to the Green's function can be written as

$$C_1(\mathbf{r}_f,t_f,\mathbf{r}_i,t_i,) = -\int_{t_i}^{t_f}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} C_0(\mathbf{r}_f,t_f,\mathbf{r}',t') \times \hat{F}_1 C_0(\mathbf{r}',t',\mathbf{r}_i,t_i) d\mathbf{r}' dt'.$$

If $(x_f,y_f)\to(x_i,y_i)$ then $C_1=0$. This reflects the fact that the quantum correction does not depend on the sign of applied magnetic field. The second-order correction to the Green's function is defined by the expression

$$C_2 = -\int_{t_i}^{t_f}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} C_0(\mathbf{r}_f,t_f,\mathbf{r}',t') \times [\hat{F}_1 C_1(\mathbf{r}',t',\mathbf{r}_i,t_i) + \hat{F}_2 C_0(\mathbf{r}',t',\mathbf{r}_i,t_i)] d\mathbf{r}' dt'. \quad (12)$$

Let us introduce the value Δg_H,

$$\Delta g_H = \Delta g(\mathbf{B}) - \Delta g(0), \quad (13)$$

where the $\Delta g(\mathbf{B})$ is the weak-localization correction to the conductance in the inhomogeneous magnetic field $\mathbf{B}(\mathbf{r})$. Then the value Δg_H is determined by the second-order correction $C_2(\mathbf{r},t_0)$,

$$\Delta g_H(\mathbf{r}) = -\frac{2e^2D}{\pi\hbar}\int_0^\infty C_2(\mathbf{r},t_0)dt_0. \quad (14)$$

In expression (14) we set the lower integration limit equal to zero because of the absence of the small t_0 divergence in the integrand. Thus, we neglect the correction of the order τ/τ_φ.

As we are interested only in spatially averaged correction to the conductance expression (14) should be integrated over $\mathbf{r}$. Performing the integration in expression (12) we obtain the averaged correction to the conductance $\langle \Delta g_H\rangle$,

$$\langle \Delta g_H\rangle = \frac{8e^4D}{\hbar^3c^2S}\int_0^\infty dt_0 e^{-t_0/\tau_\varphi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{k}\frac{H_\mathbf{k}H_{-\mathbf{k}}}{k^2} \times\left[1-\frac{2}{k\sqrt{Dt_0}}e^{-k^2Dt_0/4}\Phi\left(\frac{k\sqrt{Dt_0}}{2}\right)\right], \quad (15)$$

where S is the area of the sample and $\Phi(\xi)=\int_0^\xi e^{t^2}dt$. Since we neglect the corrections proportional to τ/τ_φ the integration in expression (15) should be performed over $|\mathbf{k}| < (D\tau)^{-1/2}$.

Note that $H_{-\mathbf{k}}=H^*_\mathbf{k}$ and for $\alpha > 1$

$$\int_0^\infty e^{-\alpha x^2}\Phi(x)dx = \frac{1}{4\sqrt{\alpha}}\ln\left(\frac{\sqrt{\alpha}+1}{\sqrt{\alpha}-1}\right). \quad (16)$$

Then expression (15) can be rewritten in the form

$$\langle \Delta g_H\rangle = \frac{2e^4D^2\tau_\varphi^2}{\hbar^3c^2S}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{k}|H_\mathbf{k}|^2F\left(\frac{k^2D\tau_\varphi}{4}\right), \quad (17)$$

where

$$F(z) = \frac{1}{z}\left(1-\frac{1}{\sqrt{z(z+1)}}\ln(\sqrt{z+1}+\sqrt{z})\right). \quad (18)$$

With the increase in the z coordinate the function $F(z)$ is monotonically decreasing from the value 2/3 at $z=0$ to zero at $z=\infty$ decaying as z^{-1} at large z values (see Fig. 2). Therefore, the spatial harmonics of magnetic field with $|\mathbf{k}| \ll L_\varphi^{-1}$ make the main contribution to the weak-localization correction. In particular, for the case of magnetic field with narrow spectrum in the momentum space (the value $H_\mathbf{k}$ is nonzero only in the spectral region $|\mathbf{k}| \ll L_\varphi^{-1}$) the value $\langle \Delta g_H\rangle$ is defined by the expression

$$\langle \Delta g_H\rangle = \frac{4e^4D^2\tau_\varphi^2}{3\hbar^3c^2S}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|H_\mathbf{k}|^2d^2\mathbf{k}. \quad (19)$$

Using the properties of Fourier transformation, we can rewrite expression (19) in the form

$$\langle \Delta g_H\rangle = \frac{e^4D^2\tau_\varphi^2}{3\pi^2\hbar^3c^2S}\int_{S_0} H_z^2d^2\mathbf{r}. \quad (20)$$

It is seen from Eq. (20) that in the case of weak nonhomogeneous field with spatial scale larger than L_φ the averaged correction to conductance is defined only by the square of magnetic field averaged over the sample. This result corresponds to the local approximation.

