Cross correlations between charge noise and critical-current fluctuations in a four-level tunable Josephson system

Lara Faoro[1,2,3] and Frank W. J. Hekking[3]
[1]*Laboratoire de Physique Théorique et Hautes Énergies, CNRS UMR 7589, Universités Paris 6 et 7, 4 place Jussieu, 75252 Paris Cedex 05, France*
[2]*Department of Physics and Astronomy, Rutgers University, 136 Frelinghuysen Rd, Piscataway, New Jersey 08854, USA*
[3]*LPMMC, Université Joseph Fourier–CNRS, 25 Avenue des Martyrs, BP 166, 38042 Grenoble, France*
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A protocol to test cross correlations between charge and critical-current noise in the small superconducting contacts of an asymmetric Cooper-pair transistor coupled to a phase qubit is presented. The superconducting circuit behaves as a tunable four-level quantum system that can be prepared in two different configurations where cross-correlation terms are, respectively, absent or present and therefore, in principle, detectable. The measurements of the cross correlations are performed either through the escape probability of the dc superconducting quantum interference device or a quantum-state tomography of a few elements of the reduced density matrix of the four-level quantum system.

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I. INTRODUCTION

A very serious obstacle in the realization of superconducting qubits is the ubiquitous decoherence caused by randomly moving charged defects responsible for low-frequency $1/f$ charge noise in Josephson charge qubits[1,2] and critical-current fluctuations in phase and flux qubits.[3,4] A complete understanding of the microscopic origin of these defects is still missing. Conventional wisdom attributes it to two-level systems (TLS) (Ref. 5) located in the oxide layers of the substrate and the junction barriers. Indeed, strongly coupled TLS were directly observed in the spectroscopy of phase qubits with large Josephson junctions[6] and more recently of charge qubits.[7] However, there is a problem with this model. TLS and similar objects have a constant density of states at low energies. This would give a linear T dependence of the $1/f$ noise power spectrum at low frequency and a white spectrum at high frequency. In contrast, both low-frequency charge-noise spectra measured in small Josephson contacts[8] and low-frequency critical-current fluctuations spectra observed in large superconducting contacts[9] deviate from the linear T dependence. Moreover, a linear frequency dependence (i.e., *ohmic* behavior) of the charge-noise spectrum at high frequency has been inferred from measurements of the relaxation time of Josephson charge qubits.[1]

Recently, a novel microscopic origin for noise types has been identified.[10,11] Kondo-type traps (KT) formed by localized spins at the superconductor insulator (SI) interfaces. This theory overcomes the difficulties with TLS and gives low-frequency noise spectra in agreement with experimental data; moreover, it predicts *ohmic* behavior for the noise power spectra at high frequency ($\omega > T$, T is the temperature) both of charge and critical-current noise. The hallmark of the KT mechanism is the appearance of *additional* contributions to the charge and critical-current noise spectra in the superconducting state. Realistically, one expects that in different superconducting devices and materials, there might be competition between TLS and KT mechanisms. In order to prove this conjecture, it would be important (i) to have direct noise measurements for the normal-state resistance and the critical current of small and large Josephson junctions; (ii) to be able to measure cross correlations between charge and critical-current noise in a superconducting circuit containing small Josephson junctions, where the density of TLS in the barrier is rather low and both charge noise and critical-current fluctuations are mainly due to the KT mechanism. Measuring cross correlations between charge and critical-current noise is a rather difficult task. We stress that testing such correlations has important implications for future designs of fault tolerant error-correction schemes[12] for those superconducting qubits where charge noise is responsible for flip errors and critical-current noise for phase errors. In fact, the presence of significant cross correlations translate into qubit errors that are *not* independent.

In this Brief Report, we show that the circuit recently studied in Ref. 13, consisting of an asymmetric Cooper-pair transistor (ACPT) coupled to a phase qubit offers the possibility to detect cross correlations between charge and critical-current noise in the small junctions of the ACPT. The interesting feature of this circuit is the strongly tunable coupling between the ACPT and the phase qubit that allows to bias the circuit at a "sweet spot" where (i) first-order fluctuations in charge and excess low-frequency flux noise are suppressed; (ii) it behaves as a tunable four-level quantum system. Specifically, we demonstrate that it is possible to tune the circuit at two different four-level configurations where the cross-correlation terms are, respectively, absent or present and therefore, in principle, detectable. The measurement of the cross correlations is performed either through the escape probability of the dc superconducting quantum interference device (SQUID) (Ref. 14) or a quantum-state tomography[15] of a few elements of the reduced density matrix of the four-level quantum system.

