Magnetostatic fields in planar assemblies of magnetic nanoparticles

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Magnetostatic fields are studied in the model of a planar assembly of magnetic particles forming a periodic lattice infinite in the xy plane and finite along the z axis. Two types of lattices are considered: simple cubic and body-centered cubic. Exact analytical expressions of the magnetostatic fields for the magnetization perpendicular and parallel to the xy plane, $H_{\perp}(x,y,z)$ and $H_{\parallel}(x,y,z)$, are obtained in the form of Fourier series both inside the particles, and in the surrounding matrix. Exact formulas for mean magnetostatic fields inside the particles, $\bar{H}_{\perp}$ and $\bar{H}_{\parallel}$, are obtained by averaging over the particle volume. A numerical analysis of these formulas shows linear dependences of $\bar{H}_{\perp}$ and $\bar{H}_{\parallel}$ on the volume filling factor c. A comparison of these exact solutions with the results of earlier works where such dependences were obtained using different approximate approaches, confirms the correctness of some of them. The correctness of methods allowing the estimation of the filling factor c from both magnetic resonance and magnetometric data is also confirmed.

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I. INTRODUCTION

Assemblies of ferromagnetic nanoparticles embedded in nonmagnetic matrices attract much attention because their properties are significantly different from those observed in bulk ferromagnets and interesting for applications. The properties of substances containing magnetic nanoparticles result from both intrinsic particle properties and interparticle interactions. The dipolar interaction has an important role in determining fundamental properties of magnetic nanoparticle assemblies such as superparamagnetic relaxation, energy barriers, blocking temperature, as well as their behavior in external magnetic fields.

A number of numerical calculations and computer simulations concerned with these problems have been carried out (e.g., see Refs. 1–7). In particular, the dipolar interaction between spherical particles as well as between particles with shape anisotropy (for instance, ellipsoidal particles) has been considered. In the case of uniform orientation of the particle easy axes, an average anisotropy arises in a sample because of the particle shape anisotropy. The limiting case for this situation is the magnetic needle (wire) structure directed along one of the coordinate axes. For a periodic lattice of infinitely long magnetic needles of rectangular cross section the problem of magnetostatic field has been solved exactly.[8,9] The dipolar interaction between two particles of arbitrary shape has been considered.[10] In the case of random distribution of the particle easy axes for both the shape and crystalline anisotropy an isotropic magnetization loop occurs. The shape, remanence, and coercivity of this loop are described by the classical Stoner-Wohlfarth theory[11] in the absence of dipolar interaction. The latter interaction results in narrowing of the loop which still remains isotropic at the condition of the isotropic matrix.

The actual ferromagnetic nanoparticle systems are very complex for a theoretical analysis because of both stochastic spatial arrangement of the particles and random orientations of their magnetic anisotropy axes. Therefore, as a rule, model particle assemblies used in calculations are infinite in all directions which allows to avoid additional difficulties due to surface demagnetizing fields. However, in some important cases the sample shape greatly effects physical phenomena which are due to the dipolar interaction. In particular, this is the case of ferromagnetic resonance fields and magnetization curves in planar assemblies of magnetic particles. For such samples the external field can be applied in the plane of the sample (parallel resonance or parallel magnetization curve, respectively) or perpendicular to the plane (perpendicular resonance or perpendicular magnetization curve). In a ferromagnetic resonance study of particles in discontinuous magnetic films, Netzelmann[12] found that the difference between the perpendicular and parallel resonance fields, $H_{o\perp}$ and $H_{o\parallel}$, was considerably less than for homogeneous magnetic films. To explain these experimental result, Netzelmann suggested to approximate the effective magnetostatic energy density F_M by a sum of two terms corresponding to limiting cases of the volume filling factor of magnetic particles c, the first one, corresponding to $c \to 0$ being the magnetostatic energy of an isolated particle and the second one, corresponding to $c=1$ being the energy of homogeneous magnetic film. Assuming the transition between these limiting cases to be linear in c, he obtained a simple expression for F_M by introducing the factors $c-1$ and c before the first and second terms of the sum, respectively. Comparing with the experimental data the expressions for the resonance magnetic fields $H_{o\perp}(c)$ and $H_{o\parallel}(c)$ obtained from the latter formula, Netzelmann proposed an original method to determine c from the difference between $H_{o\perp}$ and $H_{o\parallel}$. Later on, Netzelmann's formula for the magnetostatic energy and his method of evaluating c have been used by many authors, e.g., see Refs. 13 and 14.

Dubowik[15] considered the magnetostatic energy and shape anisotropy for an ellipsoidal matrix containing magnetic particles regularly spaced forming a three-dimensional array. Using the mean-field approach he obtained approximate expressions containing both macroscopic and microscopic demagnetizing tensors and considered particular cases of these expressions for magnetic multilayers, granular films, etc. For the granular films, see erratum to Ref. 15, he obtained an expression for the magnetostatic energy coinciding with the Netzelmann's formula. Thus, the theory developed in the mean-field approximation confirmed the Netzelmann's simple interpolation considerations.

Recent experimental studies of nanoparticle planar assemblies, such as magnetic particles dispersed in nonmagnetic films or implanted at the surface of bulk materials[16–19] show that in all cases the resonance fields as well as the magnetization curves in parallel and perpendicular magnetic fields possess a shape anisotropy depending on the filling factor c. Meanwhile, new approaches to describing this anisotropy emerge; for instance, the authors of Ref. 20, seemingly unaware of the earlier results,[12–15] put forward approximate expressions for the magnetostatic field which differ from those given in Refs. 12–15 (this issue will be discussed in more detail in Sec. IV of the present paper).

