Gilbert damping and current-induced torques on a domain wall: A simple theory based on itinerant 3*d* electrons only

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Many electronic theories of Gilbert damping in ferromagnetic metals are based on the *s-d* exchange model, where localized 3*d* magnetic spins are exchanged-coupled to itinerant 4*s* electrons, which provide the needed spin relaxation. Recently, Tserkovnyak *et al.* have obtained Gilbert damping from itinerant 3*d* electrons alone, which have their own spin relaxation. We show that simple semiclassical equations of motion for precessing itinerant 3*d* spins predict exactly the same formula $\alpha=1/(\omega_d\tau_{sr}^d)$ for the Gilbert damping constant as the full Green's function quantum treatment by Tserkovnyak *et al.* Here, ω_d is the precession frequency of 3*d* spins in the *d-d* mutual exchange field, and τ_{sr}^d the 3*d* spin-relaxation time. A correct form for the spin-relaxation torque is crucial for success: The spins relax toward an instantaneous direction which is that of the vector sum of external field and *d-d* exchange field. Remarkably, *d-d* exchange torques disappear completely from the equations of motion for the total 3*d* magnetization, and exchange plays only an indirect role through the spin relaxation. This purely 3*d* model is simpler than the traditional *s-d* model. We also present a theory of current-induced torques on a domain wall, based on the 3*d* model. We find equivalents to the so-called adiabatic and nonadiabatic torques. They are given by formulas similar to those holding for the *s-d* exchange model.

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I. INTRODUCTION

Damping of the motion of magnetic spins in ferromagnets is of the kind described by Gilbert, where the damping rate is proportional to the spin precession frequency. Many electronic theories for metallic ferromagnets are based on the *s-d* exchange model,[1] where localized 3*d* magnetic spins **S** are coupled to itinerant 4*s* electron spins **s** by an interaction $E=-2J_{sd}\mathbf{S}\cdot\mathbf{s}$, where $J_{sd}\simeq 0.1-0.2$ eV.

Because of the momentum gap existing[2] between spin-up and spin-down Fermi surfaces, no damping is obtained at $T=0$ unless spin relaxation,[3] connected with a combination of spin-orbit interaction and electron scattering, is introduced for the 4*s* electrons. It is represented by a spin-relaxation time $\tau_{sr}^s\simeq 10^{-12}-10^{-13}$ s. One exception is the theory of Mills *et al.*,[4] who showed that spin orbit can be replaced here by *s-d* exchange itself.

Using *s-d* exchange and coupled semiclassical equations of motion for **S** and **s**, Turov[5] derived the value of the ferromagnetic resonance linewidth. It is directly related to the dimensionless Gilbert damping parameter α. In the limit $\omega_s\tau_{sr}^s\gg 1$, this reduces to

$$\alpha=\frac{s}{(S+s)(\omega_s\tau_{sr}^s)}. \tag{1}$$

Here, S and s are the magnitudes of **S** and **s**, with units of atom^{-1}. The quantity $\omega_s=2J_{sd}S/\hbar$ would represent the **s** precession frequency in the *s-d* exchange field set up by **S**, if that field had a constant direction.

Later, Heinrich *et al.*[6] treated this problem with a Green's function formalism. Remarkably, this quantum treatment yields exactly the same expression for α [Eq. (1)] as the simple equations of Turov[5] for the classical precession of **S** and **s**.

Recently, Tserkovnyak *et al.*[7] obtained Gilbert damping from itinerant 3*d* electrons alone, assumed to have their own spin-relaxation time τ_{sr}^d.This purely 3*d* model leads to

$$\alpha=\frac{1}{\omega_d\tau_{sr}^d}, \tag{2}$$

where ω_d would be the 3*d*-spin precession frequency in the Stoner exchange field generated by all the other 3*d* itinerant spins, if that field had a fixed direction.

The purpose of the present paper is to show that a simple classical equation of motion for a precessing 3*d* spin predicts exactly the same formula for α [Eq. (2)] as the full quantum treatment by Tserkovnyak *et al.*[7] which uses Keldysh Green's functions combined with the Boltzmann equation. The present approach also provides a clear physical picture [Fig. 1(b)] of processes involved in Gilbert damping.