II. JOSEPHSON-COUPLED CIRCUIT

The circuit consists of a charge qubit (an ACPT) coupled to a phase qubit (a current-biased dc SQUID), see Fig. 1. This circuit and the Hamiltonian describing its dynamics

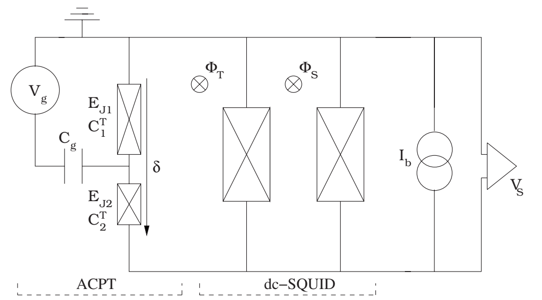

FIG. 1. Electrical schematic of the coupled circuit (Ref. 13). The working point is controlled by the dc gate voltage V_g, the bias current I_b, and the fluxes Φ_T and Φ_S.

have been studied in Ref. 16. In the following, we follow the notation of this work.

The dynamics of the current-biased dc SQUID is described by the Hamiltonian of an anharmonic oscillator, $H_S = \frac{\hbar\omega_p}{2}(\hat{P}^2+\hat{X}^2)-\varkappa\hbar\omega_p\hat{X}^3$, where ω_p is the plasma frequency of the SQUID and $\varkappa$ is the relative magnitude of the cubic term compared to the harmonic term. $\hat{P}$ and $\hat{X}$ are the reduced charge and phase conjugate operators. At low energy, only two levels (say $|\bar{0}\rangle$, $|\bar{1}\rangle$), corresponding to the zero and one plasmon states are relevant and the dc-SQUID dynamics reads

$$H_S = \hbar\omega_0\left(\hat{a}^\dagger\hat{a}+\frac{1}{2}\right) = \frac{1}{2}\hbar\omega_0(|\bar{0}\rangle\langle\bar{0}| + 3|\bar{1}\rangle\langle\bar{1}|). \quad (1)$$

The frequency $\omega_0=\omega_p(1-\frac{15}{2}\varkappa^2)$ depends on the bias current I_b and the dc flux Φ_S through the SQUID.

The ACPT consists of a superconducting island connected to two different Josephson junctions. The dynamics is described by the Hamiltonian $H_T=4E_C(\hat{n}-\frac{Q_g}{2e})^2-E_{J1}\cos\hat{\theta}-E_{J2}\cos(\hat{\theta}-\delta)$; $E_C=e^2/2C_\Sigma$ is the charging energy, E_{J1}, E_{J2} are the Josephson energies. $Q_g=C_gV_g$ denotes the gate charge, $\hat{n}$ is the number operator of (excess) Cooper pairs on the island and $\hat{\theta}$ is its quantum mechanically conjugate phase, $\hat{n}=-i\hbar\partial/\partial\hat{\theta}$. The variable δ denotes the phase difference across the transistor and it is controlled by the dc flux Φ_T inside the loop. For values of the gate charge close to e, only two charge states $(n=0,1)$ play a role while all other charge states, having a much higher energy can be ignored. As a result, the Hamiltonian reads

$$H_T = -\frac{\delta E_c}{2}\sigma_z - \frac{E_{J1}}{2}\sigma_x - \frac{E_{J2}}{2}e^{-i\delta}(\sigma_+ + e^{i2\delta}\sigma_-), \quad (2)$$

where $\delta E_c=4E_C(1-Q_g/e)$ and the charge basis $\{|0\rangle,|1\rangle\}$ is expressed using the Pauli matrices.

