Effective circuit theory for the cusplike zero-bias anomaly in tunneling magnetoresistance

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An effective circuit approach is developed to investigate inelastic tunneling of electrons through magnetic tunnel junctions. Electrons tunneling via impurities may dissipate their energy through interaction with the collective modes of the environment, which are effectively modeled by an infinite lumped *LC* circuit. The present theory can well reproduce the cusplike zero-bias anomaly of the tunneling magnetoresistance observed in magnetic tunnel junctions, the energy dissipation and spin-flip scattering playing a critical role.

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The spin-dependent electronic transport through a magnetic tunnel junction, consisting of two ferromagnetic electrodes separated by an insulating layer, has attracted much theoretical and experimental interest in recent years, especially the tunneling magnetoresistance (TMR) effect, due to its potential applications in spintronic devices.[1–17] The TMR represents the effect that the tunneling conductance changes as the magnetizations of the two electrodes alter their relative orientation under an applied magnetic field. An important characteristic of the TMR is that it decreases dramatically with increasing bias voltage, exhibiting a peak at zero bias, as observed in many experiments. This behavior is often called the zero-bias anomaly (ZBA) of the TMR.[5–13]

A number of theoretical models have been suggested to explain the ZBA in magnetic tunnel junctions. The simplest theory attributed the ZBA to the energy dependence of elastic transmission matrix elements,[3,18,19] which however predicted a much slower decrease of the TMR at low bias voltages. Zhang *et al.*[6] suggested that hot electrons from the emitting electrode may be scattered by local magnetic moments at the interfaces, which leads to voltage dependence of the conductance and TMR. Another type of models were based upon electron tunneling assisted by magnons or phonons.[7,20–22] Interestingly, in the experiment of the TMR through a vacuum barrier, the ZBA did not occur for bias voltage up to a volt, suggesting that the ZBA might have its origin in impurities in the insulating barrier.[15] Some authors investigated the elastic electron tunneling via impurities in the insulating barrier, and showed that its contribution to the conductance and TMR could be dependent of bias voltage.[9–11,22–27] Most of the theories above indicated that the conductance $G(V)$ of the tunnel junctions has a power-law dependence on small bias voltages, i.e., $G(V)-G(0)\sim|V|^p$ with $p>1$, except that $p=1$ in the theory.[6] As pointed out in a later work,[28] these models could explain some experimental data without cusplike feature at zero bias, but failed to interpret the cusplike feature of the ZBA observed in many experiments.[5–9] Explanation of the cusplike peak of the TMR requires that the slope of the conductance $dG(V)/dV\sim|V|^p/V$ be discontinuous at $V=0$; i.e., $p<1$. A theory[28] taking into account electron tunneling via impurities, which are coupled to acoustic phonons, was presented. The theory predicted that $G(V)-G(0)\sim|V|^{2g}$. With the dimensionless coupling constant g used as an adjustable parameter, the theory could possibly explain both cusplike $(2g<1)$ and noncusplike $(2g>1)$ features of the ZBA. However, its success seemed to be overshadowed by the complex nonperturbative treatments as well as the detailed assumptions of the impurity scattering potential and properties of the phonons adopted in the theory. So far, a widely accepted theory of the ZBA is still absent. A simpler but more general model that can capture the essential mechanisms of the ZBA is highly desirable.

In this Brief Report, we present a phenomenological effective circuit theory for the ZBA of the TMR. The spin-dependent tunneling for electrons through the insulating barrier is divided into two types of processes: direct tunneling between the ferromagnetic electrodes and indirect tunneling via impurities which are randomly distributed in the insulating barrier. In the latter, the tunneling electrons are considered to dissipate energy due to coupling to certain collective modes of the environment. Such an inelastic tunneling process is described effectively by connecting the elastic tunneling conductance to an infinite lumped *LC* circuit. We show that this circuit model can capture essential features of the ZBA, and the calculated result is well consistent with experimental data.

