Interplay between the Fulde-Ferrell-like phase and Larkin-Ovchinnikov phase in the superconducting ring pierced by an Aharonov-Bohm flux

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We study the phase diagram of a superconducting ring threaded by an Aharonov-Bohm flux and an in-plane magnetic Zeeman field. The simultaneous presence of both the external flux and the in-plane magnetic field leads to the competition between the Fulde-Ferrell (FF)-like phase and the Larkin-Ovchinnikov (LO) phase. Using the Bogoliubov-de Gennes equation, we investigate the spacial profile of the order parameter. Both the FF-like phase and the LO phase are found to exist stably in this system. The phase boundary is determined by comparing the free energy. The distortion of the phase diagrams due to the mesoscopic effect is also studied.

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I. INTRODUCTION

In recent years, one of the inhomogeneous superconducting states, known as Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state, has received a lot of interest. This superconducting state with periodical spacial variation in order parameter (OP) was first proposed independently by Fulde and Ferrell[1] and by Larkin and Ovchinnikov[2] in 1960s. The possible evidence of its existence has been reported in certain unconventional superconductors[3–5] and the possibility of its realization in trapped cold atoms.[5–10] In literature, the state is collectively known as the FFLO state.[11] Actually, they are two kinds of states with slight difference: the OP of the LO state is real and spatially inhomogeneous, which breaks the translational symmetry, while the OP of the FF state has a uniform magnitude, but an inhomogeneous phase similar to that of a plane wave, breaks the time-reversal symmetry. According to previous studies, the FF state is usually unstable and unfavorable in comparison with the LO state. Although the LO to FF phase transition was predicted in Ref. 12, a more recent study[13] shows that there is no stable FF phase in such a system and there is no LO to FF phase transition either. The authors in Ref. 14 mention a possible FF state in a momentum space study, but as to the best of our knowledge, a realization of stable FF state in the presence of a Zeeman field has not been reported yet in a real-space calculation.

As is well known when a Zeeman field is added to a superconductor, the LO state becomes favorable in comparison with the Bardeen-Cooper-Schrieffer (BCS) state, irrespective of the geometry of the superconductor. Meanwhile, we notice that in a superconducting ring, which is threaded by a magnetic flux, the Aharonov-Bohm (AB) flux breaks the time-reversal symmetry in much the same spirit as that in the FF phase.[15] As a result the FF-like state comes out. We note that the original FF state proposed by Fulde and Ferrell[1] has no net current, but the state in Ref. 15 has nonzero current. We call the state induced by AB flux FF-like state to distinguish it from the original FF state proposed by Fulde and Ferrell.[1] An interesting question is then if we add both the magnetic flux and an in-plane magnetic field, how will the two phases compete with each other? Motivated by this observation, we study in this paper the interplay between this AB flux-driven FF-like phase and the Zeeman field-induced LO phase. It is of great interest to study the phase transitions and phase diagram in such a system. The investigation is carried out in a tight-binding model for a superconducting ring pierced by an AB magnetic flux, and in the presence of a Zeeman magnetic field. We solve self-consistently the Bogliubov-de Gennes (BdG) equation for the superconducting OP and determine the phase diagram by comparing the total energy. We find that for this system, there are four different phases when we vary the two parameters, magnetic flux Φ and the Zeeman field h. More interestingly, we also study the mescscopic effect.

The paper is organized as follows: in Sec. II, we introduce the tight-binding model and present the mean-field treatment. In Sec. III, we numerically carry out the calculation of superconducting OP as a function of the magnetic flux and Zeeman field, and determine the phase diagram by comparing the free energies. Section IV is the discussion and conclusion.