The coupling between the ACPT and the phase qubit consists of two terms $H_{coupl}=H_c+H_p$, where $H_c=i\frac{E_{cc}}{2}(\hat{a}^\dagger-\hat{a})(\hat{n}-Q_g/2e)$ couples the charge on the dc SQUID with the charge on the ACPT while $H_p=i\frac{E_{cj}}{2}(\hat{a}+\hat{a}^\dagger)(e^{-i\delta}\sigma_+-e^{i\delta}\sigma_-)$ couples the phase of the dc SQUID with the phase $\hat{\theta}$ of the ACPT via the second Josephson junction. The coupling constants are given by $E_{cc}=(1-\lambda)\kappa\omega_p$ and $E_{cj}=(1-\mu)\kappa E_{JT}$. The constant $\kappa=\sqrt{\frac{E_c^S}{\omega_p}}$ depends on $E_c^S\approx e^2/2C^s$, with C^S the SQUID capacitance. The parameters $\lambda=\frac{C_1^T-C_2^T}{C_1^T+C_2^T}$ and $\mu=\frac{E_{J1}-E_{J2}}{E_{JT}}$, where C_1^T and C_2^T are the capacitances of the ACPT junctions and $E_{JT}=E_{J1}+E_{J2}$. The coupling H_{coupl} is tunable since the Josephson coupling E_{cj} can be varied by changing the Josephson energies of the junctions in the ACPT. For the circuit of Ref. 13, $E_c=26.7$ GHz, $E_{JT}=21$ GHz, $E_{cc}=3.66$ GHz, $E_{cj}=0.7$ GHz, $\lambda=37.7\%$, $\mu=41.6\%$, and $\kappa=0.06$.

III. NOISE MODEL

The KT mechanism is related to the presence of weak Kondo subgap states (traps) located at the SI interface of the Josephson junctions, whose formation is due to a large Coulomb repulsion between two electrons in the same trap.[10] The characteristic energy scale for these resonances is given by the Kondo temperature T_K which depends on the bare level width Γ and the bare level position ϵ_0: $T_K\propto\exp(-\pi\epsilon_0/2\Gamma)$. The assumption that Γ is distributed in a broad range leads to a very wide distribution of Kondo temperatures in the normal state, $P(T_K)\propto 1/T_K$. In the superconductor, the resonances having $T_K^*\approx\Delta$ become localized low-energy levels with a constant surface density of states, $\nu(\epsilon)=\rho_{2D}/T_K^*$, characterized by the small weight $\zeta=T_K^*/\epsilon_0$. Here ρ_{2D} is the bare surface density of traps ($\rho_{2D}\approx 10^{13}$ cm^{-2}). Both charge and critical-current noise are due to quasiparticle tunneling processes between pairs of KT located on the same side of the SI interface in the Josephson-junction barriers. Depending on the distance r between two traps, one can distinguish fast processes in which electrons move a short distance $r<\xi$ which are responsible for high-frequency *ohmic* behavior of the noise power spectrum and exponentially slow ones where the distance is large $r\gg\xi$ which are responsible for the low-frequency $1/f$ noise power spectrum (ξ is the coherence length of the superconductor). Charge and critical-current noise spectra can be expressed as a function of the parameters of the microscopic theory. In particular, it has been shown that charge-noise spectra read, respectively, $S_{Q_g}(\omega)=\frac{\alpha^2}{\omega}e^2$ at low frequency and $S_{Q_g}(\omega)=\frac{\alpha^2}{T^2}e^2\omega$ at high frequency, with $\alpha\approx\zeta(\frac{\rho_{2D}AT}{T_K^*})$ (Ref. 10) while critical-current fluctuations spectra read, respectively, $S_{I_c}(\omega)=\gamma^2\frac{T^2}{A\omega}I_c^2$ at low frequency and $S_{I_c}(\omega)=\frac{\gamma^2}{AT^2}I_c^2\omega$ at high frequency, with $\gamma=\zeta(\frac{\rho_{2D}\delta A_{eff}\xi}{T_K^*})$.[11] Here $S_O(\omega)=\int_{-\infty}^{\infty}dt\langle\delta O(t)\delta O(0)\rangle e^{i\omega t}$, A and T denote, respectively, the Josephson-junction area and the temperature while δA_{eff} represents the change in the *effective* area of the junction due to a random movement of a charged fluctuator blocking the critical-current channel in the barrier. In the coupled circuit we consider, the ACPT has two different junctions with areas, respectively, $A_1=0.05$ μm^2 and $A_2=0.02$ μm^2. Only the intensity of the $1/f$ charge-noise power spectrum was measured, $\alpha^2=2\times10^{-6}$.[13] The contacts being very small, we can assume that KT mechanism is dominant over TLS and express the spectra of the low-frequency critical-current fluctuations in the junctions as a function of the intensity α of the low-frequency charge noise, $S_{I_{ci}}(\omega)=\frac{(\xi\alpha\delta A_{eff})^2}{(A_1^2+A_2^2)}\frac{I_{ci}^2}{A_i\omega}$ for $i=1,2$. The parameter δA_{eff} is unknown and expected to depend on material preparation; in very small junctions a value of $\delta A_{eff}\sim 10$ nm^2 was reported in 1980s.[17] We estimate $\gamma_1\simeq\gamma_2=\gamma\approx 10^{-6}$ in agreement with typical values estimated in Ref. 3.