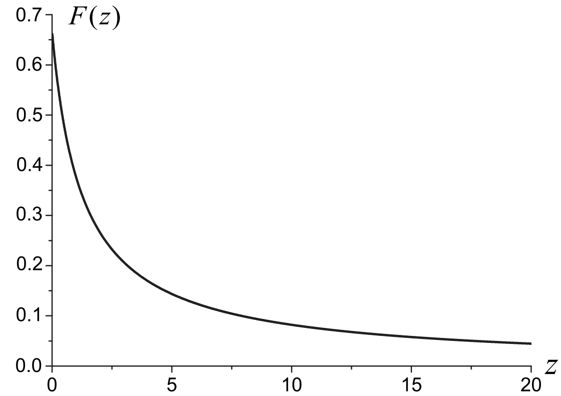

FIG. 2. The function $F(z)$ defined by expression (18).

Note that expression (15) is correct also in the case of magnetic field with nonzero spatial average H which satisfies the condition $HL_\varphi^2 \ll \Phi_0$. In this case the expression for the spatially averaged weak-localization correction to the conductance has the form

$$\langle g(H)\rangle = g_D - \frac{e^2}{2\pi^2\hbar}\ln\frac{\tau_\varphi}{\tau} + \langle \Delta g_H\rangle + \frac{e^2}{3\pi^2\hbar}\left(\frac{eHD\tau_\varphi}{\hbar c}\right)^2, \quad (21)$$

where $\langle \Delta g_H\rangle$ is defined by expression (17) for magnetic field with zero average. Thus the homogeneous component of the magnetic field makes small additional contribution to the averaged correction.

Expression (14) for the local conductance value can be further simplified for the particular case of one-dimensional field which depends on the x coordinate. In this case the magnetic field can be written in the form

$$H_z = \int_{-\infty}^{\infty} H_k e^{ikx}dk,$$

where k is the scalar Fourier variable. Performing integration in Eq. (12) we obtain an analytical expression for the Green's function $C_2(\mathbf{r})$,

$$C_2 = \frac{4e^2e^{-t_0/\tau_\varphi}}{\pi\hbar^2c^2(Dt_0)^{3/2}}\int_{-\infty}^{\infty}dk\int_{-\infty}^{\infty}dq\frac{H_kH_q}{k^2q^2(k+q)}e^{i(k+q)x} \times\left[e^{-(q+k)^2Dt_0/4}\Phi\left(\frac{(k+q)\sqrt{Dt_0}}{2}\right) - e^{-Dk^2t_0/4}\Phi\left(\frac{k\sqrt{Dt_0}}{2}\right) - e^{-q^2Dt_0/4}\Phi\left(\frac{q\sqrt{Dt_0}}{2}\right) + \frac{kqDt_0}{2}e^{-(k+q)^2Dt_0/4}\Phi\left(\frac{(k+q)\sqrt{Dt_0}}{2}\right)\right]. \quad (22)$$

This expression can be made even more transparent in the special case of the magnetic field with the sinusoidal profile.

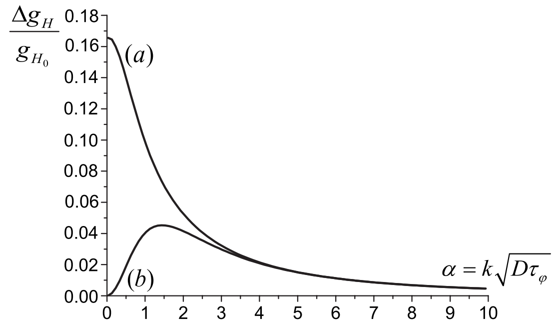

FIG. 3. The dependence of the value Δg_H vs parameter α for (a) $x=d/2$ and (b) $x=d/2$.

2. *Quantum correction to the conductance in low sinusoidal magnetic field*

Let the magnetic field has the form

$$H_z(x) = H_0 \cos(kx). \tag{23}$$

Then for the value Δg_H we obtain the following expression:

$$\Delta g_H = \frac{2e^4 H_0^2}{\pi^2 \hbar^3 c^2 k^4} \int_0^\infty e^{-\beta^2/k^2 D\tau_\varphi} \left\{ \beta - 2e^{-\beta^2/4} \Phi\left(\frac{\beta}{2}\right) + \cos(2kx) \frac{e^{-\beta^2}}{\beta^2} \left[4e^{(3/4)\beta^2} \Phi\left(\frac{\beta}{2}\right) - (\beta^2+2)\Phi(\beta) \right] \right\} d\beta. \tag{24}$$

One can see that the only dimensionless parameter $\alpha = k\sqrt{D\tau_\varphi} = \pi L_\varphi / d$ defines the value of the integral in expression (24).

It is interesting to consider the dependence of the value Δg_H on the period of magnetic field d. The dependencies of the value Δg_H on the parameter $\alpha = \pi L_\varphi / d$ for two interesting cases $x=0$ and $x=d/2$ are shown in Fig. 3, where we introduced the value

$$g_{H_0} = \frac{2e^2}{\hbar} \left(\frac{H_0 D \tau_\varphi}{\Phi_0} \right)^2.$$

Note that the value $\Delta g_H(\mathbf{r})$ is defined by the module of the averaged magnetic flux through all possible closed trajectories of the size L_φ which are passing through the point $\mathbf{r}$. In the maxima of the magnetic field ($x=nd$; n is an integer) the averaged flux is decreasing with d decreasing (increasing α) and this leads to the decrease in the Δg_H value. At the zeros of the magnetic field ($x=d/2+nd$) when the scales d and L_φ are comparable one can observe a maximum in the dependence Δg_H vs α [see curve (b) in Fig. 3]. In this case the averaged module of the magnetic flux through the closed trajectories is maximal.