Actually, the fact that the present model uses only one kind of electron is more important than the exact *d* or *s* nature of such electrons.

II. *S-D* EXCHANGE MODEL

The equations of motion for the localized 3*d* magnetic spin **S** and the itinerant 4*s* conduction-electron spin **s** [Fig. 1(a)] are[5]

$$\hbar\frac{d\mathbf{s}}{dt}=-g\mu_0\mu_B\mathbf{s}\times(\mathbf{H}+\mathbf{H_{sd}})-\hbar\frac{(\mathbf{s}-\mathbf{s_0})}{\tau_{sr}^s}$$

$$\hbar\frac{d\mathbf{S}}{dt}=-g\mu_0\mu_B\mathbf{S}\times(\mathbf{H}+\mathbf{H_{ds}}). \tag{3}$$

Here, **H** is the external static field, $\mathbf{H_{sd}}=-2J_{sd}\mathbf{S}/g\mu_0\mu_B$ the exchange field exerted by **S** on **s**, μ_0 the vacuum perme-

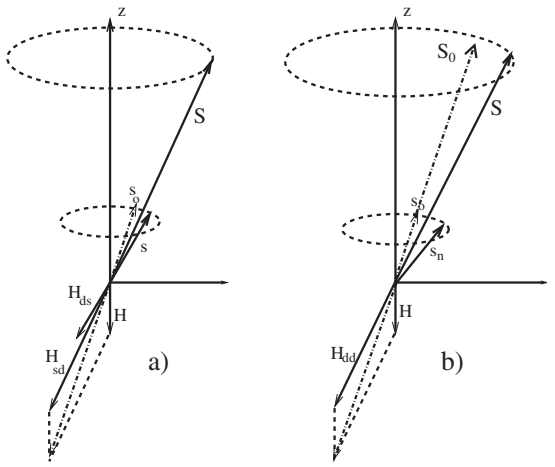

FIG. 1. (a) 4*s* conduction-electron spin **s** and 3*d* magnetic-electron spin **S** precessing around the magnetic field **H**. The *s-d* exchange field $\mathbf{H}_{sd}$ is antiparallel to **S** and acts on **s**; and vice versa for $\mathbf{H}_{ds}$. The vector $\mathbf{s}_0$ is antiparallel to the total field $\mathbf{H}+\mathbf{H}_{sd}$ acting on **s**, and is the direction toward which **s** is relaxing. (b) 3*d* individual spin $\mathbf{s}_n$ and total 3*d* spin $\mathbf{S}=\Sigma_n\mathbf{s}_n$ precessing around the magnetic field **H**. The *d-d* mutual exchange field $\mathbf{H}_{dd}$ is antiparallel to **S** and acts on $\mathbf{s}_n$. The vector $\mathbf{s}_0$ is antiparallel to $\mathbf{H}+\mathbf{H}_{dd}$ and is the direction toward which $\mathbf{s}_n$ is relaxing.

ability in the SI system of units (Systeme International), and $\mathbf{H_{ds}}=-2J_{sd}\mathbf{s}/g\mu_0\mu_B$ the field exerted by **s** on **S**. Also, μ_B is the Bohr magneton. The *g*-factor *g* is assumed for simplicity to have the same value for **S** and for **s**.

In Eqs. (3), $\mathbf{s_0}$ is the instantaneous direction toward which **s** is relaxing. As discussed a long time ago by Hasegawa,[8] this direction should be antiparallel to the total field $\mathbf{H}+\mathbf{H_{sd}}$ acting on **s** [Fig. 1(a)]

$$\mathbf{s_0}=-s\frac{\mathbf{H}+\mathbf{H_{sd}}}{|\mathbf{H}+\mathbf{H_{sd}}|}. \tag{4}$$

This choice of $\mathbf{s_0}$ represents the instantaneous direction where the total Zeeman energy of **s** would be minimum. It is a logical choice since, during spin relaxation, the Zeeman energy is lost to the lattice through emission of phonons.

Choices which differ from Eq. (4) would lead[5,8] to shifts in the **S** precession frequency, away from the usual value; such shifts are not observed in actual resonance experiments. Note also that Walker[9] has derived Eq. (4) on the basis of Fermi-liquid theory.