The above overview shows the necessity of developing an exact theory of dipolar interactions in some appropriate planar assembly of nanoparticles. Such a theory would offer the possibility of validating the results of different approximate approaches (or of their limiting cases) as well as to describe more correct experimental data obtained for analogs planar assemblies. Besides, this theory gives expressions for magnetostatic fields not only inside the particles but also in the interparticle space, what would be of interest for the studies of magnetic field effects on the electron-transport properties in the matrix.

The aim of the present paper is to give an exact solution of the magnetostatic problem for the model of cubic shape particles located at the nodes of a three-dimensional periodic lattice of finite size along the z axis and infinite in the xy plane. This model is a generalization of the two-dimensional model proposed in Ref. 8 for the study of magnetostatic fields in the system of infinitely long wires (needles). The paper is arranged as follows. In Sec. II analytical expressions of magnetostatic fields in a periodic nanoparticle monolayer are obtained. In Sec. III two types of lattices are considered: simple cubic (sc) and body-centered cubic (bcc), and the corresponding analytical expressions for the magnetostatic fields are found. In Sec. IV a numerical analysis of these analytical formulas is carried out. In Sec. V the results are summarized and compared with previous approximate approaches.

II. MONOLAYER MODEL

Consider one layer of ferromagnetic particles of cubic shape with the edge length $2a$. The particles are arranged periodically with the same period T along the x and y axes (Fig. 1). We denote by $\psi(x,y,z)$, $\psi^{+}(x,y,z)$, and $\psi^{-}(x,y,z)$ the magnetostatic potential inside the layer $(-a \le z \le a)$, above $(z > a)$, and below the layer $(z < -a)$, respectively. We consider two cases: all particles are magnetized homogeneously either along the z axis (the magnetization **M** is perpendicular to the layer) or along the x axis (**M** is parallel to the layer).

Perpendicular orientation of the magnetization. In this case the magnetostatic potential both inside and outside the layer is described by Laplace's equations

$$\nabla^2 \psi = 0, \quad \nabla^2 \psi^{\pm} = 0, \tag{1}$$

and the boundary conditions have the form

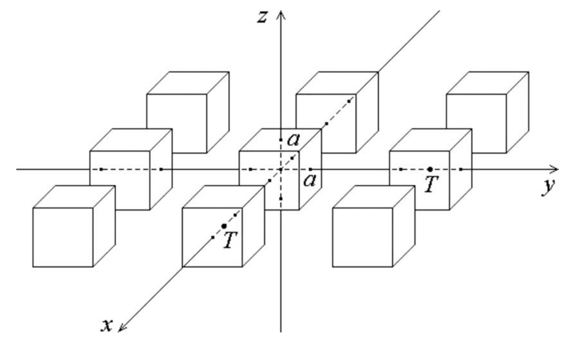

FIG. 1. The monolayer model of particles arrangement.

$$\frac{\partial \psi}{\partial z} - \frac{\partial \psi^{\pm}}{\partial z} = 4\pi M_z \Big|_{z=\pm a}. \tag{2}$$

We represent the potentials, ψ and $\psi^{\pm}$, and the magnetization projections M_z as two-dimensional Fourier series

$$\psi(x,y,z) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \psi_{nm}(z) e^{iq(nx+my)},$$

$$\psi^{\pm}(x,y,z) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \psi_{nm}^{\pm}(z) e^{iq(nx+my)},$$

$$M_z(x,y,z) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} M_{nm}(z) e^{iq(nx+my)}, \tag{3}$$

where $q = 2\pi/T$.

The equations and boundary conditions for the Fourier transforms ψ_{nm} and $\psi_{nm}^{\pm}$ take the form

$$\frac{d^2\psi_{nm}}{dz^2} - (n^2+m^2)q^2\psi_{nm} = 0,$$

$$\frac{d^2\psi_{nm}^{\pm}}{dz^2} - (n^2+m^2)q^2\psi_{nm}^{\pm} = 0, \tag{4}$$

$$\psi_{nm} = \psi_{nm}^{\pm}\Big|_{z=\pm a}, \quad \frac{d\psi_{nm}}{dz} - \frac{d\psi_{nm}^{\pm}}{dz} = 4\pi M_{nm}\Big|_{z=\pm a}. \tag{5}$$

We obtain the magnetization Fourier transform M_{nm} for the structure shown in Fig. 1 as

$$M_{nm} = \frac{M}{\pi^2 nm} \sin qna \sin qma. \tag{6}$$

We search the solutions of Eq. (4) in the form

$$\psi_{nm} = a_{nm}e^{pqz} + b_{nm}e^{-pqz}, \quad \psi_{nm}^{\pm} = a_{nm}^{\pm}e^{\mp pqz}, \tag{7}$$

where $p \equiv p_{nm} = \sqrt{n^2+m^2}$.

