Robust states in semiconductor quantum dot molecules

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Semiconductor quantum dots coherently driven by pulsed laser are fundamental physical systems which allow studying the dynamical properties of confined quantum states. These systems are attractive candidates for a solid-state qubit, which open the possibility for several investigations in quantum-information processing. In this work we study the effects of a specific decoherence process, the spontaneous emission of excitonic states, in a quantum dot molecule. We model our system considering a three-level Hamiltonian and solve the corresponding master equation in the Lindblad form. Our results show that the spontaneous emission associated with the direct exciton helps to build up a robust indirect exciton state. This robustness against decoherence allows potential applications in quantum memories and quantum gate architectures. We further investigate several regimes of physical parameters, showing that this process is easily controlled by tuning of external fields.

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I. INTRODUCTION

The advance on the manipulation and dynamical control of quantum states under the action of coherent radiation has recently become a subject of intense research in condensedmatter physics. Dynamical control is a necessary step for the implementation of any protocol associated with quantuminformation processing.¹ In this sense, semiconductor quantum dots (ODs) have been proved to be an ideal candidate. Using strong resonant laser pulses and different probe techniques, several different groups have successfully demonstrated coherent manipulation of the exciton population of a single QD.²⁻⁷ They demonstrated a process known as Rabi oscillation which is indeed a proof of the exciton qubit rotation. Unlike atoms, however, QDs suffer from unavoidable variation in their size, and the presence of a surrounding environment with which they may interact strongly, making the entire system to lose its phase quickly.⁸⁻¹⁰ The main interest in QDs arises from their characteristic discrete-energy spectrum, and its great flexibility in change it, not only by manipulation of their geometric structure but also with the application of external gates. A natural next step for the development of such system is to put two quantum dots together and allowing them to couple. A lot of work has been done in this direction, where beautiful examples of a molecule formation have been achieved.¹¹⁻²² However, the coherent dynamics of such objects under strong laser pumping remains largely unexplored experimentally, and our work can give further insight to help the experimental development.

In this paper, we study the effects of the spontaneous decay in the excitonic states of a self-assembled semiconductor quantum dot molecule (QDM) coupled by tunneling and under the influence of an external electromagnetic field. We use a standard density-matrix approach in the Lindblad form to describe the system dynamics and our results indicate that the spontaneous decay of the direct exciton helps to build up a coherent population of the indirect exciton (electron and hole in different dots), which has a longer lifetime due to its spatial separation with small overlap of the wave function. This effect is robust to the changes in external parameters

and in order to describe it we describe the physical system and the detailed theoretical model in Sec. II, then in Sec. III we show the results of numerical calculations, followed by our conclusions in Sec. IV.

II. THEORY AND MODEL

The physical system we consider here is an asymmetric double quantum dot coupled by tunneling. Electrons and holes can be confined in either dot and we can use a nearresonant optical pulse to promote electrons from the valence to the conduction band, creating an electron-hole correlated state known as exciton. Electrons or holes can then tunnel to the other dot, creating an indirect exciton. An external electric field, applied in the growth direction, brings the individual levels of electrons or holes into resonance, favoring the tunneling. Nevertheless, in asymmetric QDM structures it is even possible to control which type of carrier, electron or holes, tunnels.²³ In this situation, we can safely neglect the tunneling of holes as the electric field brings one level (conduction band) more into resonance while makes the other (valence band) more out of resonance. With this assumption, the dynamics of the QDM can be modeled by a simple threelevel system, where the ground state $|0\rangle$ is a molecule without any excitation, $|1\rangle$ is the system with one exciton in the left dot, while $|2\rangle$ is the system with one indirect exciton, after the electron has tunneled. The schematic configuration of levels and physical parameters are shown in the Fig. 1, where we also include the decoherence channels associated with spontaneous emission of excitonic states (Γ_0^1, Γ_0^2) .