We introduce coordinates *x*, *y*, and *z*, with *z* antiparallel to **H** [Fig. 1(a)], and look for solutions of Eqs. (3) and (4) of the form $s_+(t)=s_+(0)e^{-(\Gamma+i\omega)t}, S_+(t)=S_+(0)e^{-(\Gamma+i\omega)t}$, where $s_+=s_x+is_y$. We assume $H\ll H_{sd},H_{ds}$ and $|s_+|\ll s,|S_+|\ll S$. We introduce the quantity $\omega_s=2J_{sd}S/\hbar$. It would represent the **s** precession frequency around $\mathbf{H_{sd}}$ if the latter had a fixed direction. We obtain in the limit $\omega_s\tau_{sr}^s\gg 1$

$$\omega\simeq\frac{g\mu_0\mu_BH}{\hbar};\quad\Gamma\simeq\frac{s\omega}{(s+S)\omega_s\tau_{sr}^s}. \tag{5}$$

Then, the Gilbert damping parameter, defined as $\alpha=\Gamma/\omega$, is given by Eq. (1) in agreement with Refs. 5 and 6. Interestingly, starting with a Bloch-type spin-relaxation term in the equations of motion [Eqs. (3)], we arrived nevertheless [Eq. (5)] to a Gilbert form for the damping rate Γ, i.e., with $\Gamma\propto\omega$. The **H** term in Eq. (4) is responsible for this.

III. ITINERANT *D-D* MODEL

In this model, we consider only itinerant 3*d* electrons, in Bloch waves with various wave vectors and spin states, labeled with the index $n=1,2,3,\ldots$. Paired spin-up and spin-down electrons of same wave vector give zero total spin, and can be ignored. Only the remaining unpaired spin-up states matter. Since they all have different wave vectors, they can have nonorthogonal spin parts while still being orthogonal and obeying the exclusion principle. This makes possible a classical picture of individual precessing 3*d* spins, pointing in different directions, with increased exchange energy.

As mentioned before, the fact that only one kind of electron appears in the model is more important than the exact *d* or *s* nature of such electrons. Actually, the two kinds of states are significantly mixed through *s-d* hybridization. This question will be discussed in more detail in Sec. VI.

As in the last section, we write a classical equation of motion for the spin $\mathbf{s}_n$ [Fig. 1(b)] of an individual 3*d* electron

$$\hbar\frac{d\mathbf{s}_n}{dt}=-g\mu_0\mu_B\mathbf{s}_n\times(\mathbf{H}+\mathbf{H_{dd}})-\hbar\frac{\mathbf{s}_n-\mathbf{s}_0}{\tau_{sr}^d}. \tag{6}$$

Here, $\mathbf{H_{dd}}$ is the *d-d* (Stoner) exchange field [Fig. 1(b)] acting on $\mathbf{s}_n$, generated by all other itinerant 3*d* electrons, and $\tau_{sr}^d\simeq 10^{-13}-10^{-14}$ s the 3*d* spin-relaxation time. Also, **S** is the total spin of 3*d* electrons in the system, with $\mathbf{S}=\Sigma_n\mathbf{s}_n$. The total exchange energy is $-2J_{dd}\Sigma_{n>m}\Sigma_m\mathbf{s_n}\cdot\mathbf{s_m}$. Then, $\mathbf{H_{dd}}$ is given by $\mathbf{H_{dd}}=-J_{dd}\mathbf{S}/g\mu_0\mu_B$. For simplicity, we assume the *d-d* exchange integral J_{dd} to have the same value between all pairs of 3*d* states. Band-structure calculations are consistent[10] with $J_{dd}\simeq 0.5$ eV.