The analysis of the expression for Δg_H shows that in the limit of $d \ll L_\varphi$ the value Δg_H is proportional to d^2,

$$\Delta g_H \approx \frac{e^2}{\pi^2 \hbar} \left(\frac{L_\varphi}{L_{H_0}} \right)^4 \left(\frac{d}{L_\varphi} \right)^2,$$

where $L_{H_0} = \sqrt{\Phi_0 / H_0}$.

In case of smooth field variation ($d \gg L_\varphi$, but $H_0 d L_\varphi \ll \Phi_0$) keeping the corrections $\propto \alpha^2$ we find

$$\Delta g_H = \frac{e^2 L_\varphi^4}{6\hbar L_{H_0}^4} \left((1+\cos 2kx) - \frac{\alpha^2}{5}(1 + 7\cos 2kx) \right). \tag{25}$$

Expression (25) differs strongly from the correspondent expression for the case of homogeneous field since even at the points of zero magnetic field the value Δg_H is positive. This fact is quite natural since the averaged module of the flux through the closed trajectories does not vanish even at these points.

B. Dephasing time in spatially periodic magnetic fields

Expressions (17) and (21) obtained within the perturbation theory diverge at low temperatures as the dephasing time τ_φ tends to infinity. Thus, to describe the behavior of the conductance at low temperatures one should take account of the renormalization of the dephasing time caused by the magnetic field.

In this section we consider such renormalization procedure for a periodic magnetic field

$$H_z(\mathbf{r} + \mathbf{R}_n) = H_z(\mathbf{r}), \tag{26}$$

where $\mathbf{R}_n$ are translational vectors which generate a two-dimensional lattice.

The expression for the electron probability of return obtained from the solution of Eq. (3) can be written in the following form:

$$W(\mathbf{r}, t_0) = e^{-t_0/\tau_\varphi} \sum_j |\psi_j(\mathbf{r})|^2 e^{-\varepsilon_j D t_0}. \tag{27}$$

Here ε_j and $\psi_j(\mathbf{r})$ are the eigenvalues and normalized eigenfunctions of the operator

$$\hat{H}(\mathbf{r}) = \left(-i\nabla - \frac{2e}{\hbar c}\mathbf{A}(\mathbf{r}) \right)^2, \tag{28}$$

$$\hat{H}(\mathbf{r})\psi_j(\mathbf{r}) = \varepsilon_j \psi_j(\mathbf{r}),$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \psi_j(\mathbf{r})\psi_{j'}^*(\mathbf{r}) d^2\mathbf{r} = \delta_{j,j'}. \tag{29}$$

Substituting expression (27) into Eq. (1) we find the quantum correction to the conductance

$$\Delta g(\mathbf{r}) = -\frac{2e^2}{\pi\hbar} \sum_j \frac{|\psi_j(\mathbf{r})|^2}{\varepsilon_j + \frac{1}{D\tau_\varphi}} e^{-D\tau(\varepsilon_j + 1/D\tau_\varphi)}. \tag{30}$$

Thus, the correction to the conductance is defined only by the spectrum and by the set of eigenfunctions of the operator $\hat{H}(\mathbf{r})$. Further we will be interested only in spatially averaged quantum correction to the conductance which defines the voltage between the sample contacts. Then taking into account the normalization condition for eigenfunctions we obtain

$$\langle \Delta g \rangle = -\frac{2e^2}{\pi\hbar}\sum_j \frac{e^{-D\tau(\varepsilon_j+1/D\tau_\varphi)}}{\varepsilon_j+\dfrac{1}{D\tau_\varphi}}. \tag{31}$$

To calculate the spectrum of the operator $\hat{H}(\mathbf{r})$ let us expand the magnetic field into the Fourier series

$$B_z(\mathbf{r}) = H + \sum_{\mathbf{b}_n\neq 0} H_n e^{i\mathbf{b}_n\mathbf{r}}. \tag{32}$$

We start our analysis from the case of magnetic field with zero spatial average, i.e., we set $H=0$. The corresponding vector potential can also be written in the form of Fourier series

$$\mathbf{A}(\mathbf{r}) = \sum_{\mathbf{b}_n\neq 0} \mathbf{A}_n e^{i\mathbf{b}_n\mathbf{r}}. \tag{33}$$