Determining the arbitrary constants a_{nm}, b_{nm}, and $a_{nm}^{\pm}$ from Eqs. (5) we obtain

$$\psi_{nm} = \frac{4\pi M_{nm}}{pq} e^{-pqa} \sinh pqz,$$

$$\psi_{nm}^{\pm} = \pm \frac{4\pi M_{nm}}{pq} \sinh pqa e^{\mp pqz}. \quad (8)$$

Substituting Eq. (8) in Eq. (3) and taking into account Eq. (6) give explicit expressions for the potentials $\psi(x,y,z)$ and $\psi^{\pm}(x,y,z)$ in the entire space. Correspondingly, the magnetic fields both inside and outside the particles in the layer are determined from these potentials through the equations,

$$\mathbf{H} = -\nabla\psi, \quad \mathbf{H}^{\pm} = -\nabla\psi^{\pm}. \quad (9)$$

Only the z component of the magnetostatic fields in the case of $\mathbf{M}$ perpendicular to the layer will be of interest in this paper. From Eq. (9) we obtain for this component inside the layer (at $-a \le z \le a$)

$$H_z(x,y,z) = -4\pi \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} M_{nm} e^{-pqa} \cosh pqz e^{iq(nx+my)} \quad (10)$$

and outside the layer (at $z \ge a$, and $z \le -a$, respectively)

$$H_z^{\pm}(x,y,z) = 4\pi \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} M_{nm} \sinh pqa e^{\mp pqz} e^{iq(nx+my)}. \quad (11)$$

By averaging the field $H_z(x,y,z)$ over the particle volume, $V=(2a)^3$,

$$\bar{H}_z = \frac{1}{V} \int_V H_z(x,y,z) dx dy dz, \quad (12)$$

we obtain the mean magnetostatic field inside each particle induced by all particles of the layer and including the demagnetizing field of the particle:

$$\frac{\bar{H}_z}{4\pi M} = \xi^2 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{\sin^2 n\varphi}{(n\varphi)^2} \frac{\sin^2 m\varphi}{(m\varphi)^2} \frac{\sinh p\varphi}{p\varphi} e^{-p\varphi}, \quad (13)$$

where $\xi = 2a/T$ and $\varphi = \pi\xi$.

Parallel orientation of the magnetization. In this case the magnetostatic potential is described by the Poisson equation inside the layer and by the Laplace's equation outside the layer. We suppose that the magnetization vector $\mathbf{M}$ is directed along the x axis in all particles. The equations and boundary conditions for the magnetostatic potential have the form, respectively,

$$\nabla^2\psi = 4\pi \frac{\partial M_x}{\partial x}, \quad \nabla^2\psi^{\pm} = 0, \quad (14)$$

$$\psi^{\pm} = \psi|_{z=\pm a}, \quad \frac{\partial\psi}{\partial z} = \left.\frac{\partial\psi^{\pm}}{\partial z}\right|_{z=\pm a}. \quad (15)$$

For the Fourier transforms of the potential we obtain

$$\frac{d^2\psi_{nm}}{dz^2} - p^2q^2\psi_{nm} = 4\pi i q n M_{nm},$$

$$\frac{d^2\psi_{nm}^{\pm}}{dz^2} - p^2q^2\psi_{nm} = 0, \quad (16)$$

where the Fourier transform M_{nm} of the M_x projection is determined by Eq. (6), as in the previous case. The boundary conditions for the Fourier transforms have the same form as those for the potential [Eqs. (15)]. We obtain the solutions of Eq. (16), satisfying these boundary conditions,

$$\psi_{nm} = -4\pi M_{nm} \frac{in}{p^2q}(1 - e^{-pqa} \cosh pqz),$$

$$\psi_{nm}^{\pm} = -4\pi M_{nm} \frac{in}{p^2q} \sinh pqa e^{\mp pqz}. \quad (17)$$

Substituting Eq. (17) in Eq. (3) and taking into account Eq. (6) give explicit expressions for the potentials $\psi(x,y,z)$ and $\psi^{\pm}(x,y,z)$ in the entire space. The magnetostatic fields in the entire space are determined by Eq. (9). Only the x component of the magnetostatic field will be of interest in the case of $\mathbf{M}$ parallel to the layer. From Eq. (9) we obtain for this component inside the layer

$$H_x(x,y,z) = -4\pi \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} M_{nm} \frac{n^2}{n^2+m^2} \times [1 - e^{-pqa} \cosh pqz] e^{iq(nx+my)} \quad (18)$$

and outside the layer

$$H_x^{\pm}(x,y,z) = -4\pi \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} M_{nm} \frac{n^2}{n^2+m^2} \times \sinh pqa e^{\mp pqz} e^{iq(nx+my)}. \quad (19)$$

By averaging the field $H_x(x,y,z)$ over the particle volume, $V=(2a)^3$, we obtain the mean magnetostatic field inside each particle induced by all particles of the layer and including the demagnetizing field of the particle:

$$\frac{\bar{H}_x}{4\pi M} = -\xi^2 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{n^2}{n^2+m^2} \frac{\sin^2 n\varphi}{(n\varphi)^2} \frac{\sin^2 m\varphi}{(m\varphi)^2} \times \left(1 - \frac{\sinh p\varphi}{p\varphi} e^{-p\varphi}\right). \quad (20)$$

III. MULTILAYER MODELS

By applying the superposition principle, the expressions for the magnetostatic fields inside the layer $\mathbf{H}$ and outside the layer $\mathbf{H}^{\pm}$ obtained in the previous section, can be used to calculate the magnetostatic fields for different models including a finite number of layers. We consider two such models, viz., a sc and a bcc lattice of the magnetic particles.

A. Simple-cubic lattice

Consider the model of magnetic particles arranged in a sc lattice (Fig. 2). We calculate the sum of magnetic fields inside a particle of the initial layer induced by all other par-

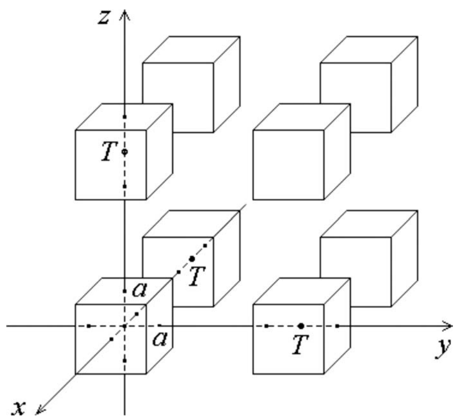

FIG. 2. The simple-cubic particle lattice.