II. MODEL AND MEAN-FIELD TREATMENT

We consider a one-dimensional superconducting ring threaded by an external magnetic flux Φ (see Fig. 1). Meanwhile, there is an in-plane magnetic field B, which generates

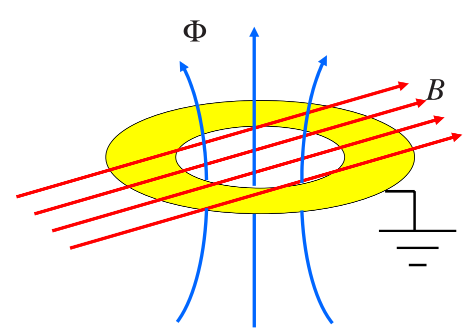

FIG. 1. (Color online) Schematic illustration of the setup. A superconducting ring is threaded by an external magnetic flux, denoted by Φ. A magnetic field B is applied in the plane of the ring. The ring is connected to the ground to ensure that the chemical potential is fixed, but the electron number may fluctuate.

the Zeeman spliting and gives rise to the inhomogeneous pairing. The system is described by the following Hamiltonian:

$$H = -\sum_{i,j,\sigma} \tilde{t}_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + g\mu_B B \sum_{i,\sigma} \sigma c_{i\sigma}^{\dagger} c_{i\sigma} - V\sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i,\sigma} c_{i\sigma}^{\dagger} c_{i\sigma}. \quad (1)$$

Here $\tilde{t}_{ij}=t_{ij}e^{i2\pi\Phi/N\Phi_0}$, where t_{ij} is the bare hopping coefficient, $\Phi_0=hc/e$ is the normal-state flux quantum, and N is number of lattice sites for the ring. $c_{i\sigma}^{\dagger}$ ($c_{i\sigma}$) is the creation (annihilation) operator on the ith lattice site with spin $\sigma=\pm 1$ for spin-up and spin-down electrons, arising from the interaction between the magnetic field and the spin of the electrons; $n_{i,\sigma}=c_{i\sigma}^{\dagger}c_{i\sigma}$ is the particle number on the ith site with spin σ; μ_B is the Bohr magneton; B is the strength of the in-plane magnetic field; V is the strength of the on-site pairing interaction; and μ is the chemical potential. For simplicity, we define $h=g\mu_B B$ as the strength of the Zeeman field; g is equal to 2. In the present work, we take t_{ij} to be t between nearest-neighboring sites and zero otherwise. Within the mean-field approximation, the Hamiltonian (1) is reduced to

$$H = -\sum_{i,j,\sigma} \tilde{t}_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + h\sum_{i,\sigma} \sigma c_{i\sigma}^{\dagger} c_{i\sigma} - \mu \sum_{i,\sigma} c_{i\sigma}^{\dagger} c_{i\sigma} + V\sum_{i} (\Delta_i c_{i\uparrow}^{\dagger} c_{i\downarrow}^{\dagger} + \text{H.c.}) + \sum_{i} \frac{|\Delta_i|^2}{V}, \quad (2)$$

where $\Delta_i \equiv V\langle c_{i\uparrow} c_{i\downarrow}\rangle$ is the pair potential. To diagonalize this Hamiltonian, we employ the following Bogoliubov transformation:

$$c_{i\sigma} = \sum_{\nu} [u_{i\sigma}^{\nu}\gamma_{\nu} - \sigma(v_{i\sigma}^{\nu})^{*}\gamma_{\nu}^{\dagger}],$$

$$c_{i\sigma}^{\dagger} = \sum_{\nu} [(u_{i\sigma}^{\nu})^{*}\gamma_{\nu}^{\dagger} - \sigma v_{i\sigma}^{\nu}\gamma_{\nu}], \quad (3)$$

corresponding to the eigenvalues E_{ν}^{σ}, where γ_{ν} and $\gamma_{\nu}^{\dagger}$ are the quasiparticle operators. The coefficients $(u_{i\sigma}^{\nu}, v_{i\sigma}^{\nu})$ satisfy the BdG equation[16]

$$\sum_{j} \begin{bmatrix} H_{ij\sigma} & \Delta_i \delta_{ij} \\ (\Delta_i)^{*}\delta_{ij} & -\bar{H}_{ij\sigma} \end{bmatrix} \begin{bmatrix} u_{j\sigma}^{\nu} \\ v_{j\bar{\sigma}}^{\nu} \end{bmatrix} = E_{\nu}^{\sigma} \begin{bmatrix} u_{i\sigma}^{\nu} \\ v_{i\bar{\sigma}}^{\nu} \end{bmatrix}, \quad (4)$$