Consider a magnetic tunnel junctions with two ferromagnetic electrodes separated by an insulating barrier. In the absence of an applied magnetic field, the magnetic moments of the electrodes are assumed to be in an antiparallel (AP) configuration. When an external magnetic field is applied to the system, the magnetic moments are aligned into a parallel (P) configuration. We consider some impurity clusters or impurities with relatively long-range scattering potential distributed in the insulating barrier,[26] whose energies constitute a continuum of levels in the insulator gap. The total conductance $G^P(G^{AP})$ in the P (AP) configuration is the sum of the conductances due to the two types of tunneling processes,

$$G^{P(AP)}(V)=G_D^{P(AP)}+G_I^{P(AP)}(V), \tag{1}$$

where $G_D^{P(AP)}$ and $G_I^{P(AP)}(V)$ stand for the conductances due to the direct and indirect tunneling, respectively. G_D^P and G_D^{AP} depend mainly on the spin-dependent electron density of states in the electrodes and the electron-tunneling matrix elements between the electrodes, which are approximately taken to be constants independent of bias voltage.

As an electron hops into an impurity state, it induces local charge fluctuations in the barrier, exciting collective oscillation modes of the environment, such as phonons, magnons,

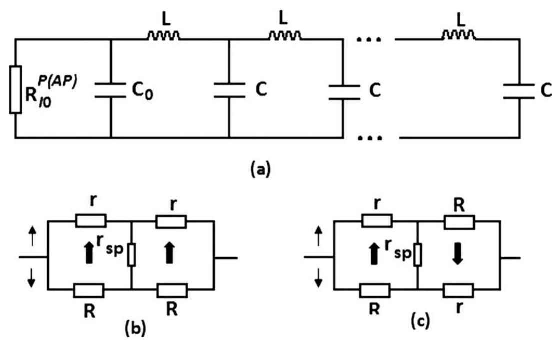

FIG. 1. (a) Effective circuit for electron tunneling via impurities. $R_{I0}^{P(AP)}$ is the resistance in the absence of coupling to the environment in the P (AP) configuration. The infinite lumped circuit consisting of inductors L and capacitors C models oscillation modes of the environment, and C_0 is a capacitor, which modulates the coupling strength between tunneling electrons and the lumped LC circuit. (b) and (c) Expanded circuit diagrams for R_{I0}^{P} and R_{I0}^{AP}, respectively.

and electromagnetic modes. They in turn play a scattering role in the electron tunneling via impurities. We propose a simplified quantum circuit model to describe these inelastic-scattering processes, as shown in Fig. 1(a). The total resistance $R_I^{P(AP)}(V)=1/G_I^{P(AP)}(V)$ of the indirect tunneling is considered to be a parallel connection of the bare tunneling resistance $R_{I0}^{P(AP)}$, a coupling capacitance C_0, and an infinite lumped LC circuit. This circuit model is an extension of the theory that was first proposed by Girvin *et al.*[29] to study electron transport in a single tunnel junction coupled to its environment by a transmission line, where the lumped LC circuit was used to model the transmission line. Here, we introduce the LC circuit to phenomenologically describe the collective modes of the environment, such as phonons. The lumped LC circuit is equivalent to a series of coupled harmonic oscillators with the charges of the capacitors as the canonical coordinates.[29] The phonons are essentially related to the harmonic oscillations of atoms. As an electron tunnels, it displaces the harmonic LC oscillators suddenly, and dissipates its energy to the environment by exciting the LC oscillation mode, which mimics emission of phonons. The capacitor C_0 effectively modulates the strength of the interaction between the tunneling electrons and collective oscillation modes. For infinite C_0, the coupling between the tunneling electrons and the LC circuit vanishes, since charge fluctuations induced by the tunneling electrons are fully screened out by the infinite charging capacity of C_0. On the other hand, $C_0=0$ corresponds to the strongest coupling limit.