A distinctive feature of the KT mechanism is that the density of Kondo traps is huge, i.e., $\nu(\epsilon)\approx\rho_{2D}/\Delta\approx 5\times 10^{18}$ cm^{-2} eV^{-1}, and that each quasiparticle tunneling event between the traps gives rise to a very weak fluctuation of charge and critical current.[10] It is thus reasonable to model the effect in the ACPT Hamiltonian (2) as *classical* fluctuations both in the gate charge and in the critical current of the junctions,

$$H(t) = -2E_C\frac{\delta Q_g(t)}{e}\sigma_z - \frac{\Phi_0}{4\pi}\sum_{j=1,2}\delta I_{c_j}(t)(e^{-i\delta_j}\sigma_+ + \text{H.c.})$$

here $\delta_1=0$, $\delta_2=\delta$. It is convenient to rewrite this Hamiltonian using three fluctuating fields χ_i, with $i=0$ for charge noise and $i=1,2$ for critical-current noise in junction 1 and 2, respectively,

$$H(t) = -v_0\chi_0(t)\sigma_z - \sum_{j=1,2} v_j\chi_j(t)(e^{-i\delta_j}\sigma_+ + \text{H.c.}). \quad (3)$$

In the coupled circuit, typical values for the noise coupling strengths are $v_0=2E_C\approx 53.4$ GHz, $v_1=\frac{\Phi_0}{4\pi}I_{c1}\approx 14.5$ GHz, and $v_2=\frac{\Phi_0}{4\pi}I_{c2}\approx 6.5$ GHz. The noise power spectra are defined as

$$\Gamma_{j,k}(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t}\langle\chi_j(t)\chi_k(0)\rangle \quad j,k=0,1,2. \quad (4)$$

The noise being classical, $\Gamma_{j,k}(\omega)$ is a real function and $\Gamma_{j,k}(\omega)=\Gamma_{j,k}(-\omega)$ for $j,k=0,1,2$. While $\Gamma_{0,0}(\omega)=S_{Q_g}(\omega)/e^2$ and $\Gamma_{j,j}(\omega)=S_{I_{cj}}(\omega)/I_{cj}^2$ with $j=1,2$, $\Gamma_{j,k}(\omega)$ for $j=0$ and $k=1,2$ quantify the cross correlations between charge and critical-current noise and it is called the cospectrum.

IV. CROSS CORRELATIONS AT THE SWEET SPOT

We tune the circuit at the optimal point, $(Q_g/e,\delta)=(1,0)$ so that first-order fluctuations in charge and flux noise are suppressed and we set the frequency of the phase qubit in resonance with the energy splitting of the ACPT, i.e., $\omega_0=E_{JT}$. Under these conditions, the idle Hamiltonian of the coupled system $H_0=H_S+H_T+H_{coup}$ is diagonalized as $H_0=\Sigma_{i=1,4}E_i|w_i\rangle\langle w_i|$ with eigenvalues,

$$E_1 = E_{JT} - \frac{E_{cc}-2E_{cj}}{4} \qquad E_3 = E_{JT} - \frac{\Sigma}{4},$$