where $H_{ij\sigma}=-\tilde{t}_{ij}-\mu\delta_{ij}+\sigma h\delta_{ij}$ and $\bar{H}_{ij\sigma}=[-\tilde{t}_{ij}-\mu\delta_{ij}]^{*}+\bar{\sigma}h^{*}\delta_{ij}$. The self-consistent equation of the pair potential

$$\Delta_i = \frac{V}{2}\sum_{\nu=1}^{2N} u_{i\uparrow}^{\nu}(v_{i\downarrow}^{\nu})^{*}\tanh\frac{E_{\nu}^{\uparrow}}{2T} \quad (5)$$

is solved by iteration. Here T is the temperature (the Boltzmann constant $k_B=1$ has been taken). Notice that the quasiparticle energy is measured with respect to the chemical potential.

III. NUMERICAL RESULTS

In our numerical calculation, we take the energy unit $t=1$, and the chemical potential $\mu=-0.5$, the interaction strength $V=2$, and the ring size $N=50$. Though the system size is far from the thermodynamic limit, it already gives the same phase boundary as that of infinite N. The OP structure depends not only on the Zeeman field h but also the magnetic flux Φ. We note[15,17] that all physical quantities have already been a function of Φ with a period of Φ_0 even in the normal state. Therefore, it is sufficient for us to consider the magnetic flux in the range $\Phi \in [0,\Phi_0]$. In the absence of the magnetic flux $\Phi=0$, the BCS OP $\Delta=0.351$ for $h=0$ and the LO state is stable for $h_{c1}<h<h_{c2}$ with $h_{c1}=0.23$ and $h_{c2}=1.56$. The system becomes normal ($\Delta=0$) for $h>h_{c2}$. In the presence of the magnetic flux, the magnetic flux can induce a change in the structure of the BCS state in an s-wave superconductor, namely, a crossover from the BCS state in the absence of a magnetic flux to a FF-like state with a magnetic flux when the Zeeman field is low $h<h_{c1}$. When the Zeeman field increases, the LO becomes favorable and both BCS and FF-like states give in. If we continue to increase the Zeeman field, the amplitude of the pairing potential of the LO phase will be suppressed by the Zeeman field until it disappears finally, and the system enters the normal state. To better demonstrate our ideas, we illustrate the OP of normal state, BCS state, LO state, and FF-like state in Fig. 2. It can be seen that the OP of the LO and BCS states have only real component. Moreover, the OP of LO state is sinusoidal. The OP of FF-like sate has both real and imaginary components, and both of them are sinusoidal. We would like to mention that in a recent publication,[5] solitonlike solutions of the OP are obtained in annular disks, which means the OP contains higher order Fourier components. But in our current calculations, there is no higher order Fourier components. Our result is in agreement with the original work of Larkin and Ovchinnikov.[2] In the following, we will numerically construct the phase diagrams.

A. Phase boundary in h-Φ plane

We first focus on the low-temperature case $\beta=1/T=200$ (corresponding to $T=0.005$). In the absence of the magnetic flux, there are three different phases: BCS, LO, and normal. In the presence of the magnetic flux, there are also three phases, FF-like, LO, and normal states. In the following, we study the phase transitions and the phase boundaries for fixed temperature when varying h and Φ.