In Fig. 1(a), $R_{I0}^{P(AP)}$ is the bare resistance for electron tunneling via impurities for the P (AP) magnetization configuration in the absence of energy dissipation. Within the two-channel model, $R_{I0}^{P(AP)}$ can be modeled by the circuits drawn in Figs. 1(b) and 1(c) for P and AP configurations, respectively. Here, R (r) stands for a larger (smaller) resistance for the electrons in the minority (majority) spin subband tunneling between an electrode and the impurities in the insulating layer, due to the smaller (larger) electron density of states in the ferromagnetic electrode. We further take into account the fact that the electrons may be subject to spin-flip scattering when tunneling via the impurities, and a short-circuit resistance r_{sp} between the two spin channels is introduced. In the magnetic tunnel junctions, spin-flip effect can arise as a result of electron scattering by local magnetic moments of the impurities[26] or by spin-dependent collective excitations. The higher the spin-flip rate, the smaller r_{sp}. In terms of these simple circuits, the expressions for R_{I0}^{P} and R_{I0}^{AP} can be obtained as[30]

$$R_{I0}^{P}=\frac{2Rr}{R+r}, \tag{2}$$

$$R_{I0}^{AP}=\frac{2Rr+Rr_{sp}+rr_{sp}}{R+r+2r_{sp}}. \tag{3}$$

The corresponding conductances are denoted as $G_{I0}^{P}=1/R_{I0}^{P}$ and $G_{I0}^{AP}=1/R_{I0}^{AP}$.

By following the work of Girvin *et al.*,[29] in which the LC circuit is treated quantum mechanically, the conductance for indirect tunneling is obtained as

$$G_I^{P(AP)}(V)=G_{I0}^{P(AP)}\int_0^{eV/\hbar}\frac{d\omega}{2\pi}A(\omega)\left(1-\frac{\hbar\omega}{eV}\right). \tag{4}$$

Here, $A(\omega)=-2\ \mathrm{Im}\ \mathcal{G}(\omega)$ is the spectrum density function of the environment with $\mathcal{G}(\omega)$ as the Green's function, which is the Fourier transform of

$$\mathcal{G}(t)=-i\theta(t)\exp\left[\int_0^{\infty}\frac{d\omega}{\omega}\frac{2\ \mathrm{Re}[Z_t(\omega)]}{R_Q}(e^{-i\omega t}-1)\right]. \tag{5}$$

Here, $R_Q=h/e^2$ is the resistance quantum, and $Z_t(\omega)$ is the total impedance of the parallel connection of C_0 and the lumped LC circuit $Z_t(\omega)=1/[i\omega C_0+1/Z(\omega)]$, with $Z(\omega)=(i\omega L+\sqrt{-\omega^2L^2+4L/C})/2$ as the impedance of the LC circuit. $\mathcal{G}(t)$ describes the coherent evolution of an electron from an initial state into a final state due to the excitation of the LC mode in the circuit model. $A(\omega)$ is the probability for a tunneling electron to emit energy $\hbar\omega$ to the LC circuit. Obviously, the dressed conductances $G_I^{P(AP)}(V)$ are dependent of bias voltage, in contrast to the bare ones $G_{I0}^{P(AP)}$.

In Eq. (5), since the dominant contribution to the integral comes from small ω, $Z(\omega)$ can be approximately replaced by $Z(0)=\sqrt{L/C}$, so that one can take $Z(\omega)=gR_H/2$ with g as a dimensionless constant.[29] The parameter g effectively measures the hardness of exciting the collective modes of the environment, and is a material-dependent quantity. The smaller g, the less energy cost it takes on average to excite the collective modes. For small bias voltage, an approximate expression for the conductance can be obtained as $G_I^{P(AP)}(V)=c|V/V_0|^g G_{I0}^{P(AP)}$, where $V_0=e/\pi g C_0$ is a characteristic voltage, and $c=e^{-\gamma g}/2\pi\Gamma(g+2)$ with γ the Euler constant and $\Gamma(z)$ the gamma function. It shows that $G_I^{P(AP)}(V)$ is completely suppressed at zero-bias voltage. When the exponent $g<1$, the slope of the conductance is discontinuous at $V=0$, showing cusplike bias anomaly. For large $|V|$, $G_I^{AP}(V)$ saturates to a constant value $G_{I0}^{P(AP)}$ asymptotically. The calculated result for $G_I^{AP}(V)$ of the indirect tunneling in the AP configuration is plotted in Fig. 2(a) for different values of g, where only the part of positive bias voltage is shown because