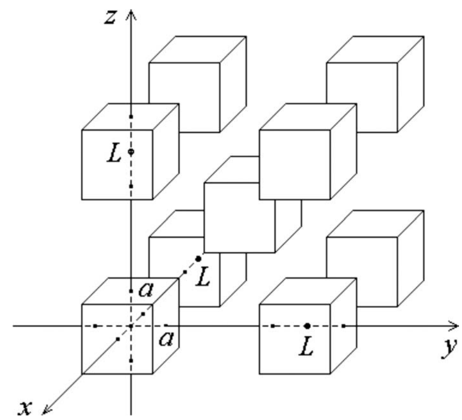

FIG. 3. The body-centered-cubic particle lattice.

ticles of the same layer as well as by the layers situated above and below this layer. We suppose that the interlayer distances are multiples of a period T. As in Sec. II, we consider the orientations of the magnetization $\mathbf{M}$ perpendicular and parallel to the layers.

Perpendicular orientation of the magnetization. Consider the kth layer situated above the initial layer at the distance kT, where $k=1,2,3,\ldots$. The field induced by this layer at the initial layer is equal to the field H_z^+ induced by the initial layer at the kth layer. So, this field is determined by Eq. (11) for H_z^+ with $z+kT$ in place of z. We take the average of this field over the volume V of the particle in the initial layer:

$$\bar{H}_z^+ = \frac{1}{V}\int_V H_z^+(x,y,z+kT)dxdydz. \tag{21}$$

A field of same value, $\bar{H}_z^- = \bar{H}_z^+$, is induced at the initial layer by the kth layer situated below the initial layer. By summing the fields $\bar{H}_z^+$ over k from $k=1$ to $k=N^+$ and the fields $\bar{H}_z^-$ from $k=1$ to $k=N^-$, where N^+ and N^- are the numbers of layers above and below the initial layer, respectively, we obtain the mean field inside a particle of the initial layer induced by all particles of outside layers. By summing the latter field with the mean field $\bar{H}_z$ determined by Eq. (13), we obtain the total mean field inside a particle of the initial layer,

$$\frac{\bar{H}_\perp^{sc}}{4\pi M} = -\xi^2 \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} \frac{\sin^2 n\varphi}{(n\varphi)^2}\frac{\sin^2 m\varphi}{(m\varphi)^2}\frac{\sinh p\varphi}{p\varphi} \times (e^{-p\varphi} - \sinh p\varphi \Phi^{nm}), \tag{22}$$

where

$$\Phi^{nm} = \sum_{k=1}^{N^+} e^{-2\pi kp} + \sum_{k=1}^{N^-} e^{-2\pi kp}. \tag{23}$$

The superscript sc in $\bar{H}_\perp^{sc}$ stands for the simple-cubic lattice.

Parallel orientation of the magnetization. In this case the field induced at the initial layer by a kth above- or below-lying layer is described by Eq. (19) for H_x^+ with $z+kT$ in place of z. We take the average of this field over the volume V of a particle in the initial layer using an equation analogous to Eq. (21). By summing the fields $\bar{H}_x^+$ induced by N^+ layers above the initial layer and the fields $\bar{H}_x^- = \bar{H}_x^+$ induced by N^- layers below the initial layer and adding the mean field $\bar{H}_x$, Eq. (20), we obtain the total mean field inside a particle of the initial layer,

$$\frac{H_\parallel^{sc}}{4\pi M} = -\xi^2 \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} \frac{n^2}{n^2+m^2}\frac{\sin^2 n\varphi}{(n\varphi)^2}\frac{\sin^2 m\varphi}{(m\varphi)^2} \times\left(1 - \frac{\sinh p\varphi}{p\varphi}e^{-p\varphi} + \frac{\sinh^2 p\varphi}{p\varphi}\Phi^{nm}\right), \tag{24}$$

where Φ^{nm} is determined by Eq. (23).

B. Body-centered-cubic lattice

Consider an arrangement of magnetic particles in a bcc lattice (Fig. 3). In our case this lattice consists of two identical cubic sublattices. Both sublattices have period L and are shifted from one another through $L/2$ along all three coordinate axes x, y, and z. The mean field inside the particles of the initial layer induced by all particles of the first sublattice (including those of the initial layer), depending on the orientation of the vector $\mathbf{M}$, and with L in place of T. The field induced by the second sublattice is calculated in the following way.

Perpendicular orientation of the magnetization. We shift the origin of coordinates in Eq. (11) for the field H_z^+ through $L/2$ along the x and y axes and through $L/2+(l-1)L$ along the z axis, where $l=1,2,3,\ldots$. As the result, we obtain the field induced at the initial layer of the first sublattice by the lth layer of the second sublattice

$$\frac{H_z^+}{4\pi M} = \eta^2 \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} (-1)^n(-1)^m \frac{\sin n\psi}{n\psi}\frac{\sin m\psi}{m\psi} \times \sinh p\psi e^{-p(2l-1)\pi} e^{-p\pi} e^{-pq'z} \cos nq'x \cos mq'y \tag{25}$$

where $\eta=2a/L$, $\psi=\pi\eta$, and $q'=2\pi/L$.