Choosing the vector potential in the Lorentz gauge div $\mathbf{A}=0$ so that

$$\mathbf{A}_n = i\frac{[\mathbf{b}_n,\mathbf{H}_n]}{\mathbf{b}_n^2}, \tag{34}$$

we obtain the following expression for the operator $\hat{H}(\mathbf{r})$:

$$\hat{H}(\mathbf{r}) = -\nabla^2 - \frac{4e}{\hbar c}\sum_{\mathbf{b}_n\neq 0} H_n e^{i\mathbf{b}_n\mathbf{r}}\alpha_n\nabla + \frac{4e^2}{\hbar^2 c^2}\sum_{\mathbf{b}_n\neq 0} Q_n e^{i\mathbf{b}_n\mathbf{r}} + \frac{4e^2}{\hbar^2 c^2}\sum_{\mathbf{b}_n\neq 0}\frac{|H_n|^2}{\mathbf{b}_n^2}, \tag{35}$$

where

$$\alpha_n = \frac{[\mathbf{b}_n,\mathbf{z}_0]}{\mathbf{b}_n^2}, \tag{36}$$

$$Q_n = \sum_{m\neq 0, m\neq n}\frac{H_m H_{n-m}}{(\mathbf{b}_n-\mathbf{b}_m)^2}\left(1-\frac{(\mathbf{b}_n,\mathbf{b}_m)}{\mathbf{b}_m^2}\right). \tag{37}$$

The Hamiltonian is translationally invariant $[\hat{H}(\mathbf{r}+\mathbf{R}_n)=\hat{H}(\mathbf{r})]$ and its eigenfunctions satisfy the *Bloch* theorem,

$$\psi_{\mathbf{k}}(\mathbf{r}) = \sum_{\mathbf{b}_n} u_n e^{i(\mathbf{k}+\mathbf{b}_n)\mathbf{r}}. \tag{38}$$

Substituting expression (38) into Eq. (29) and introducing the value

$$\varepsilon' = \varepsilon - \frac{4e^2}{\hbar^2 c^2}\sum_{\mathbf{b}_n\neq 0}\frac{|H_n|^2}{\mathbf{b}_n^2}, \tag{39}$$

we find the following equation for the amplitudes u_n:

$$[(\mathbf{k}+\mathbf{b}_n)^2-\varepsilon']u_n + \frac{4e^2}{\hbar^2 c^2}\sum_{\mathbf{b}_m\neq 0} Q_m u_{n-m} - \frac{4ie}{\hbar c}\sum_{\mathbf{b}_m\neq 0} u_{n-m}H_m\alpha_m(\mathbf{k}+\mathbf{b}_n-\mathbf{b}_m) = 0. \tag{40}$$

In the absence of the magnetic field only the amplitude u_n corresponding to $n=0$ is nonzero ($u_{n=0}\neq 0$ and $u_{n\neq 0}=0$). Therefore, in this case the spectrum has the form $\varepsilon(\mathbf{k})=\mathbf{k}^2$.

Further in this section we consider the solution of Eq. (40) in the nearly free electron approximation, which means that we will restrict ourselves to the case of low amplitude of the magnetic field so that $L_{H_0}\gg d$ ($L_{H_0}=\sqrt{\Phi_0/H_0}$ and d is the characteristic scale of the magnetic field inhomogeneity). Moreover we will focus mainly in the effect of the weak periodic magnetic field on the zero-temperature divergence of the weak-localization correction to the conductance [see Eq. (4)]. So we will consider the case of low temperatures when $L_\varphi\gg d$ and will take into account only the corrections to the argument of the logarithm in expression (4) as these corrections dominate in the zero-temperature limit. The condition of the nearly free electron approximation applicability and the assumption of low temperature correspond to the white square in Fig. 1.

As it is seen from Eq. (31), the zero-temperature divergence of the conductance correction comes from the region of low ε which corresponds to the region of low $|\mathbf{k}|$. Taking into account the condition $d\ll L_{H_0}$ we will assume that the region $|\mathbf{k}|\ll|\mathbf{b}_n|$ gives the main contribution to the low temperature correction. In this case the spectrum of the operator $\hat{H}(\mathbf{r})$ can be calculated in the nearly free electron approximation.

In the presence of magnetic field the spectrum can be written in the form $\varepsilon'(\mathbf{k})=\mathbf{k}^2+\varepsilon^{(1)}(\mathbf{k})+\varepsilon^{(2)}(\mathbf{k})$, where $\varepsilon^{(1)}$ is proportional to the field amplitude and $\varepsilon^{(2)}$ is proportional to the square of the field's amplitude.

The first-order correction to the zero field spectrum is equal to the second term in expression (39) but with the opposite sign. Thus, the correction $\varepsilon^{(1)}$ in the spectrum ε' is zero. The second-order correction $\varepsilon^{(2)}$ has the following form:

$$\varepsilon^{(2)} = -\frac{16e^2}{\hbar^2 c^2}\sum_{\mathbf{b}_n\neq 0}\frac{|H_n|^2(\alpha_n,\mathbf{k})^2}{\mathbf{b}_n^2}. \tag{41}$$

Thus, the spectrum reads

$$\varepsilon(\mathbf{k}) = \mathbf{k}^2 - \frac{16e^2}{\hbar^2 c^2}\sum_{\mathbf{b}_n\neq 0}\frac{|H_n|^2(\mathbf{k},[\mathbf{b}_n,\mathbf{z}_0])^2}{\mathbf{b}_n^6} + \frac{4e^2}{\hbar^2 c^2}\sum_{\mathbf{b}_n\neq 0}\frac{|H_n|^2}{\mathbf{b}_n^2}. \tag{42}$$