In order to check if the FF-like state becomes the ground state, we assign a periodic phase to the OP at each site as an initial condition. Similarly, we assign a constant phase to see if BCS state becomes the ground state. For a set of fixed parameters (h,Φ,N,T), different stable solutions (with different OP textures) could be obtained from different initial configurations. For example, one may find both stable LO-type OP and FF-type OP for the same set of parameters ($N=50$, $\beta=200$, $\Phi=0.25\Phi_0$, $h=0.25$). Even there are more than one stable LO-type solutions for the same set of parameters, which means different net momentum of the Cooper pair. To distinguish one state from other competing

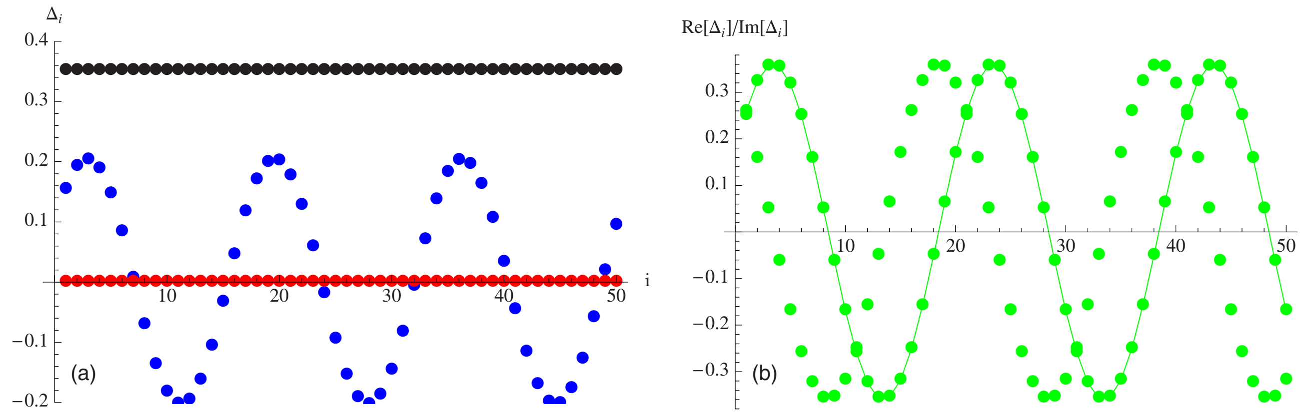

FIG. 2. (Color online) (a) Order parameters of BCS state, normal state, and LO state. Here we choose the ring size $N=50$, the temperature $\beta=200$, the magnetic flux $\Phi_T=0$, and the Zeeman field for BCS, LO, and normal states are $B=0.01$, 0.35, and 1.58, respectively. (b) The real and imaginary parts of the order parameter of FF-like state. They are represented by joined dots and unjoined dots, respectively. The magnetic flux and the Zeeman field for FF-like state is chosen to be $\Phi_T=0.25\Phi_0$ and $B=0.01$.

states (including BCS state and FF-like state), we choose the energetically most favored one by comparing their free energies. For the model in Eq. (1), the free energy is given by

$$F = -\frac{1}{\beta}\sum_{\nu} \ln(1 + e^{-\beta E_{\nu}^{\uparrow}}) + \sum_{i} \frac{|\Delta_i|^2}{V} - \sum_{i} (\mu + h). \quad (6)$$

Here, we just compare the summation of the first two terms, because the third term is a constant for all solutions of different phases. In the following we determine the phase boundary between FF-like state, LO state, and normal state.