We average this field over the volume of a particle of the initial layer, summarize the averaged fields over l for N^+

layers above and N^- layers below the initial layer and add the mean field of the first sublattice described by Eq. (22) with L substituted for T, η substituted for ξ, and ψ substituted for φ. As the result, we obtain the total mean field inside a particle of the initial layer,

$$\frac{\bar{H}_\perp^{bcc}}{4\pi M} = -\eta^2 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{\sin^2 n\psi}{(n\psi)^2} \frac{\sin^2 m\psi}{(m\psi)^2} \frac{\sinh p\psi}{p\psi} \times [e^{-p\psi} - \sinh p\psi (\Phi^{nm} - (-1)^n(-1)^m \Psi^{nm})], \quad (26)$$

where

$$\Psi^{nm} = \sum_{l=1}^{N^+} e^{-\pi(2l-1)p} + \sum_{l=1}^{N^-} e^{-\pi(2l-1)p}. \quad (27)$$

The superscript in $\bar{H}_\perp^{bcc}$ stands for the bcc lattice. The value of $l=1$ corresponds to the field induced in the particles of the initial layer of the first sublattice by the nearest (both upper and lower) layers of the second sublattice.

Parallel orientation of the magnetization. Making the same shift of the origin of coordinates in Eq. (19) for the field outside the layer, H_x^+ as before for the field H_z^+, we obtain the field induced in the initial layer by the lth layer of the second sublattice,

$$\frac{H_x^+}{4\pi M} = -\eta^2 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} (-1)^n(-1)^m \frac{n^2}{n^2+m^2} \frac{\sin n\psi}{n\psi} \frac{\sin m\psi}{m\psi} \times \sinh p\psi e^{-p(2l-1)\pi} e^{-pq' z} \cos nq' x \cos mq' y. \quad (28)$$

We average this field over the volume of a particle of the initial layer, summarize the averaged fields over l for N^+ layers above and N^- layers below the initial layer, and add the mean field of the first sublattice, described by Eq. (24) with L substituted for T, η substituted for ξ, and ψ substituted for φ. As the result, we obtain the total mean field inside a particle of the initial layer,

$$\bar{H}_\parallel^{bcc} = -\eta^2 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{n^2}{n^2+m^2} \frac{\sin^2 n\psi}{(n\psi)^2} \frac{\sin^2 m\psi}{(m\psi)^2} \left[1 - \frac{\sinh p\psi}{p\psi} \times e^{-p\psi} + \frac{\sinh^2 p\psi}{p\psi} (\Phi^{nm} + (-1)^n(-1)^m \Psi^{nm})\right]. \quad (29)$$

One important remark should be made. In deriving the equations for the magnetostatic fields we have considered only two orientations, viz., perpendicular or parallel, of the magnetization vector **M** with respect to the xy plane. Thus we have avoided difficulties caused by the necessity of taking into account two magnetization components, M_z and M_x, and the fields induced by these components when solving the problem of the magnetostatic potential. Meanwhile, the x components of the fields induced by M_z and z components of the fields induced by M_x vanish after averaging over the particle volume because of symmetry of these fields. Therefore, the formulas for the mean magnetostatic fields remain valid in the more general case of arbitrary (but uniform) orientation of the vector **M** if we substitute **M** for M_z in Eqs. (13), (22), and (26) for $\bar{H}_\perp$, and for M_x in Eqs. (20), (24), and (29) for $\bar{H}_\parallel$.

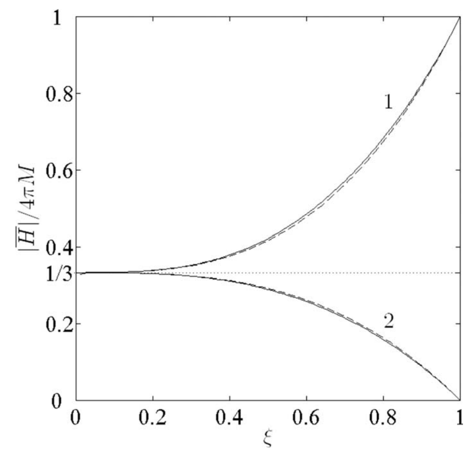

FIG. 4. The mean magnetostatic fields $\bar{H}_\perp$ (curves 1) and $\bar{H}_\parallel$ (curves 2) in the sc lattice vs the relative particle size $\xi=2a/T$.

IV. CALCULATION OF THE MAGNETOSTATIC FIELDS

We use Eqs. (22) and (24) for numerical calculations of the magnetostatic fields inside the magnetic particles arranged in a sc lattice. Figure 4 shows the results for the mean fields $\bar{H}_\perp^{sc}$ (curve 1) and $\bar{H}_\parallel^{sc}$ (curve 2) for the directions of the magnetization vector **M** perpendicular and parallel to the particle layers respectively, as functions of $\xi=2a/T$. These fields are negative for all ξ values, so, they are demagnetizing fields. (Fig. 4 and the following show absolute values of these fields.) Each layer in the multilayer structure has a different number of layers N^+ above and N^- below this layer. In order to get the exact value of the mean magnetostatic field we should sum up fields induced by all these layers at a particle of a given layer. However, one can see that the both curves 1 and 2 are almost insensitive to the numbers of layers N^+ and N^-. Indeed, in Fig. 4 the solid curves calculated for $N^+=N^-=100$ are very close to the dashed curves calculated for $N^+=N^-=0$, that is, corresponding to the monolayer model. Hence, the fields $\bar{H}_\perp^{sc}$ and $\bar{H}_\parallel^{sc}$ in a given particle, for the most part, are constituted of the own field of the particle and the external fields produced by the particles belonging to the same layer. Hence, in practice the summation over the layers N^+ and N^- is not necessary.