Similarly to the last section, and for the same reasons, $\mathbf{s}_n$ relaxes [see Fig. 1(b)] toward the direction

$$\mathbf{s}_0=-s\frac{\mathbf{H}+\mathbf{H_{dd}}}{|\mathbf{H}+\mathbf{H_{dd}}|}. \tag{7}$$

The remarks about $1/\tau_{sr}^s$ made in that section also apply to $1/\tau_{sr}^d$. The mechanism of spin relaxation of Ref. 3 works for 3*d* electrons, since these are now assumed itinerant. We sum Eq. (6) over n, to obtain an equation of motion for the total 3*d* spin **S**

$$\hbar\frac{d\mathbf{S}}{dt}=-g\mu_0\mu_B\mathbf{S}\times\mathbf{H}-\frac{\hbar}{\tau_{sr}^d}(\mathbf{S}-\mathbf{S}_0), \tag{8}$$

where $\mathbf{S}_0=-S(\mathbf{H}+\mathbf{H}_{dd})/(|\mathbf{H}+\mathbf{H_{dd}}|)$. We see that exchange torques have disappeared from Eq. (8). The reason is that these are internal to the 3*d*-electron system, not external as in the case of the *s-d* exchange model of last section. Exchange appears only indirectly, through $\mathbf{S}_0$ in the spin-relaxation term. We define the quantity $\omega_d=g\mu_0\mu_BH_{dd}/\hbar$. It would represent the $\mathbf{s}_n$ precession frequency around $\mathbf{H}_{dd}$ if the latter had a fixed direction. Similarly, we define $\omega=g\mu_0\mu_BH/\hbar$ and

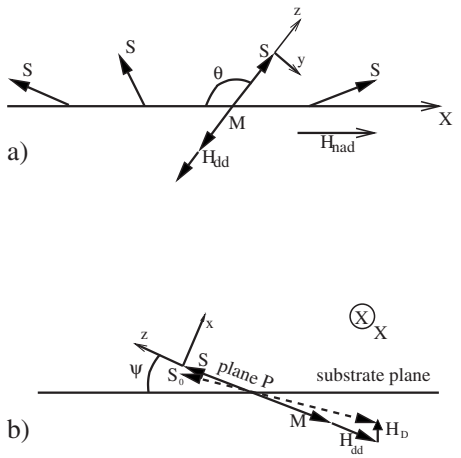

FIG. 2. (a) Simple tail-to-tail domain wall in a nanowire. The X axis runs along the length of the nanowire. The total $3d$ spin $\mathbf{S}$ makes an angle $\theta(X,t)$ with the $-X$ axis. The plane of the picture is plane P which contains all the spins $\mathbf{S}$ and makes an angle ψ with the plane of the substrate. Local spin coordinates x, y, and z have the z axis parallel to $\mathbf{S}$, and x normal to plane P and to the X axis. (b) View of the same domain wall, with the plane of the picture normal to the X axis. Plane P, which contains the spins $\mathbf{S}$, is at an angle ψ to the plane of the substrate. The vector $\mathbf{S}_0$ is antiparallel to the total field $\mathbf{H}_{dd}+\mathbf{H}_D$ and is the direction toward which $\mathbf{S}$ is relaxing.

$S_+=S_x+iS_y$, with z antiparallel to $\mathbf{H}$ [Fig. 1(b)]. After assuming $H \ll H_{dd}, |s_+| \ll s, |S_+| \ll S$, Eq. (8) gives

$$\frac{dS_+}{dt} = -i\omega S_+ - \frac{\omega}{\omega_d \tau_{sr}^d} S_+$$

$$\frac{dS_z}{dt} \simeq 0.$$

To first order in the precession amplitude $|S_+|$, the modulus of $\mathbf{S}$ is constant. Again, we look for a solution of the form $S_+(t)=S_+(0)e^{-(\Gamma+i\omega)t}$, and find immediately

$$\Gamma = \frac{\omega}{\omega_d \tau_{sr}^d}. \tag{9}$$

Then, $\alpha=\Gamma/\omega$ is given by Eq. (2) in agreement with Ref. 7. Again, and for the same reasons, Γ is of the Gilbert form.

Even when taking into account s-d hybridization, we have $\omega_s < \omega_d$ but $\tau_{sr}^s > \tau_{sr}^d$. Thus, the dimensionless parameters $\omega_{sr}^s \tau_{sr}^s$ in Eq. (1) and $\omega_{sr}^d \tau_{sr}^d$ in Eq. (2) may have comparable values $\simeq 10-100$.