Using the rotating wave approximation and dipole approximation, the system Hamiltonian is written as^{24}

$$\begin{split} \hat{H}(t) &= \sum_{j=0}^{2} \hbar \omega_{j} |j\rangle \langle j| + T_{e}(|1\rangle \langle 2| + |2\rangle \langle 1|) + \hbar \Omega(e^{i\omega_{L}t}|0\rangle \langle 1| \\ &+ e^{-i\omega_{L}t} |1\rangle \langle 0|), \end{split}$$
(1)

where ω_j are the frequencies of $|j\rangle$ th states (j=0,1,2), T_e is the tunneling coupling, ω_L is the frequency of the applied



FIG. 1. Scheme of energy levels with physical parameters on Hamiltonian (1).

laser, and the dipole coupling is $\Omega = \langle 0 | \vec{\mu} \cdot \vec{E} | 1 \rangle / 2\hbar$, where $\vec{\mu}$ is the electric dipole moment and \vec{E} is the amplitude of incident field. The intensity of incident field can be easily controlled to provide available conditions for the coherent control of the system quantum state. We also assume a low-intensity incident pulse, so the Rabi frequency is significatively smaller than the intraband excitation energy,²⁵ $\Omega \ll \omega_{10} = \omega_1 - \omega_0$ and $\omega_L \approx \omega_{10}$. Under this assumption, we might consider that only the ground-state exciton can be formed in our system.

Applying the unitary transformation²⁴

$$\hat{U} = \exp\left[\frac{i\omega_L t}{2}(|1\rangle\langle 1| - |0\rangle\langle 0| + |2\rangle\langle 2|)\right]$$
(2)

and using the *Baker-Hausdorff lemma*,²⁶ we obtain a timeindependent version of Hamiltonian (1) written as follows:

$$\hat{H}' = \frac{1}{2} \begin{pmatrix} -\delta_1 & 2\hbar\Omega & 0\\ 2\hbar\Omega & \delta_1 & 2T_e\\ 0 & 2T_e & \delta_2 \end{pmatrix},$$
(3)

where $\delta_1 = \hbar(\omega_{10} - \omega_L)$ is the detuning between the frequency of optical pulse and exciton transition, $\delta_2 = \delta_1 + 2\hbar\omega_{21}$ and ω_{ij} is the optical transition between *i* and *j* energy states.

To taking into account the effects of decoherence, we used the Liouville-Von Neuman-Lindblad equation given by^{27}

$$\frac{\partial \hat{\rho}(t)}{\partial t} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}(t)] + \hat{L}(\hat{\rho}).$$
(4)

Here, $\hat{\rho}(t)$ is the density-matrix operator. The Liouville operator, $\hat{L}(\hat{\rho})$, describes the dissipative process. Assuming the Markovian approximation, Liouville operator can be written as²⁸

$$\hat{L}(\hat{\rho}) = \frac{1}{2} \sum_{i} \Gamma_{j}^{i}(2|j\rangle\langle i|\hat{\rho}|i\rangle\langle j| - \hat{\rho}|i\rangle\langle i| - |i\rangle\langle i|\hat{\rho}), \qquad (5)$$

where Γ_j^i corresponds to the decoherence rates due spontaneous decay from the state $|i\rangle$ to the state $|j\rangle$. In order to investigate the dynamics associated with this physical system, we solve the Eq. (4), and found the density-matrix coefficients at certain time *t*. Writing Eq. (4) in the basis defined by $|0\rangle$, $|1\rangle$, and $|2\rangle$ states, we obtain a set of nine coupled linear differential equations written as