The limit of $\xi \to 0$ corresponds to large interparticle distances in comparison with the particle dimensions. In this case the dipolar interaction between the particles vanishes and both $\bar{H}_\perp^{sc}$ and $\bar{H}_\parallel^{sc}$ fields tend to the same limit corresponding to the mean demagnetizing field of a cube which is equal to the demagnetizing field of a sphere, $-4\pi M/3$.[21,22] With the increase in ξ the absolute value of $H_\perp^{sc}$ increases, and for $\xi=1$ it reaches the value of $4\pi M$ corresponding to $H_\perp$ of a homogeneous plate magnetized along its normal. The $H_\parallel^{sc}$ field decreases with the increase in ξ and it vanishes at $\xi=1$.

The results of calculations of the mean fields $\bar{H}_\perp^{bcc}$ and $\bar{H}_\parallel^{bcc}$ in the particles forming a bcc lattice are shown in Fig. 5

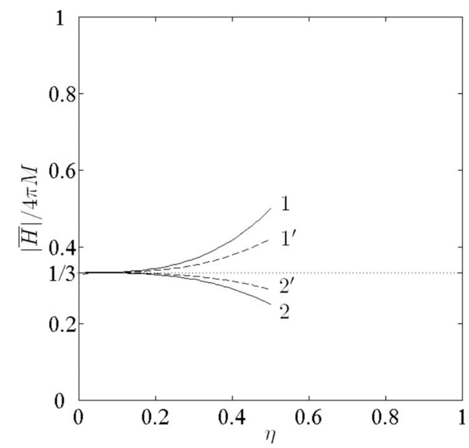

FIG. 5. The mean magnetostatic fields $\bar{H}_\perp$ (curves 1 and 1′) and $\bar{H}_\parallel$ (curves 2 and 2′) in the bcc lattice as functions of the relative particle size $\eta = 2a/L$.

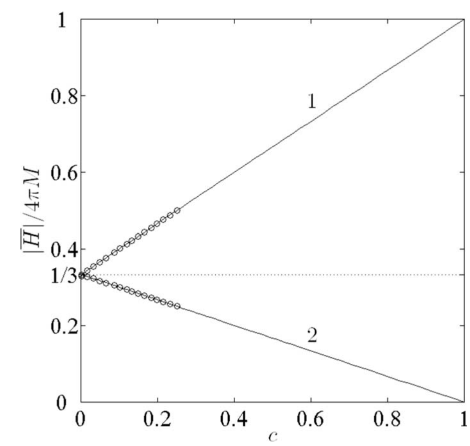

FIG. 6. The mean magnetostatic fields for the sc lattice ($\bar{H}_\perp^{sc}$ and $\bar{H}_\parallel^{sc}$, curves 1 and 2, respectively) and the bcc lattice ($\bar{H}_\perp^{bcc}$ and $\bar{H}_\parallel^{bcc}$, circles along the curves 1 and 2, respectively) as functions of the filling factor c.

as functions of $\eta = 2a/L$. In this case both magnetostatic fields, Eqs. (26) and (29), are the sums of the fields induced by two cubic lattices shifted one from another. Curves 1′ and 2′ correspond to the fields $\bar{H}_\perp^{bcc}$ and $\bar{H}_\parallel^{bcc}$ induced by the first lattice only. Therefore, these curves coincide (assuming $L = T$) with the corresponding parts of the curves shown in Fig. 4. Adding the second sublattice changes significantly the dependences of $\bar{H}_\perp^{bcc}$ and $\bar{H}_\parallel^{bcc}$ on η (curves 1 and 2 in Fig. 5). However, these changes are for the most part due to the two nearest layers of the second sublattice located at the distance $L/2$ from the initial layer of the first sublattice, described by the terms $l=1$ in Eqs. (26) and (29). Taking into account the subsequent layers of the second sublattice located on the distances $3L/2$, $5L/2$, etc., practically does not change the form of the curves 1 and 2. The limit of $\eta \to 0$ for the bcc lattice, as the limit of $\xi \to 0$ for the sc lattice, vide supra, corresponds to the intrinsic mean demagnetizing field of a given particle $\bar{H}_\perp^{bcc} = \bar{H}_\parallel^{bcc} = -4\pi M/3$. With the increase in η the $\bar{H}_\perp^{bcc}$ field increases and the $\bar{H}_\parallel^{bcc}$ field decreases. In contrast to that of the sc lattice, the model of the bcc lattice remains correct only up to $\eta = 0.5$; indeed, at this value the vertices of cubes belonging to the first and the second sublattices come into contact so that any further increase in η becomes meaningless.

According to general properties of the dipolar interaction, the magnetostatic fields depend only on the relation between the particle size and the interparticle distance. Therefore, the results described above are applicable to particles of any size. However, real planar particle assemblies satisfying the conditions of application of the above calculations (the dimension along the z axis being much less than those in the xy plane) are nanoscopic or mesoscopic particles in thin films or implanted layers.

The volume filling factor of magnetic particles is defined as

$$c = \frac{v_m}{v_m + v_0}, \tag{30}$$

where v_m is the volume occupied by the particles and v_0 is the volume of the nonmagnetic matrix. For the sc and bcc lattices c is expressed in terms of ξ and η, respectively,

$$c = \xi^3 = 2\eta^3. \tag{31}$$

The dependences of $\bar{H}_\perp$ and $\bar{H}_\parallel$ on c are shown in Fig. 6 for both the sc (solid lines) and the bcc lattice (circles).