1. First-order transition between the FF-like and LO phases

When determining the pair potential self-consistently by iteration, we find that in certain range of the strength of the in-plane Zeeman field h, different initial configurations of the pair potential lead to different stable solutions. In another word, there are more than one stable solutions through iteration. For example, when we fix $\Phi=\Phi_0/4$, and vary the magnetic field in the range $0.08<h<0.29$, stable solutions of both the FF type and the LO type can be arrived at through iteration. The free energies of these two types of stable solutions are listed in Table I. It can be seen that the LO state becomes energetically favorable when the magnetic field is equal to or greater than $h_{c1}=0.21$. In addition, the free energy at h_{c1} is continuous, but its first-order derivative is not continuous. Hence, we conclude that for a fixed magnetic flux $\Phi=\Phi_0/4$, there is a first-order phase transition between the FF-like and LO states at h_{c1}. Similarly, we fix magnetic flux Φ at different values and we can find the threshold value of h at which the system changes from the FF-like state to the LO state or vise versa. Thus for a fixed temperature $\beta=200$ and fixed system size $N=50$, the phase transition line between FF-like and LO states is determined by comparing the free energy of the FF-like phase and the LO phase, and we plot it in Fig. 3. To ensure that the phase boundary given by $N=50$ is close to that of the thermodynamic limit, we change the system size to $N=200$, and we find the phase boundary does not change. For $N=50$ and 200, the magnitude of the OP Δ_i in BCS phase is the same. Hence the result based on $N=50$ can be regarded as in thermodynamic limit. It can be seen that the first-order transition line is not parallel to the Φ axis, so we can tune the flux to make the system change from the LO phase to the FF-like phase or vise versa keeping the Zeeman field constant. We call this phase transition AB-effect-induced phase transition. We can also see that the phase boundary between the LO and FF-like states is symmetric around $\Phi=\Phi_0/4$, and the period of FF-like phase is $\Phi_0/2$. Specifically, the LO state exists in the range [0.23,1.56], [0.21,1.52], and [0.23,1.51] for $\Phi=0$, $\Phi_0/4$, and $\Phi_0/2$, respectively.

2. Second-order transition between the LO- and normal-state phases

If we continue to increase the in-plane magnetic field above the value $h=0.29$ for Φ fixed at $\Phi_0/4$, all initial configurations of the pair potential will lead to the LO state, or

TABLE I. Free energies [up to a constant $-\Sigma_{i=1}^{N}(\mu+h)$] for stable solutions of FF-like state and LO state. Here the ring size is $N=50$, the magnetic flux $\Phi=\Phi_0/4$, and the temperature $\beta=200$. It can be seen that there is a first-order phase transition from the FF-like state to LO state when the in-plane magnetic field is tune across $h=0.21$.

	$h=0.20$	$h=0.21$	$h=0.22$	$h=0.23$	$h=0.24$	$h=0.25$
FF	−56.1487	−55.6487	−55.1487	−54.6487	−54.1487	−53.6487
LO	−56.1019	−55.6319	−55.1619	−54.6919	−54.2219	−53.7519

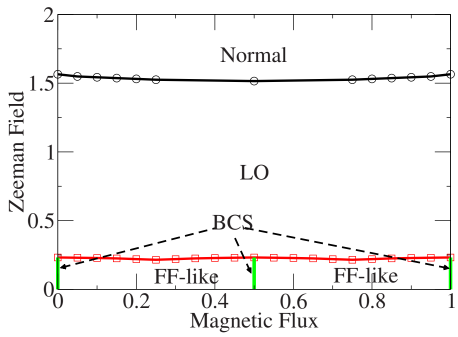

FIG. 3. (Color online) Phase diagram of the superconducting ring in the h-Φ plane. Here the ring size is $N=50$ and the temperature is $T=0.005$. Notice that the boundary line between the FF-like and LO phases has the periodicity in Φ with a period of $\Phi_0/2$ while that between the LO- and normal-state phases has the periodicity in Φ with a period of Φ_0.

only the LO state becomes stable. Meanwhile the amplitude of the pair potential decreases and the period of the modulation of the pair potential is shortened continuously. Further increase in the Zeeman field leads to the reduction in the pair potential until it vanishes gradually. When the magnetic field reaches $h_{c2}=1.52$, the amplitude of the pairing potential vanishes or the LO state is completely compressed by the in-plane magnetic field, and the system changes from the LO state to the normal state. If we do the iteration from zero pair potential, we will find that when $h>h_{c2}$ the stable solution for the pair potential is zero (normal state). When $h\leq h_{c2}$, the stable solution is an LO state. There is no coexistence area of the LO and the normal states in the h axis. Hence we conclude that the phase transition at h_{c2} for a fixed Φ is a second-order phase transitoin. Our result is consistent with previous studies.[11,18,19]