$$\begin{split} \dot{\rho}_{00} &= -i\Omega(\rho_{10} - \rho_{01}) + \Gamma_{0}^{1}\rho_{11} + \Gamma_{0}^{2}\rho_{22}, \\ \dot{\rho}_{01} &= \frac{i}{\hbar} \Big[\delta_{1}\rho_{01} + \hbar\Omega(\rho_{00} - \rho_{11}) + T_{e}\rho_{02} \Big] - \frac{1}{2}\Gamma_{0}^{1}\rho_{01}, \\ \dot{\rho}_{02} &= \frac{i}{\hbar} \Big[\frac{\rho_{02}}{2} (\delta_{1} + \delta_{2}) - \hbar\Omega\rho_{12} + T_{e}\rho_{01} \Big] - \frac{1}{2}\Gamma_{0}^{2}\rho_{02}, \\ \dot{\rho}_{10} &= \frac{i}{\hbar} \Big[- \delta_{1}\rho_{10} + \hbar\Omega(\rho_{11} - \rho_{00}) - T_{e}\rho_{20} \Big] - \frac{1}{2}\Gamma_{0}^{1}\rho_{10}, \\ \dot{\rho}_{11} &= \frac{i}{\hbar} \Big[\hbar\Omega(\rho_{10} - \rho_{01}) + T_{e}(\rho_{12} - \rho_{21}) \Big] - \Gamma_{0}^{1}\rho_{11}, \\ \dot{\rho}_{12} &= \frac{i}{\hbar} \Big[\frac{\rho_{12}}{2} (\delta_{2} - \delta_{1}) - \hbar\Omega\rho_{02} + T_{e}(\rho_{11} - \rho_{22}) \Big] \\ &\quad - \frac{1}{2} (\Gamma_{0}^{1} + \Gamma_{0}^{2})\rho_{12}, \\ \dot{\rho}_{20} &= \frac{i}{\hbar} \Big[- \frac{\rho_{20}}{2} (\delta_{2} + \delta_{1}) + \hbar\Omega\rho_{21} - T_{e}\rho_{10} \Big] - \frac{1}{2}\Gamma_{0}^{2}\rho_{20}, \\ \dot{\rho}_{21} &= \frac{i}{\hbar} \Big[\frac{\rho_{21}}{2} (\delta_{1} - \delta_{2}) + \hbar\Omega\rho_{20} + T_{e}(\rho_{22} - \rho_{11}) \Big] \\ &\quad - \frac{1}{2} (\Gamma_{0}^{1} + \Gamma_{0}^{2})\rho_{21}, \\ \dot{\rho}_{21} &= \frac{i}{\hbar} \Big[\frac{\rho_{21}}{2} (\delta_{1} - \delta_{2}) + \hbar\Omega\rho_{20} + T_{e}(\rho_{22} - \rho_{11}) \Big] \\ &\quad - \frac{1}{2} (\Gamma_{0}^{1} + \Gamma_{0}^{2})\rho_{21}, \\ \dot{\rho}_{21} &= \frac{i}{\hbar} \Big[\frac{\rho_{21}}{2} (\delta_{1} - \delta_{2}) + \hbar\Omega\rho_{20} + T_{e}(\rho_{22} - \rho_{11}) \Big] \\ &\quad - \frac{1}{2} (\Gamma_{0}^{1} + \Gamma_{0}^{2})\rho_{21}, \\ \dot{\rho}_{21} &= \frac{i}{\hbar} \Big[\frac{\rho_{21}}{2} (\Gamma_{0}^{1} + \Gamma_{0}^{2})\rho_{21}, \\ \dot{\rho}_{21} &= \frac{i}{\hbar} \Big[\frac{\rho_{21}}{2} (\Gamma_{0}^{1} + \Gamma_{0}^{2})\rho_{21} \Big] \\ \dot{\rho}_{21} &= \frac{i}{\hbar} \Big[\frac{\rho_{21}}{2} (\Gamma_{0}^{1} + \Gamma_{0}^{2})\rho_{21} \Big] \\ \dot{\rho}_{21} &= \frac{i}{\hbar} \Big[\frac{\rho_{21}}{2} (\Gamma_{0}^{1} + \Gamma_{0}^{2})\rho_{21} \Big]$$

$$\dot{\rho}_{22} = \frac{i}{\hbar} T_e(\rho_{21} - \rho_{12}) - \Gamma_0^2 \rho_{22}.$$
(6)

In order to solve the set of Eqs. (6), we rewrite as $\dot{\rho} = A\rho$, considering ρ as a column vector and A being a square matrix associated with the coefficients of the coupled system above. The solution can be written as

$$\rho_{ij}(t) = \sum_{j=0}^{8} S_{ij} e^{\lambda_j t} [S_{ij}^{-1} \rho_{ij}(0)], \qquad (7)$$

where, λ_j and S_{ij} are the eigenvalues and the matrix formed by the eigenvectors of matrix A, respectively. $\rho_{ij}(0)$ are the elements of the density-matrix operator at t=0.