The second term in Eq. (42) is of the order of $k^2(d/L_{H_0})^4 \ll k^2$ and leads to renormalization of the "effective mass" in a quadratic spectrum $\varepsilon(\mathbf{k})\propto k^2$. This change in the effective mass does not affect the zero-temperature divergence of the weak-localization correction to the conductance and further will be neglected. Thus, the resulting spectrum has the form

$$\varepsilon(\mathbf{k}) = \mathbf{k}^2 + \frac{4e^2}{\hbar^2 c^2}\sum_{\mathbf{b}_n\neq 0}\frac{|H_n|^2}{\mathbf{b}_n^2}. \tag{43}$$

Note that the second term in Eq. (43) makes an important contribution to the dephasing time. The effective dephasing time has the form

$$\frac{1}{\tau_B} = \frac{1}{\tau_\varphi} + \frac{4e^2 D}{\hbar^2 c^2} \sum_{\mathbf{b}_n \neq 0} \frac{|H_n|^2}{\mathbf{b}_n^2}. \tag{44}$$

From expression (44) one can see that for arbitrarily small magnetic field the low temperature divergence of quantum correction to the conductance is cut off by the finite effective dephasing time τ_B. The corresponding expression for quantum correction to the conductance reads

$$\langle \Delta g \rangle \approx \frac{e^2}{2\pi^2 \hbar} \ln \left[\frac{\tau}{\tau_\varphi} + \frac{4e^2 D\tau}{\hbar^2 c^2} \sum_{\mathbf{b}_n \neq 0} \frac{|H_n|^2}{\mathbf{b}_n^2} \right]. \tag{45}$$

The logarithmic term dominates in the weak-localization correction which allows us to consider the contribution of magnetic field only in the argument of logarithmic function, and we neglect small additional corrections to expression (45), which are also caused by the magnetic field.

In the limit of zero temperature the value τ_B^{-1} is proportional to the square of amplitude of magnetic field in a sharp contrast to the case of homogeneous field where $t_H^{-1} \sim H$. This analytical result is in agreement with the qualitative estimate obtained in Ref. 9.

Now we proceed with the analysis of the case of periodic magnetic field with nonzero but small spatial average (i.e., $H \neq 0$ and $HS_0 \ll \Phi_0$, S_0 is the area of the unit cell defined by the basic vectors $\mathbf{a}_1$ and $\mathbf{a}_2$) at low temperatures when $L_\varphi \gg d$. As previously we assume here that $L_{H_0} \gg d$ to use the nearly free electron approximation. The vector potential is given by

$$\mathbf{A}(\mathbf{r}) = \frac{1}{2}[\mathbf{H}, \mathbf{r}] + \sum_{\mathbf{b}_n \neq 0} \mathbf{A}_n e^{i\mathbf{b}_n \mathbf{r}}, \tag{46}$$

where $\mathbf{H} = H\mathbf{z}_0$. To obtain the spectrum of the operator $\hat{H}(\mathbf{r})$ we can first admit that the vector potential $\mathbf{A}_0(\mathbf{r}) = \frac{1}{2}[\mathbf{H}, \mathbf{r}]$ corresponding to the homogeneous component H of magnetic field is constant on the characteristic scales of a periodic magnetic field.[23] The spectrum of the operator $\hat{H}(\mathbf{r})$ has the following form [see Eq. (43)]:

$$\varepsilon(\mathbf{k}) = \left(\mathbf{k} - \frac{2e}{\hbar c} \mathbf{A}_0 \right)^2 + \frac{4e^2}{\hbar^2 c^2} \sum_{\mathbf{b}_n \neq 0} \frac{|H_n|^2}{\mathbf{b}_n^2}. \tag{47}$$

Proceeding with the analysis in the momentum space one needs to restore the commutation relations for the components of quasimomentum $\mathbf{k}$ and the components of the radius vector operator $\hat{\mathbf{r}} = i\partial/\partial \mathbf{k}$. Then spectrum (47) transforms into a new effective operator, which can be reduced to the harmonic oscillator Hamiltonian. The spectrum of this effective Hamiltonian has the form

$$\varepsilon_m = \frac{2eH}{\hbar c}(2m+1) + \frac{4e^2}{\hbar^2 c^2} \sum_{\mathbf{b}_n \neq 0} \frac{|H_n|^2}{\mathbf{b}_n^2}, \tag{48}$$

where m is a nonzero integer number. Note that in expression (48) we neglect the renormalization of the effective mass in a way similar to the derivation of spectrum (43).

Finally carrying out the summation over m in Eq. (31) we obtain

$$\langle \Delta g \rangle = -\frac{e^2}{2\pi^2 \hbar} \left\{ \psi\left(\frac{1}{2} + \frac{\hbar c}{4eHD\tau} \right) - \psi\left(\frac{1}{2} + \frac{\hbar c}{4eHD\tau_B} \right) \right\}, \tag{49}$$

where the value τ_B is defined by expression (44). Expanding expression (49) in the limit $HD\tau_B/\Phi_0 \ll 1$, we obtain

$$\langle \Delta g \rangle \approx -\frac{e^2}{2\pi^2 \hbar} \ln\left(\frac{\tau_B}{\tau} \right) + \frac{e^2}{3\pi^2 \hbar} \left(\frac{eHD\tau_B}{\hbar c} \right)^2. \tag{50}$$

Expressions (49) and (50) formally coincide with the ones for the homogeneous field, but in a modulated magnetic field the dephasing time τ_B is determined by the amplitude of modulation.