B. Phase boundary in *h*-*T* plane

In the Sec. III A 2, we study the phase transitions when we vary the magnetic flux Φ or the in-plane magnetic field h. The temperature is fixed at a very low value. Hence these phase transitions can be regarded as quantum phase transitions. In this section, we will study the phase transitions induced by thermal fluctuations and determine their phase boundaries. We will fix the magnetic flux Φ and vary the temperature β or the in-plane magnetic field h. First we consider the case in the absence of the magnetic flux $\Phi=0$. We fix the temperature at $T=0.05$, 0.10, 0.15, 0.20, and 0.25, respectively, and do the iteration separately. The phase boundary between the BCS and LO states is determined in a similar way to that in Sec. III A. It can be seen that when we fix the magnetic flux to be zero, and tune the in-plane magnetic field or the temperature, the system will change between the BCS, LO, and normal states. As can be seen from Fig. 4(a), the LO phase emerges below the critical temperature $T\approx 0.12$. We note that the BCS to the LO state is first order and the LO to normal is second order. When the magnetic flux is nonzero, the BCS state will be replaced by FF-like state with the phase diagram, as shown in Fig. 4(b), very similar to the zero-flux case.

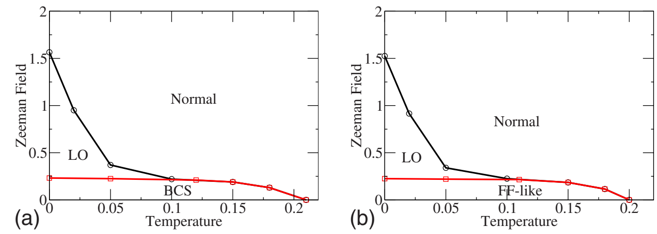

FIG. 4. (Color online) Phase diagram of the superconducting ring in the h-T plane. Here the ring size is $N=50$ and the magnetic fluxes are (a) $\Phi=0$ and (b) $\Phi=\Phi_0/4$, respectively. The LO-state to normal-state transition (black with open circles) is of second order, while the BCS-state (or FF-like) to the normal-state transition (red with open squares) is of first order. The zero-field transition temperature is around $T_c=0.21$ and 0.20 for $\Phi=0$ and $\Phi_0/4$, respectively.

C. Mesoscopic effect

Another interesting question is the mesoscopic effect. In this section we will study the mesoscopic effect by fixing the temperature and decreasing the ring size. As mentioned in the above discussion, the ring size $N=50$ already gives the same phase boundary as that of $N\rightarrow\infty$. A simple check is that when we increase the ring size to $N=100$ and 200, we find that the phase boundaries do not change in comparison with that for $N=50$. This means that for the current model, $N=50$ can be treated as in the thermodynamic limit. However, if we decrease the ring size, for example, to $N=20$, the mesoscopic effect will occur. First, in the h-Φ plane, the LO phase will shrink dramatically and the FF-like phase will expand [see Fig. 5(a)]. This is because (1) the influence of the magnetic flux on the system will increase and the influence of the in-plane Zeeman field will decrease relatively and (2) having a finite size restricts the periodicity of the LO OP. At a given Zeeman field, if the period is not commensurate with the corresponding ring, solutions of the LO state will have to be modified to be commensurate with system size, which results in some energy cost. Therefore, the LO state will shrink. Second, the periodicity of the magnetic flux changes from $\Phi_0/2$ to Φ_0. This is because the system size is so small that the Cooper pair can no longer be treated as a whole and can only be treated as two separate electrons. We show in Fig. 5(b) the phase boundary for $N=10$ and the re-entrant behavior of various phases can be seen in the phase diagram.