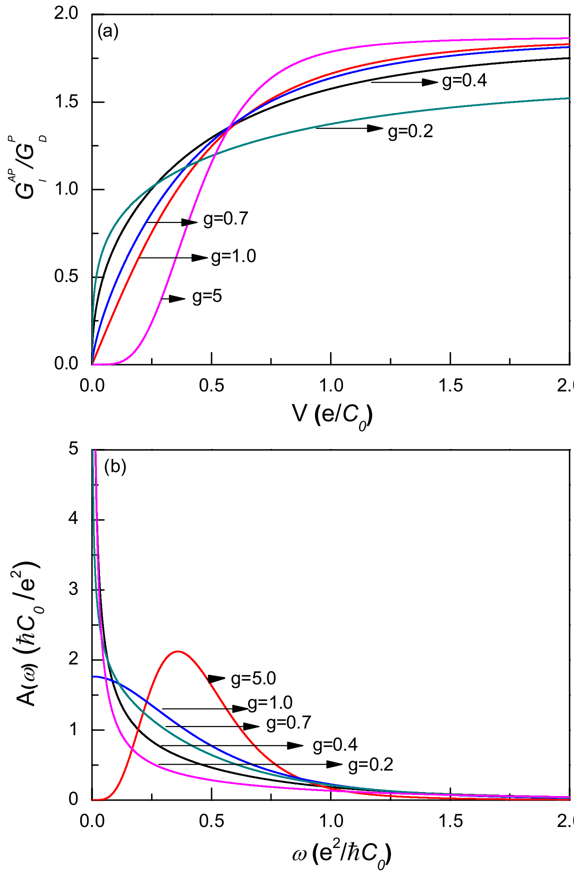

FIG. 2. (Color online) (a) Conductance $G_I^{AP}(V)$ of indirect electron tunneling, normalized by conductance G_D^P of direct tunneling (G_D^P being a constant), as a function of bias voltage V, and (b) spectrum density $A(\omega)$, for different values of g. The parameters used are $R_D^{AP}=5R_D^P/3$, $r_{sp}=0.01R_D^P$, $r=0.4R_D^P$, and $R=0.8R_D^P$.

$G_I^{AP}(V)$ is an even function of bias voltage. It is found that the conductance $G_I^{AP}(V)$ increases monotonously with bias voltage V. The conductance $G_I^P(V)$ in the P configuration exhibits similar behavior with changing bias voltage, which is not shown in the figure.

The bias voltage dependence of the conductance $G_I^{P(AP)}(V)$ can be understood by the following argument. If the tunneling process is decoupled from the environment, we have $Z_t(\omega)=0$ and $A(\omega)=2\pi\delta(\omega)$, so that electrons in the energy window between the Fermi levels of the two electrodes are allowed to tunnel due to the Pauli principle. The number of such electrons is proportional to $e|V|$, leading to a constant conductance $G_I^{P(AP)}(V)=G_{I0}^{P(AP)}$ independent of bias voltage. In the present dissipative model ($g\neq 0$), however, $A(\omega)$ distributes in a finite range of ω [see Fig. 2(b)], whose linewidth is determined by the values of g and C_0. The number of electrons allowable to tunnel is thus reduced. This reduction is particularly prominent at small bias voltage, because there is a large probability for a tunneling electron to lose energy $\hbar\omega$ greater than $e|V|$ to the environment and such tunneling processes are forbidden. As a result, the conductance is greatly suppressed, and $G_I^{P(AP)}(V)\to 0$ as V tends towards vanishing. With increasing $|V|$, the conductance increases rapidly because of increased energy difference between the Fermi levels of the two electrodes. For large $|V|$, $G_I^{AP}(V)$ saturates to a constant value G_{I0}^{AP}.