$$E_2 = E_{JT} + \frac{E_{cc}-2E_{cj}}{4} \qquad E_4 = E_{JT} + \frac{\Sigma}{4}, \quad (5)$$

where $\Sigma=\sqrt{(E_{cc}+2E_{cj})^2+16E_{JT}^2}$. The noisy part of the Hamiltonian given in Eq. (3) can be written as $H(t)=\Sigma_{i=0,1,2}v_iA_i\chi_i(t)$, where the error operators A_i acting in the space generated by the eigenvectors $\{|w_1\rangle,|w_2\rangle,|w_3\rangle,|w_4\rangle\}$ read

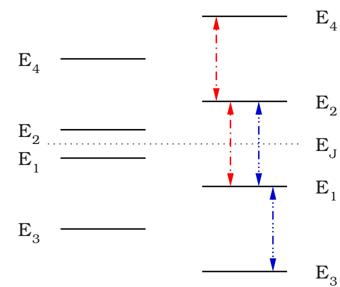

FIG. 2. (Color online) Two different eigenvalues configurations for the tunable four-level circuit. The probability $P_1(t)$ is insensitive to cross correlations between charge and critical-current noise for configuration (i). However, for configuration (ii), cross-correlations terms enter in K_{21}^{13} and K_{42}^{21} once energy constraints $E_{13}=E_{21}$ and $E_{21}=E_{42}$ are met.

$$A_0 = \frac{1}{\sqrt{1+\eta^2}}\begin{pmatrix} 0 & 0 & 1+\eta & \eta-1 \\ 0 & 0 & 1-\eta & 1+\eta \\ 1+\eta & 1-\eta & 0 & 0 \\ \eta-1 & 1+\eta & 0 & 0 \end{pmatrix},$$

$$A_{1(2)} = \frac{1}{1+\eta^2}\begin{pmatrix} 0 & -(1+\eta^2) & 0 & 0 \\ -(1+\eta^2) & 0 & 0 & 0 \\ 0 & 0 & 1-\eta^2 & \eta \\ 0 & 0 & \eta & \eta^2-1 \end{pmatrix},$$

where $\eta=\frac{4E_{JT}}{E_{cc}+2E_{cj}}$. The physical characteristics of the noise model allow us to derive the evolution equation for the matrix elements $\rho_{ij}(t)=\langle w_i|\rho(t)|w_j\rangle$ of the reduced density operator $\rho(t)$ of the coupled system by solving a master equation (ME) under the assumptions of weak coupling and Markov approximations. We find that[18]

$$\dot{\rho}_{ab}(t) = i\langle w_a|[\rho(t),H_0]|w_b\rangle + \sum_{mn=1}^{4}\Big\{\delta(E_{nb}^{ma})K_{bn}^{am}\rho_{mn}(t) - \frac{1}{2}[\delta(E_{an}^{mn})K_{an}^{nm}\rho_{mb}(t) + \delta(E_{mn}^{bn})K_{nm}^{nb}\rho_{am}(t)]\Big\} \quad (6)$$

with $K_{bn}^{am}=\Sigma_{jk=0}^{2}v_jv_k\Gamma_{jk}(E_{ma})\langle w_a|A_k|w_m\rangle\langle w_b|A_j|w_n\rangle^*$, $E_{kl}^{ij}=E_{ij}-E_{kl}$, and $E_{ij}=E_i-E_j$ and δ is the Dirac delta.

In order to study the cross correlations between charge and critical-current noise in the coupled circuit, we suggest to initially prepare the system in the state $|-,\bar{0}\rangle$ (here $|-\rangle$ is the ground state of the ACPT in the rotated basis) and to measure the escape probability $P_1(t)=\frac{1}{2}\Sigma_{i,j=1,2}\rho_{ij}(t)$ to find the dc SQUID in the state $|\bar{1}\rangle$ at time t. In the absence of noise, $P_1(t)=[1-\cos(E_{21}t)]/2$. In the presence of noise, by looking at the ME and at the block structures of the error operators, it is evident that different noise spectra are relevant at different eigenvalue configurations for the coupled circuit. In particular, the circuit can be tuned in two configurations where cross correlations are, respectively, absent or present; the eigenvalues given in Eq. (5) are (i) nondegenerate and (ii) nondegenerate and *equidistant*, see Fig. 2.