IV. STRONG MAGNETIC FIELDS: POSITIVE MAGNETORESISTANCE

We now proceed with the consideration of the strong field limit and focus on the possibility to change the sign of magnetoresistance of 2DEG in the presence of a modulated magnetic field. Specifically, we consider a ferromagnetic film/2DEG system placed in the external magnetic field perpendicular to the 2DEG plane. Let the ferromagnetic film have a periodic stripe domain structure. We will assume that the film of 2DEG is thin enough to consider only the z component H_z of magnetic field depending only on x coordinate along the sample surface. We will denote the external field as H and the absolute value of the periodic field of stripe structure by H_0. Further the description of the weak-localization correction to the conductance will be developed on the basis of local approximation. This approximation is correct when the electron dephasing length is less than the characteristic scale of inhomogeneous magnetic field. These conditions mean that $d \gg \min\{L_\varphi, L_B\}$ (the gray region in Fig. 1).

Note that the effect of positive magnetoresistance can be observed only in the region of parameters where the local approximation is applicable. Indeed, Eqs. (21) and (50) show that in the opposite limit the second derivative $\langle \Delta g(H) \rangle$ at $H=0$ is positive and, as a result, the magnetoresistance is negative.

The effect of positive magnetoresistance strongly depends on the magnetic field configuration. We assume for simplicity the thickness of ferromagnetic layer to exceed strongly the period of stripe structure d. In this case for hybrid ferromagnetic/2DEG structure the spatial configuration of magnetic field in the region of 2DEG depends mostly on the thickness of the spacer between 2DEG and ferromagnetic film. If the spacer is much thinner than the period of stripe structure d then the distribution of the z-component $H_z(\mathbf{r})$ of the magnetic field in 2DEG approximately has the form of meander (see Fig. 4). In the opposite case, when the spacer thickness is much larger than the spatial period of the domain structure, the magnetic field profile is smeared. On a qualitative level one can describe this limit considering a sinusoidal field profile.

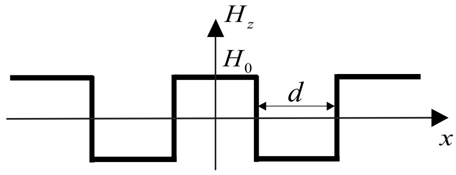

FIG. 4. The profile of the z component of magnetic field in the form of meander.

A. Periodic magnetic field in the form of meander

The periodic magnetic field in the form of meander

$$H_z(x) = H_0 \,\mathrm{sgn}[\cos(\pi x/d)]$$

is the simplest configuration of magnetic field which reveals the effect of positive magnetoresistance. The external homogeneous magnetic field applied to the system leads to the suppression of weak localization in the regions where the sign of the external field coincides with the one of the periodic field components. In opposite, the external field results in the increase of the interference corrections in the domains where the signs of these field components are different. The competition between these two effects defines the resulting dependence of the averaged conductance vs an external homogeneous field. If the increase in the weak-localization correction dominates then the resulting dependence of averaged conductance vs external homogeneous magnetic field is decreasing. In this case one can conclude that 2DEG has positive magnetoresistance.

Let us search for the region of parameters, where the effect of positive magnetoresistance can be observed. Within the local approximation the averaged conductance is defined by the following expression:

$$\langle g_m(h)\rangle = \tfrac{1}{2}[g(h+h_0) + g(h-h_0)], \tag{51}$$

where $h_0 = 4eH_0 D\tau_\varphi/\hbar c$,

$$g(H) = g_D - \frac{e^2}{2\pi^2\hbar}\left\{\psi\left(\frac{1}{2} + \frac{\hbar c}{4e|H|D\tau}\right) - \psi\left(\frac{1}{2} + \frac{\hbar c}{4e|H|D\tau_\varphi}\right)\right\},$$

and g_D is the Drude conductance. Introducing dimensionless variables we obtain

$$g(h) = g_D - g_0\left\{\psi\left(\frac{1}{2} + \frac{\xi}{|h|}\right) - \psi\left(\frac{1}{2} + \frac{1}{|h|}\right)\right\}, \tag{52}$$

where $g_0 = e^2/2\pi^2\hbar$, $\xi = \tau_\varphi/\tau$, and $h = 4eHD\tau_\varphi/\hbar c$. Expanding expression (51) into the Taylor's series for $h \ll h_0$ we find

$$\langle g_m(h)\rangle \approx g(h_0) + \tfrac{1}{2} g''(h_0) h^2.$$

One can see that the positive magnetoresistance is realized for h_0, which satisfies the condition

$$g''(h_0) < 0. \tag{53}$$

This condition is realized when the amplitude of periodic magnetic field is larger than some critical value h_c, which depends on the parameter ξ. The dependence $h_c(\xi)$ for the meander configuration of periodic magnetic field is shown in Fig. 5(a). For $\xi \gg 1$ the boundary of the positive magnetoresistance region is defined by the condition $h_c \approx 3$.

These conclusions are based on Eq. (3) and, thus, are valid only in the diffusive limit. The domain of applicability of the diffusive approximation is defined by the condition $L_B \gg l$, where l is an elastic scattering length. In the limit $\lambda_F \ll L_B \ll l$ (where λ_F is the Fermi wavelength) the weak localization is fully suppressed and the conductance approaches the Drude value.