IV. DISCUSSION AND CONCLUSION

Based on a tight-binding model, we study a one-dimensional s-wave superconducting ring subject to an in-plane Zeeman field and a magnetic flux by solving the BdG equation in real space. In the presence of a magnetic flux, a crossover from the BCS state to the FF-like state is obtained when the in-plane magnetic field is not very strong. If we increase the strength of the in-plane magnetic field, the LO state becomes favorable, and a FF-like to LO phase transition occurs. With the further increase in the in-plane magnetic field strength, the magnitude of the pair potential of the

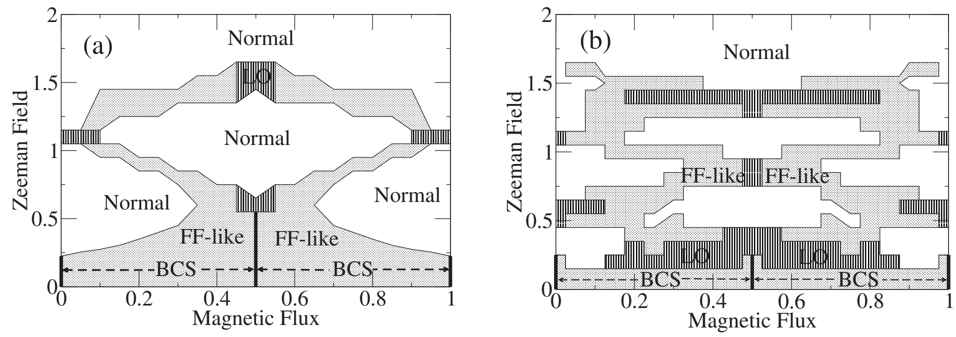

FIG. 5. Phase diagram in the h-Φ plane. All the parameters are the same as that in Fig. 3 except that the ring size is (a) $N=20$ and (b) $N=10$. The empty area represents the normal state. The gray area represents the FF-like state, and the area covered by the thin black lines represents the LO state. The thick black lines represent the BCS state. It can be seen that with the decrease in the ring size, the FF-like phase expand a lot and the LO phase shrink dramatically. The period of FF-like phase also changes from $\Phi_0/2$ to Φ_0.

LO state is suppressed and disappears finally with the system entering the normal state. In the absence of the magnetic flux, there is no FF-like phase, and the Zeeman field induces the transitions between the BCS, LO, and normal states, which has been studied extensively.[18–27] Our results agree well with the previous studies in a two-dimensional system that the energetically favorable state for s-wave superconductor is a one-dimensional stripelike LO state. This suggests the first-order transition between the BCS and LO states while a second-order transition between the LO to normal states. Our study goes beyond that and indicates a stable FF-like state due to the magnetic flux. The mesoscopic effects are also studied. When the system size decreases, two mesoscopic effects arise: (1) the LO phase in the h-Φ plane shrinks, and the FF-like state expands due to the enhancement of the Aharanov-Bohm effect; (2) the periodicity of the external magnetic flux will change from $\Phi_0/2$ to Φ_0.

The theoretical results predicted in this paper could possibly be verified by experiments. In a realistic experimental setup, probably only one external field can be applied. In this situation, the magnetic field orientation should be tuned with respect to the plane, on which the ring lies, such that the magnetic flux is varied while the exchange field remains fixed. Therefore the setup is close to the problem under consideration.

The following remarks are in order. (1) Though we study a one-dimensional model, the system should not be regarded as a mathematically one dimensional. The current study can be easily extended to the two dimensional and other geometry, such as a torus configuration threaded by a magnetic flux. It can be expected that a similar phase transition between LO state and FF-like state will occur. (2) For the one-dimensional case, the LO state exists in a broader range of parameters space (h-T space, see Fig. 3) than that of two-dimensional and three-dimensional cases,[11,28,29] which makes it easier to access experimentally. (3) In Ref. 30, it is reported that a trap potential with arbitrary configuration can be achieved. Hence, we expect that the result presented in this paper should be able to be observed experimentally in cold Fermions under current experiment technique. (4) In the thermodynamic limit, $N\rightarrow\infty$, the FF-like state reproduces the BCS state, because the phase gradient of the OP is vanishingly small. This result agrees with our intuition that when the ring size becomes infinity, the influence of the magnetic flux can be neglected. (5) If we change V from 2 to a smaller number, e.g., $V=1$, we obtain qualitatively the same phase diagram in the large N limit. Quantitatively, the normal-LO and the LO-FF-like phase transitions lines will be lower in the Zeeman field. (6) The effect of the impurity is not included in the current study and will be given in our future studies.

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