As the mean fields $\bar{H}_\perp$ and $\bar{H}_\parallel$ show the same dependences on c for both lattices, below we omit the superscripts sc and bcc in the corresponding expressions. These dependences can be approximated with good accuracy by the following equations:

$$\bar{H}_\perp = -\frac{4\pi}{3}(1+2c)M_z,$$

$$\bar{H}_\parallel = -\frac{4\pi}{3}(1-c)M_x. \tag{32}$$

The M_z and M_x components are used in these formulas instead of $\mathbf{M}$ in accordance with our previous remark, see the ending paragraph in Sec. III.

Equation (32) allows introduction of effective demagnetizing factors depending on the filling factor c and characterizing assemblies of spherical or cubic magnetic particles in a planar matrix,

$$N_x^e = N_y^e = \frac{4\pi}{3}(1-c), \quad N_z^e = \frac{4\pi}{3}(1+2c), \tag{33}$$

which obey to the usual relationship

$$N_x^e + N_y^e + N_z^e = 4\pi. \tag{34}$$

As one can see from Eq. (33), $N_z^e \geq N_{x,y}^e$ so that at $c \neq 0$ an effective magnetic anisotropy of the "easy plane" type occurs in such systems. The intrinsic magnetic fields are determined by

$$H_\perp^{in} = H_z - N_z^e M_z, \quad H_\parallel^{in} = H_x - N_x^e M_x, \tag{35}$$

where H_x and H_z are the projections of the external magnetic field on the corresponding axes.

To discuss the physical meaning of various terms in Eq. (35) for the intrinsic magnetic fields, we rewrite them as follows:

$$H_{\perp}^{in} = H_z - \frac{4\pi}{3}M_z + \frac{4\pi}{3}cM_z - 4\pi cM_z,$$

$$H_{\parallel}^{in} = H_x - \frac{4\pi}{3}M_x + \frac{4\pi}{3}cM_x. \tag{36}$$

The second terms in the right-hand parts of Eq. (36) are the demagnetizing fields in a separate spherical particle (or the mean demagnetizing fields in a cubic particle). The third terms describe positive fields induced in each particle by all other particles of the planar matrix, with the mean magnetization components cM_z and cM_x. For $M_z = M_x$ the second and third terms of $H_\perp$ and $H_\parallel$ coincide. The last term in Eq. (36) for $H_{\perp}^{in}$ is due to charges created by the surfaces of the plate with the mean magnetization cM_z.

As mentioned above, the approximate interpolation approach[12] and the mean-field theory[15] lead to one and the same expression for the magnetostatic energy density in the assembly of ellipsoidal magnetic particles in an ellipsoidal matrix

$$F_M = \frac{1}{2}[(1-c)\mathbf{M}\hat{n}\mathbf{M} + c\mathbf{M}\hat{N}\mathbf{M}], \tag{37}$$

where $\hat{n}$ and $\hat{N}$ are, respectively, the particle and matrix demagnetizing tensors.

The intrinsic magnetic fields in the particles determined from the sum of the magnetostatic energy density, Eq. (37), and the Zeeman energy density have the form

$$H_{\perp}^{in} = H_z - [n_z(1-c) + N_z c]M_z,$$

$$H_{\parallel}^{in} = H_x - [n_x(1-c) + N_x c]M_x. \tag{38}$$

It can be easily shown that for spherical particles ($n_x = n_y = n_z = 4\pi/3$) in a planar matrix ($N_x = N_y = 0$, $N_z = 4\pi$), Eq. (38) coincides with Eq. (36) and the effective demagnetizing factors obtained from Eq. (38) coincide with those given in Eq. (33). Because Eqs. (33) and (36) follow from the exact solution of the problem, this agreement shows a good accuracy of Eqs. (37) and (38).

As follows from Eq. (37), the frequencies of the perpendicular and parallel resonances in an ellipsoidal assembly of nanoparticles are described by the same equations as in a homogeneous sample[23,24] with the demagnetizing tensor of the sample $\hat{N}$ replaced by the effective demagnetizing tensor $\hat{N}^e$,

$$\omega_\perp = g\{[H - (N_z^e - N_x^e)M][H - (N_z^e - N_y^e)M]\}^{1/2},$$

$$\omega_\parallel = g\{[H - (N_x^e - N_y^e)M][H - (N_x^e - N_z^e)M]\}^{1/2}. \tag{39}$$

In the case of spherical particles in a planar matrix, substituting Eq. (33) to Eq. (39), we obtain

$$\omega_\perp = g(H - 4\pi cM), \quad \omega_\parallel = g[H(H + 4\pi cM)]^{1/2}. \tag{40}$$

Since the ferromagnetic resonance (FMR) measurements most often carried out by the scanning the magnetic field at a constant angular frequency ω, the expressions for the perpendicular and parallel resonance fields resulting from Eq. (40) can also be written as

$$H_{o\perp} = \frac{\omega}{g} + 4\pi cM,$$

$$H_{o\parallel} = \left[\left(\frac{\omega}{g}\right)^2 + (2\pi cM)^2\right]^{1/2} - 2\pi cM. \tag{41}$$

At $c=0$ the resonance frequencies $\omega_\perp = \omega_\parallel$ and fields $H_{o\perp} = H_{o\parallel}$; i.e., they correspond, respectively, to resonance frequencies and fields of noninteracting spherical particles. At $c=1$, Eqs. (40) and (41) describe, respectively, the resonance frequencies and fields of a homogeneous plate magnetized perpendicularly or parallel to its plane.