III. RESULTS AND DISCUSSION

For our calculations, we consider the following values of physical parameters: $\hbar\omega_{10} \approx 1.6 \text{ eV},^{3,29} \quad \Omega \approx 0.05$ -1.0 meV,^{25,30} $\Gamma_0^1 \approx 0.33 - 6.6 \mu \text{eV},^{30,31}$ and $\Gamma_0^2 \approx 10^{-4} \Gamma_0^{1.32}$ The tunneling coupling, which depends on the barrier characteristics and the external electric field, was selected as: $T_e \approx 0.01 - 0.1 \text{ meV}$ (Ref. 33) or $T_e \approx 1 - 10 \text{ meV},^{34}$ for weak and strong tunneling regimes, respectively. The system dynamics depends also from the detunings δ_1 and δ_2 . Experimentally, δ_1 is controlled by varying the frequency of external laser. The value of δ_2 is changed by varying δ_1 and the



FIG. 2. Dynamics of populations, P_i , of level $|i\rangle$ (i=0,1,2) as function of time for different choices of spontaneous emission rate Γ_0^1 with parameters $\delta_1=0$, $\omega_{21}=0$, $\Omega=50$ μ eV, and $T_e=10$ μ eV. We use gray line for P_0 , light gray line for P_1 , and black line for P_2 . (a) Nondissipative dynamics ($\Gamma_0^1=0$); (b) $\Gamma_0^1=1.0$ μ eV; (c) $\Gamma_0^1=2.0$ μ eV; and (d) $\Gamma_0^1=4.4$ μ eV.

frequency transition ω_{21} , which can be done by manipulation of external electric field that changes the effective confinement potential. By varying this set of parameters we are able to perform a coherent manipulation of the wave function of the system. For all simulations, we consider $|\Psi(0)\rangle = |0\rangle$ as initial condition.

Our first task is to analyze the effect on population dynamics of the decoherence process associated with spontaneous emission of direct exciton. In Fig. 2, we plot the probability of occupation associated with each of the three levels considering different values of spontaneous emission rate Γ_0^1 . For the physical parameters considered here, the nondissipative dynamics ($\Gamma_0^1=0$) shows that there are Rabi oscillations between the three levels of the system. This is illustrated in Fig. 2(a). The population of indirect exciton, state $|2\rangle$, depends directly on the parameter T_e , although the value of the coupling Ω and detunings δ_1 and ω_{21} , has important effects on dynamics.²⁴ The situation changes when spontaneous emission is taken into account. As we expected, the Rabi oscillations become damped. This can be seen in Figs. 2(b)–2(d). For long times and values of Γ_0^1 high enough, as shown in the inset of Fig. 2(d), the Rabi oscillations are suppressed and the electron wave function tends to an asymptotic state.

Now we focus our attention on the formation of a stationary state with high population of indirect excitonic level, $|2\rangle$. With a lifetime significatively longer (about 10⁴ times the direct exciton),³² this particular state shows more potential for quantum-information processing than the direct exciton, $|1\rangle$ state. In order to study the effects of several physical parameters on Hamiltonian (1) and the decoherence, we study the behavior of average occupation of state $|2\rangle$, defined as



FIG. 3. (Color online) Average population of state $|2\rangle$, $\overline{P_2}$, as function of detuning δ_1 and frequency ω_{21} for $\Omega = 50 \ \mu eV$, $T_e = 10 \ \mu eV$, and $\Gamma_0^2 \approx 10^{-4}\Gamma_0^1$. (a) $\Gamma_0^1 = 4.4 \ \mu eV$. Inset: $\overline{P_2}$ for $\Gamma_0^1 = 0$ and the same values of Ω and T_e . (b) $\Gamma_0^1 = 0.66 \ \mu eV$.

$$\overline{P_2} = \frac{1}{t_{\infty}} \int_0^{t_{\infty}} P_2(t) dt.$$

In Fig. 3, we plot our results for P_2 , as function of laser detuning δ_1 and frequency ω_{21} , considering two different values of the direct exciton spontaneous emission rate, Γ_0^1 . Bright colors are associated with high values of P_2 , which means an efficient transference of the electron from the first to the second dot. From our results, it is possible to conclude that a large occupation probability of $|2\rangle$ is obtained if the detuning δ_1 is balanced with the applied electric field so that $\delta_1 + \omega_{21} \simeq 0$. We will named this condition as balanced detuning. The behavior considering full resonance between the three levels $(\delta_1 \simeq \omega_{21} \simeq 0)$ deserves more attention. Let us define an area associated with the full resonance condition $|\delta_1|, |\omega_{21}| \leq 50 \ \mu eV$: when spontaneous emission is not considered ($\Gamma_0^1=0$), the average population $\overline{P_2}$ is near to zero, as shown in Ref. 24 and in the inset of Fig. 3(a). Thus, full resonance condition is not a good experimental choice for an optimal creation of indirect excitonic state. Considering the effects of spontaneous emission Γ_0^1 , we can observe a differ-ent behavior: the values of $\overline{P_2}$ at point $(\omega_{21}, \delta_1) = (0, 0)$ in-crease from 0.05 (for $\Gamma_0^1 = 0$) to $\simeq 0.6$ (for $\Gamma_0^1 = 4.4 \ \mu eV$) and $\simeq 0.2$ (for $\Gamma_0^1 = 0.66 \ \mu eV$). This shows that for realistic direct excitons, with a nonzero spontaneous emission rate, the transfer of the electron between dots is more efficient.