We find that at configuration (i),

$$P_1(t) = \frac{1}{2}\left[e^{-1/2\Gamma_d t} \cosh\left(\frac{\Gamma_d}{2}t\right) - e^{-\Gamma_o t} \cos(E_{21}t) \right]. \quad (7)$$

The constants $\Gamma_d = [\Gamma^{+}_{0,0}(E_{42}) + \Gamma^{-}_{0,0}(E_{32})]$ and $\Gamma_o = \frac{1}{2}\Gamma_d + \Gamma_{cc}(E_{21})$ depend on the charge-noise spectra and critical-current spectra,

$$\Gamma^{\pm}_{0,0}(\omega) = v_0^2\left[1 \pm \sqrt{1 - \left(\frac{4E_{JT}}{\Sigma}\right)^2} \right]\Gamma_{0,0}(\omega),$$

$$\Gamma_{cc}(\omega) = v_1^2\Gamma_{1,1}(\omega) + 2v_1v_2\Gamma_{1,2}(\omega) + v_2^2\Gamma_{2,2}(\omega)$$

but no cross correlations between charge and critical-current noise appear, i.e., $\Gamma_{0,j}(\omega)$, $j=1,2$.

The configuration (ii) is achieved by satisfying the constraint $|E_{21}| = \Sigma/6$, that is by varying $\mu = \frac{-5E_{cc}+8E_{JT}\kappa+\sqrt{9E_{cc}^2+32E_{JT}^2}}{8E_{JT}\kappa}$. For example, for $\kappa=0.6$ and $\lambda=37.7\%$ we find that $\mu=74.7\%$ and the eigenvalues given in Eq. (5) become equidistant with energy separation $E_{21}/2 \approx 16$ GHz. We find that

$$P_1(t) = \frac{1}{2}\left[e^{-\Gamma_d/2t} \cosh\left(\frac{\Gamma_d}{2}t\right) + e^{-\Gamma_o t}F_1(t)\cos(E_{21}t) \right], \quad (8)$$

where $F_1(t) = e^{\Gamma_1/2t}[\frac{\Gamma_1}{\Omega}\sinh(\frac{\Omega}{2}t) - \cosh(\frac{\Omega}{2}t)]$ depends on the cross correlations. Here $\Omega = \sqrt{\Gamma_1^2 + \Lambda^2}$, with $\Gamma_1 = \frac{1}{2}[\Gamma_{cc}(E_{21}) - \delta\Gamma]$ and $\Lambda = c\Sigma_{j=1,2}v_0v_j\Gamma_{0,j}(E_{21})$. $\delta\Gamma = x^2\Gamma_{cc}(0) + (1-x^2)\Gamma_{cc}(3E_{21})$ and the constant $c = \sqrt{1+x} + \sqrt{1-x}$ while $x = \frac{2E_{JT}}{3E_{21}}$. Three remarks are in order. First, the coefficient Λ depends on the cospectrum and in the absence of cross-correlation contributions ($\Lambda \to 0$), Eq. (8) reduces to Eq. (7). Second, the function $F_1(t)$ depends on the critical-current zero-frequency noise $\Gamma_{cc}(0)$. Since the noise is $1/f$, we can estimate the latter by measuring the pure dephasing decay, $\Gamma_\varphi = \pi\Gamma_{cc}(0)$, of the coherent Ramsey oscillations between eigenstates E_1 and E_2. Indeed, in the static approximation $f_R(t) = \exp[-t^2\Gamma_\varphi^2] \approx \exp[-t^2(\Sigma_{j=1,2}v_j^2\gamma_j^2)]$.[19] Third, at configuration (ii), cross-correlation contributions enter in the off-diagonal terms of the reduced density matrix $\rho(t)$. Precisely, we find that $F_2(t) = \mathcal{R}\mathrm{e}[\rho_{24}(t) + \rho_{31}(t)]$ reads