The dependencies of the averaged conductance of 2DEG vs the external homogeneous field at different amplitudes of periodic field are shown in Fig. 6(a) for $\xi = 100$. One can see that for $h_0 \gg h_c(\xi)$ these dependencies have the sharp dips with minima at $h = h_0$. For high modulated amplitude of the magnetic fields our results should be correct only near the dips because far from the dips the condition $L_B \gg l$ of the diffusive limit can be broken.

B. Periodic magnetic field in the form of cosine

As a second example we consider the effect of positive magnetoresistance in the sinusoidal profile of the z component of the magnetic field

$$H_z(x) = H_0 \cos(\pi x/d).$$

The expression for the averaged conductance does not depend on the period of magnetic field d and has the following form:

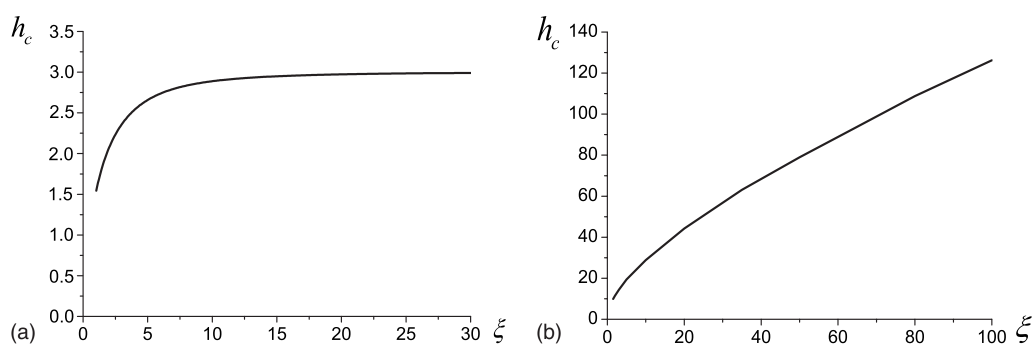

FIG. 5. The dependencies of the critical magnetic field amplitude h_c, which separates the regions of positive and negative magnetoresistances vs the parameter $\xi = \tau_\varphi/\tau$. (a) meander field profile; (b) sinusoidal field profile.

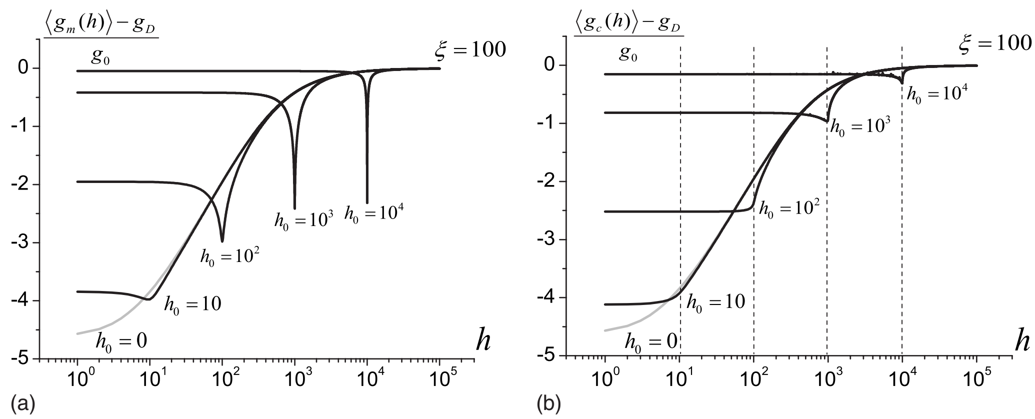

FIG. 6. The averaged conductance of 2DEG vs the external homogeneous field at different amplitudes of the periodic field h_0 and at $\xi=100$: (a) meander field profile; (b) sinusoidal field profile.

$$\langle g_c(h)\rangle = g_D - g_0\int_0^1 \left\{\psi\left(\frac{1}{2}+\frac{\xi}{|h+h_0\cos(\pi\rho)|}\right) - \psi\left(\frac{1}{2}+\frac{1}{|h+h_0\cos(\pi\rho)|}\right)\right\}d\rho. \qquad (54)$$

Here $h_0=4eH_0D\tau_\varphi/\hbar c$. The set of dependencies of $\langle g_c(h)\rangle$ for different magnitudes of amplitude h_0 is shown in Fig. 6(b). From the comparison between Figs. 6(a) and 6(b) one can see that in periodic magnetic field with sinusoidal profile the effect of positive magnetoresistance is weaker than in the case of meander profile. This is caused by the fact that for the sinusoidal profile the regions where the external and periodic fields have opposite directions shrink with the external field increasing.

In Fig. 6(b) the amplitude of the periodic magnetic field is shown by the vertical dotted line. One can see that for $h \approx h_0$ the behavior of $\langle g_c(h)\rangle$ changes qualitatively. Even for high amplitude of the periodic magnetic field h_0 in the region $h < h_0$ the conductance deviates from the Drude value. This is caused by the incomplete destruction of the interference near the field zero points even at rather high h_0 values.