In Fig. 4, we show our results for average population, P_2 , as a function of both, frequency ω_{21} and dipole coupling Ω , for different choices of Γ_0^1 considering $\delta_1=0$ µeV, Figs. 4(a), 4(c), and 4(e), and $\delta_1 = 100 \ \mu eV$, Figs. 4(b), 4(d), and 4(f). For all cases, we are able to populate the indirect exciton state, evidenced by bright regions with values of P_2 larger than 0.3. At resonance condition, Figs. 4(a), 4(c), and 4(e), this bright area have a V-like form, with higher values of P_2 concentrated on a small area associated with low values of Ω and ω_{21} . For nonresonant condition, the symmetry between negative and positive values of ω_{21} is broken. Still, the large values of P_2 are obtained when the condition δ_1 $+\omega_{21} \simeq 0$ is fulfilled. The action of spontaneous emission can be analyzed by comparing the different situations shown in Fig. 4. Higher values of parameter $\underline{\Gamma}_0^1$ are connected with higher values of average population P_2 . For example, in Fig. 4(a) when $\Gamma_0^1=0$ the maximum value of $P_2 \simeq 0.36$. Consid-



FIG. 4. (Color online) Average population of state $|2\rangle$, P_2 , as function of coupling parameter Ω and frequency ω_{21} for resonant and nonresonant conditions. (a) $\Gamma_0^1=0$ and $\delta_1=0$; (b) $\Gamma_0^1=0$ and $\delta_1=100 \ \mu\text{eV}$; (c) $\Gamma_0^1=0.66 \ \mu\text{eV}$ and $\delta_1=0$; (d) $\Gamma_0^1=0.66 \ \mu\text{eV}$ and $\delta_1=0$; (d) $\Gamma_0^1=0.66 \ \mu\text{eV}$ and $\delta_1=100 \ \mu\text{eV}$; (e) $\Gamma_0^1=4.4 \ \mu\text{eV}$ and $\delta_1=0$; and (f) $\Gamma_0^1=4.4 \ \mu\text{eV}$ and $\delta_1=100 \ \mu\text{eV}$. In all cases, $T_e=10 \ \mu\text{eV}$ and $\Gamma_0^2\approx 10^{-4}\Gamma_0^{1}$.

ering decoherence, the maximum value of $\overline{P_2}$ goes from 0.6 for $\Gamma_0^1 = 0.66 \ \mu eV$ [Fig. 4(c)] to ≈ 0.8 for $\Gamma_0^1 = 4.4 \ \mu eV$ [Fig. 4(d)]. Also, the total area for highly efficient population of $|2\rangle$ state increase as spontaneous emission increase: both, the arms of the characteristic **V** area and the region with best values of $\overline{P_2}$ become progressively large when the value of Γ_0^1 increase.

It is useful to check the combined effect of both, the tunneling and decoherence. It is expected a good transfer of population associated with higher values of \underline{T}_e parameter. This can be verified by comparing the results P_2 without the effect of decoherence process with $T_e=10 \ \mu\text{eV}$, Fig. 4(a), with the results considering a higher value of tunneling parameter $T_e=50 \ \mu\text{eV}$, Fig. 5(a). The effect of decoherence process is <u>illustrated</u> by Fig. 5(b). Notice that area in Fig. 5 with high P_2 increase by the action of decoherence and the maximum value of P_2 goes from ≈ 0.4 , in Fig. 5(a), to P_2 ≈ 0.8 , in Fig. 5(b).