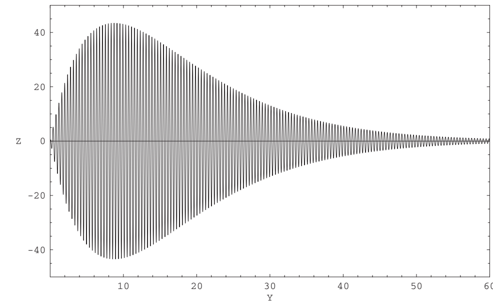

FIG. 3. Plot of $F_2(t)/\gamma$, where γ is the typical amplitude of the critical-current fluctuations depending on the effective area δA_{eff} of the junctions. In the plot, we choose the value $\delta A_{eff} = 10$ nm^2. It is likely that the latter can be bigger in small junctions, if one assumes that the conductance in the small junction is dominated by relative small areas or channels in this system (Ref. 20). This conjecture is supported by the recent experiments (Ref. 21).

$$F_2(t) = \frac{\Lambda}{\Omega}e^{-(\Gamma_0 - \Gamma_1/2)t} \sinh\left(\frac{\Omega}{2}t\right)\cos(E_{21}t). \quad (9)$$

As a result, by resorting to a state estimation tomography, it is possible to directly measure the intensity of the cross correlation between charge and critical-current noise, see Fig. 3.

Notice that additional cross-correlation contributions might enter in the measurement due to fluctuations in the gate charge V_g and the external magnetic flux. However, those terms are negligible since the coupled system is tuned at the sweet spot.

V. CONCLUSIONS

We showed that cross correlations between charge and critical-current noise in the very small junctions of an ACPT tunably coupled to a phase qubit can, in principle, be detected. We suggested that measurements of the cospectrum might be performed either through the escape probability of the dc SQUID or a quantum-state tomography of a few elements of the reduced density matrix of the coupled system.

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[1] O. Astafiev *et al.*, Phys. Rev. Lett. **93**, 267007 (2004).
[2] E. Paladino *et al.*, Phys. Rev. Lett. **88**, 228304 (2002).
[3] D. J. Van Harlingen *et al.*, Phys. Rev. B **70**, 064517 (2004).
[4] J. Eroms *et al.*, Appl. Phys. Lett. **89**, 122516 (2006).
[5] A. L. Burin, J. Low Temp. Phys. **100**, 309 (1995); P. W. Anderson *et al.*, Philos. Mag. **25**, 1 (1972).
[6] R. W. Simmonds *et al.*, Phys. Rev. Lett. **93**, 077003 (2004).
[7] Z. Kim *et al.*, Phys. Rev. B **78**, 144506 (2008); F. Wellstood *et al.*, arXiv:0805.4429 (unpublished).
[8] O. Astafiev *et al.*, Phys. Rev. Lett. **96**, 137001 (2006).
[9] F. C. Wellstood *et al.*, Appl. Phys. Lett. **85**, 5296 (2004).
[10] L. Faoro and L. B. Ioffe, Phys. Rev. Lett. **96**, 047001 (2006).
[11] L. Faoro and L. B. Ioffe, Phys. Rev. B **75**, 132505 (2007).
[12] E. Knill *et al.*, Science **279**, 342 (1998).
[13] A. Fay *et al.*, Phys. Rev. Lett. **100**, 187003 (2008).
[14] J. Claudon *et al.*, Phys. Rev. B **76**, 024508 (2007).
[15] I. L. Chuang and M. A. Nielsen, J. Mod. Opt. **44**, 2455 (1997); M. Mohseni and D. A. Lidar, Phys. Rev. Lett. **97**, 170501 (2006).
[16] O. Buisson *et al.*, Quantum Inf. Process. **8**, 155 (2009).
[17] C. T. Rogers and R. A. Buhrman, IEEE Trans. Magn. **21**, 126 (1985).
[18] S. Kryszewski *et al.*, arXiv:0801.1757 (unpublished).
[19] G. Ithier *et al.*, Phys. Rev. B **72**, 134519 (2005).
[20] J. D. R. Buchanan *et al.*, Appl. Phys. Lett. **81**, 751 (2002).
[21] L. S. Dorneles *et al.*, Appl. Phys. Lett. **82**, 2832 (2003).