Expression (54) allows us to find the condition of positive magnetoresistance in combined cosinusoidal and homogeneous magnetic fields. The magnetoresistance peak appears provided

$$\left.\frac{\partial^2 g_c(h)}{\partial h^2}\right|_{h=0} < 0,$$

which gives us the condition

$$\int_0^1 g''(h_0\beta)\frac{d\beta}{\sqrt{1-\beta^2}} < 0. \qquad (55)$$

Here the function g is defined by expression (52). Condition (55) is satisfied when the periodic magnetic field amplitude h_0 is larger than the critical value $h_c(\xi)$. The dependence $h_c(\xi)$ for sinusoidal profile of periodic magnetic field is shown in Fig. 5(b). One can observe a clear difference between two model profiles: contrary to the meander case the critical field diverges at large ξ values.

V. SUMMARY

To sum up, we have investigated the influence of inhomogeneous magnetic fields on the weak-localization phenomenon in 2DEG systems. In the low field limit we have carried out a perturbative analysis of the conductance behavior at high temperatures and developed an analytical procedure to find a renormalization of the dephasing rate at low temperatures. In the high field limit we have justified the validity of the local approximation and have used this approach to calculate the averaged conductance for particular model field profiles. It is found that the systems with modulated magnetic field profiles provide a possibility to observe the effect of positive magnetoresistance. We have showed that the positive magnetoresistance in ferromagnetic film/2DEG systems can be observed experimentally provided the amplitude of the field modulation exceeds a certain critical value depending on the system parameters.

Finally, we consider some estimates for typical 2DEG systems and define the regions of parameters in which the above effects can be observed experimentally.

Speaking about an experimentally realizable source of inhomogeneous magnetic field we keep in mind plain multilayer ferromagnetic films with domain structure. In this case the inhomogeneous distribution of the magnetic field is caused by the magnetic domains (such systems have been experimentally created, for example, in Ref. 26).

Deposition of magnetic stripes is another possibility to create a periodic magnetic field in 2DEG. Note that such stripes can induce the modulated electric field which can change the local carrier density in 2DEG. This fact should result in the modulation of the diffusive constant D. Applying our results for the description of the weak-localization phenomenon in such systems one should generalize the calculations in the spirit of Ref. 27, where the authors theoretically considered the effect of homogeneous electric field on the weak localization. Experimentally the effect of electric field has been observed in Ref. 28.

Considering the effect of positive magnetoresistance one can see that the conditions of its observation are $d \gg L_{H_0}$ (d is the period of the magnetic field and $L_{H_0}=\sqrt{\Phi_0/H_0}$) and

$L_\varphi \gg L_{H_0}$ (for low temperatures when $L_\varphi \gg l$). For typical amplitudes of the modulated magnetic field $H_0 \sim 10-10^3$ Oe induced by ferromagnetic multilayer films (see, for example, Ref. 2) the scale d of the magnetic field should be larger than 10^2-10^3 nm. Thus, the above criterion $d \gg L_{H_0}$ can be easily fulfilled for domain structures in typical experimental systems (see Ref. 2). The scale d for the specific sample can be experimentally estimated on the basis of the magnetic force microscopy measurements. To describe the weak-localization phenomenon in systems with such length scales of the magnetic field one can use the local approximation and all results of Sec. IV should be valid. The criterion $L_\varphi \gg L_{H_0}$ can be fulfilled at low temperatures (the value L_φ also should be greater than 10^2-10^3 nm). The estimates show that for the meander distribution of the magnetic field with amplitudes $H_0 \sim 10^2-10^3$ Oe the height of the magnetoresistance peak can reach one half of the weak-localization correction in zero field.

At low temperatures when $L_{H_0}^2/L_\varphi < d < L_{H_0}$ ($d \sim 10$ nm, $L_\varphi > 1-10$ μm) one can observe the effect of renormalization of the dephasing time. In this case the conductance of 2DEG is defined by expression (49).

Note that the diffusive approximation, which has been always exploited in our calculations, is valid for magnetic fields $B \ll \Phi_0/l^2$. In thin disordered films of Mg, Cu, Pd, etc., the typical value of the elastic length l is of the order $l \sim 1-10$ nm (see, for example, Refs. 29–31), and for such 2DEG systems the diffusive approximation is correct for magnetic fields, which are less than 10^5-10^7 Oe.

For high-mobility samples on the basis of GaAs/AlGaAs heterostructures (see, for example, Ref. 25) the range of validity for the diffusive approximation is narrower. In particular, for the samples with the electron mobility ($\mu > 1$ m^2 V^{-1} s^{-1}) the diffusive approximation is valid for magnetic fields, which are less than 10 Oe. Still even at such weak magnetic fields the effect of positive magnetoresistance can be observed. Indeed the height of the magnetoresistance peak depends on the ratio between the amplitude H_0 of the modulated magnetic field and the value Φ_0/l^2 (the height of the peak is large when $H_0 \gg \Phi_0/l^2$). The inelastic length L_φ strongly depends on temperature, thus, cooling the sample one can make the value Φ_0/l^2 much less than 1 Oe (see, for example, Ref. 25). Thus even in high-mobility samples the effect of positive magnetoresistance can be observed.

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