After our analysis of P_2 , it is necessary to check the actual behavior of level population P_2 . In Fig. 6, we plot P_2 considering some choices of physical parameters associated with our previous analysis (Figs. 2–5). In all cases, we limit ourselves to full resonance condition ($\delta_1 \approx \omega_{21} \approx 0$). When dynamics is associated with stationary states, the value of P_2 depends on two aspects: the final value of P_2 at stationary state and the time needed to reach this maximum value. In



FIG. 5. (Color online) Average occupation of state $|2\rangle$, P_2 , as function of coupling parameter Ω and frequency ω_{21} considering $T_e=50 \ \mu\text{eV}$, $\delta_1=0$, and $\Gamma_0^2 \approx 10^{-4}\Gamma_0^1$. (a) $\Gamma_0^1=0$ and (b) $\Gamma_0^1=4.4 \ \mu\text{eV}$.

Fig. 6(a) we plot P_2 for $\Omega = 50 \ \mu eV$ and $T_e = 10 \ \mu eV$ considering different values of Γ_0^1 . We can conclude that a higher spontaneous emission rate of the direct exciton is connected with a faster evolution to the asymptotic value of P_2 . That means, the broadening effects (short lifetime) on the direct exciton are advantageous if we are interested on manipulate electronic wave function in order to create an asymptotic state with high values of P_2 at short times.

Next, we verify that the exact maximum value of P_2 is related with coupling parameters Ω and T_e and, also, with the balanced detuning condition $\delta_1 + \omega_{21} \approx 0$. From our calculations of average occupation of indirect excitonic state, we analyze the behavior of P_2 considering a set of Ω and T_e parameters associated with maximum values of P_2 for two different Γ_0^1 rates.

The evolution of population P_2 is shown in Fig. 6(b) for two different values of $\Gamma_0^1 = 4.4 \ \mu eV(\Gamma_0^1 = 0.66 \ \mu eV)$ situation. For a fixed value of Γ_0^1 , we can define the characteristic time t_0 as the time at which the system reaches the asymptotic value of P_2 (for example, $t_0 \approx 14$ ns for Γ_0^1 =0.66 μeV). This characteristic time, which depends on the value of Γ_0^1 , allows us to distinguish two dynamical regimes:



FIG. 6. (Color online) Population of indirect exciton state, P_2 , as function of time at stationary regime. (a) P_2 for different values of spontaneous emission rate Γ_0^1 considering full resonance condition (δ_1 =0, ω_{21} =0) for coupling parameters Ω =50 μ eV and T_e =10 μ eV. (b) P_2 associated with the physical parameters for the highest values of P_2 founded in Figs. 4 and 5.

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(1) for long time, $t \ge t_0$, the population is essentially independent of time and its maximum value depends directly on the Ω/T_e rate value (the population increases when this rate increases). (2) at short times, $t < t_0$, the dynamics does not depend on the value of Ω/T_e , being governed mainly by Γ_0^1 . Comparing all cases plotted in Fig. 6(b) we can conclude that the condition to obtain an asymptotic state with large values of the occupation P_2 , associated with short characteristic times t_0 , is given by $\frac{\Omega}{T_e} \simeq \frac{T_e}{\Gamma_0^1}$. Thus, it is possible to obtain experimentally optimized values of P_2 , by adjusting appropriately the laser intensity Ω for fixed values of T_e and Γ_0^1 , which, in turn, can be obtained through optical spectroscopy.

IV. SUMMARY

In this work, we use a standard density-matrix approach in the Lindblad form to model the dynamics of a quantum dot molecule under the influences of external electric and electromagnetic fields, and in the presence of spontaneous emission. By numerically solving the density matrix we show that the spontaneous decay of the direct exciton helps to build up a coherent population of the indirect exciton, which should have important applications in quantuminformation processing due to its longer coherence time.

We further investigate the efficiency of creation of indirect exciton state as function of physical parameters of our model. For weak spontaneous emission rate, the system presents a Rabi oscillation and in the opposite limit the system rapidly build up a stationary population of the indirect exciton. Our results shown that the population of the indirect exciton is strongly influenced by the spontaneous emission of the direct exciton. We demonstrate that the indirect exciton, which has a longer lifetime, is robust against the spontaneous emission process. Finally, at maximum average population conditions, we determined a relation between the relevant parameters of the system, which allows us to obtain large populations of indirect exciton $P_2 \approx 0.9$.

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