To account for their static and dynamical experiments, the authors of Ref. 20 used expressions of $H_{\perp}^{in}$ and $H_{\parallel}^{in}$ based on simple qualitative arguments different from those of Netzelmann.[12] Replacing the magnetization M by the mean magnetization cM in the well-known expressions for a homogeneously magnetized plate they obtained for spherical particles in a planar matrix,

$$H_{\perp}^{in} = H_z - 4\pi cM_z,$$

$$H_{\parallel}^{in} = H_x. \tag{42}$$

As it can be seen from a comparison with Eq. (35) or Eq. (36), Eq. (42) is wrong, so they cannot be used to calculate $H_{\perp}^{in}$ and $H_{\parallel}^{in}$ or any related quantities. Fortunately, the authors of Ref. 20 did not apply Eq. (42) for this purpose. They proposed an experimental method of estimating the filling factor c from the difference between magnetic curves $M_x(H_x)$ and $M_z(H_z)$ corresponding, respectively, the parallel and perpendicular orientation of the external magnetic field. Based on the reasonable assumption that the same values of the magnetization $M_x = M_z = M_{x,z}$ correspond to the same values of the intrinsic magnetic fields ($H_{\perp}^{in} = H_{\parallel}^{in}$), they obtain from Eq. (42) the formula for estimating c from difference between the fields H_x and H_z. This difference is described in the same way in Eqs. (36) and (42),

$$H_z - H_x = 4\pi cM_{x,z}. \tag{43}$$

Therefore, in spite of the wrong equation (42), the method of estimating the filling factor c proposed by the authors of Ref. 20 is correct.

As it is seen from Eq. (39), the FMR frequencies in all cases depend only on the difference of the intrinsic fields, i.e., on that of the demagnetizing factors. The same is true for the resonance fields $H_{o\perp}$ and $H_{o\parallel}$. Therefore, the corresponding formulas in Ref. 20, obtained from Eq. (42), coincide with Eqs. (40) and (41) derived from Eq. (33) and with the formulas of the pioneer paper.[12]

V. CONCLUSION

In the paper the magnetostatic fields in a periodic lattice of ferromagnetic particles in a planar nonmagnetic matrix are calculated. The calculations are based on the monolayer model of cubic particles periodically arranged in the xy plane. The exact solution of the magnetostatic problem for this model allows us to obtain expressions for the magnetostatic potential and, consequently, for the magnetostatic fields in the whole space in the form of two-dimensional Fourier series. By using this solution and the superposition principle of the magnetic field, we obtain the expressions of the magnetostatic fields for multilayer models containing a finite number of layers periodically along the z axis. Both the sc and bcc particle lattices are considered. The general expressions describe the magnetostatic fields $H(x,y,z)$ both inside the particles and in the surrounding matrix for a uniform orientation of the particle magnetization **M**, directed perpendicularly or parallel to the surface of the planar matrix.

The fields inside the particles which determine the basic parameters of the magnetization curve, the magnetic-resonance frequencies, and other important characteristics of the assemblies of magnetic particles, are analyzed in detail. With this aim, the magnetostatic fields induced inside each particle by all particles of the assembly and including the demagnetizing field of a given particle are summed up and averaged over the particle volume. The expressions of the mean magnetostatic fields inside the particles, corresponding to the perpendicular and parallel orientations of the magnetization, $\bar{H}_\perp$ and $\bar{H}_\|$, are obtained and numerically analyzed.

The dependences of $\bar{H}_\perp$ and $\bar{H}_\|$ on the filling factor c are obtained. For $c \to 0$ $\bar{H}_\perp$ and $\bar{H}_\|$ are shown to coincide with each other and to be equal to the demagnetizing field of a separate sphere. With the increase in c, $\bar{H}_\perp$ increases and $\bar{H}_\|$ decreases almost linearly in the same way for both types of lattices (Fig. 6). Simple analytical expressions, Eq. (32), are introduced to approximate these dependences. We also introduce effective demagnetizing factors corresponding to these fields [Eq. (33)]. The independence of these dependences of the lattice type, demonstrated here for the sc and bcc lattices, allows us to suppose that these equations remain valid beyond the framework of the model. The physical meaning of the various terms in the expressions of $H_\perp^{in}$ and $H_\|^{in}$, Eq. (36), is explained.

The comparison of the expressions for the mean intrinsic magnetic fields obtained in this paper with those derived from the Netzelmann's formula,[12] Eq. (37), shows the good accuracy of the latter. It should be noted that the latter formula, suggested by Netzelmann in 1990 from intuitive arguments concerning the linear dependence on c of the transition between two well-known limiting cases appears to be very helpful. It has been widely used to account for experimental data in many works; 10 years later, in 2000, this formula has been confirmed by Dubowik[15] in the framework of the mean-field theory; and, at last, now it turns out that Netzelmann's formula is the best approximation for the exact expressions obtained in this our work. Therefore, Netzelmann's method allowing the determination of the filling factor c from the difference between the perpendicular and parallel resonance magnetic fields is also well justified.

Besides, in the present paper we discuss the method allowing the estimation of the filling factor c from the difference of magnetization curves in the perpendicular and parallel magnetic fields suggested in Ref. 20. We show that, in spite of wrong initial expressions for $H_\perp^{in}$ and $H_\|^{in}$, this method remains correct as far as it makes use only of the difference between these fields.

The exact expressions for the magnetostatic fields $H_z^{\pm}(x,y,z)$ and $H_x^{\pm}(x,y,z)$ in the particle-embedding matrix obtained in the present paper could be useful, for instance, for the studies of magnetic field effects on the electron-transport properties in the matrix. A detailed analysis of these expressions and their average values can be carried out at specific points or volume elements inside the matrix which are of interest for one or another specific problem.

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