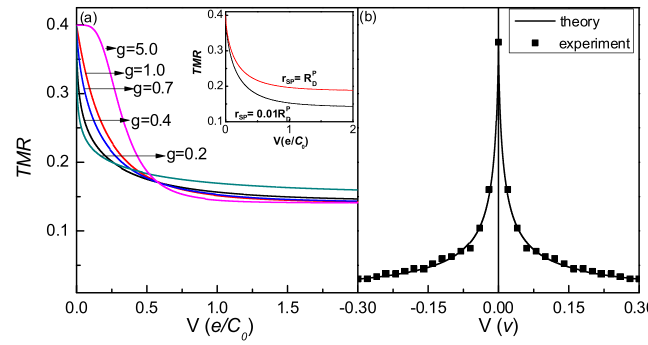

FIG. 3. (Color online) (a) TMR versus bias voltage V for different values of g. Inset: TMR for two different values of r_{sp} at g=0.7. Here, the other parameters are taken to be the same as in Fig. 2. (b) The present theoretical fitting (solid line) to the experimental data (solid squares) taken from Ref. 8 with the parameters chosen to be $R=0.1R_D^P$, $r=0.06R_D^P$, $r_{sp}=0.005R_D^P$, $g=0.8$, and $C_0=0.06$ fF.

The TMR ratio is defined as $\text{TMR}=1-G^{AP}(V)/G^P(V)$. Figure 3(a) shows the TMR ratio as a function of bias voltage for different values of g. Here, the constant resistance R_D^P is taken to be the unit of resistance, and the calculated TMR ratio is independent of the specific value of R_D^P. At zero-bias voltage, the conductance due to indirect tunneling vanishes, and the TMR takes its maximum value given by $\text{TMR}=1-G_D^{AP}/G_D^P$. For large $|V|$, the TMR gradually approaches its minimum value: $\text{TMR}=1-(G_D^{AP}+G_{I0}^{AP})/(G_D^P+G_{I0}^P)$, where the conductance due to indirect tunneling is maximized. For $g\leq 1$ the slope of TMR is discontinuous at $V=0$, leading to a cusplike peak at zero-bias voltage. The indirect tunneling disfavors the TMR mainly due to the spin-flip scattering on the impurities (finite r_{sp}), and the contribution of $G_I^{P(AP)}(V)$ to the total conductance increases with $|V|$. As a result, the TMR decreases with increasing $|V|$, as shown in Fig. 3(a). The smaller g is, the more sharply the TMR decreases. The inset shows the TMR for two different values of r_{sp} at $g=0.6$. Clearly, for smaller r_{sp}, the TMR decreases faster with $|V|$.

We finally compare the present theory with the experimental data.[8] In Fig. 3(b), the solid squares are the TMR measured at 4.2 K, taken from Fig. 4 (squares) of Ref. 8, and the solid line is the present theoretical curve fitting with the experimental data. We set $R_D^{AP}=1.6R_D^P$, such that the calculated maximum TMR at zero bias is 38%, being the same as the experimental result.[8] From Fig. 3(b), we can see that important features of the experimental data, including the cusplike peak at zero-bias voltage and the sharp drop with increasing bias voltage, are well reproduced by the present theory. We note that our model may be applicable to double-barrier junctions[8] as well. The more complicated bias dependence of the TMR ratio recently observed in tunnel junctions with MgO insulator may originate from relatively complex band structure of MgO and resonant tunneling through narrow interface states, which are not considered in the present simplified model.

In summary, we have proposed a phenomenological theory which reproduces the anomalous bias dependence of the tunneling conductance and tunneling magnetoresistance in magnetic tunnel junctions. This simple theory demonstrates clearly that energy dissipation and spin-flip effect are two key mechanisms for accounting for the zero-bias anomaly in the tunneling conductances and tunneling magnetoresistance.

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