IV. CURRENT-INDUCED TORQUES ON A DOMAIN WALL, IN THE 3*d* MODEL

We consider a tail-to-tail wall in a nanowire [Fig. 2(a)]. The spatial coordinate X runs along the length of the nanowire. The total $3d$ spin $\mathbf{S}$ at location X makes an angle $\theta(X,t)$ with the $-X$ axis. As an approximation,[11] we assume that the vector $\mathbf{S}$ in the wall is everywhere contained in a plane P parallel to the X direction, which makes an angle ψ with the substrate plane [Fig. 2(b)]. In a static wall at zero current, we have $\psi=0$. The sign convention for ψ is such that it increases when $\mathbf{S}$ turns toward the $-x$ direction. We introduce local spin coordinates x, y, and z with z parallel to $\mathbf{S}$ and x normal to X and to plane P [Fig. 2(a)].

When ψ differs from zero, the canted magnetization creates[11] in the wall a demagnetizing field $\mathbf{H}_D$. If the nanowire thickness is much less than the width, this field is normal to the substrate plane. The component of $\mathbf{H}_D$ along the normal to plane P is $H_D^x=-H_D \cos\psi=-M \sin\theta \sin\psi \cos\psi$. The torque exerted by H_D^x on the total $3d$ spin $\mathbf{S}$ is in plane P and is

$$\tau_y = (\mu_0 M_s^2/2)\sin(2\psi)\sin\theta. \tag{10}$$

The usual energy eigenstates of an itinerant electron are plane waves where the spin direction is the same at all locations. However, more general "spiral states" have been introduced[12] to represent itinerant electrons in domain walls. As long as the wall width is much larger than an electron wavelength, the spatial variation in the direction of $\mathbf{S}$ is slow and there is no difference with the usual theory of domain walls based on localized electrons. The structure of a simple transverse wall is given[13] by $\theta=f(X-v_w t)/\Delta)$ where v_w and Δ are the wall speed in the laboratory frame and the wall width, and f(u) is a certain function.

In earlier sections, there was no electric current. We introduce now the current density $j_\uparrow$ carried by spin-up $3d$ electrons, as seen from the laboratory frame. The existence of such a $3d$ current will be discussed further in Sec. VI.

The effect on $\mathbf{S}$ of torque τ_y is evaluated in the simplest manner[14] in a moving frame where the electron gas is at rest and, therefore, the spin current vanishes and causes no additional torque. The torque itself is the same in all frames. In the case of spin-up electrons, the speed of that moving frame is $v_e^\uparrow=-j_\uparrow/n_e^\uparrow e$, where $n_e^\uparrow$ is the spin-up electron density. In that frame, the spin-up parts of τ_y and $\mathbf{S}$ are related by

$$\tau_y^\uparrow = \hbar S_z^\uparrow \partial\theta/\partial t = -\hbar S_z^\uparrow (f'/\Delta)(v_w - v_e^\uparrow), \tag{11}$$

where $f'(u)=df/du$, and where $v_w-v_e^\uparrow$ is the apparent speed of the wall as seen from the moving frame.

It is also possible to derive Eq. (11) in the laboratory frame. In that frame, the apparent wall speed is v_w, not $v_w - v_e^\uparrow$. Also, the current density $j_\uparrow$ present in that frame generates a $3d$ spin current $\mathbf{j}_\mathbf{s}^\uparrow$, leading to an extra term $-div\mathbf{j}_\mathbf{s}^\uparrow$ in Eq. (11). These two changes cancel each other, so that we obtain the same Eq. (11) as before.

By working in the moving frame, we have shown that the case with current can be reduced to the case without current, by a simple change in frame. Also, we have avoided the introduction of the spin current.

We also write a expression similar to Eq. (11) for the contribution $\tau_y^\downarrow$ of spin-down electrons. Because of the exclusion principle and of orthogonality, the spins $\mathbf{S}_\uparrow$ and $\mathbf{S}_\downarrow$ of spin-up and spin-down electrons stay closely antiparallel. By equating $\tau_y^\uparrow+\tau_y^\downarrow$ to τ_y of Eq. (10), and using the fact[13] that $f'=\sin\theta$ for a uniaxial anisotropy, we obtain finally

$$(1/2)\sin(2\psi) = -[v_w - (P/P_n)v_e]/\omega_D\Delta$$

$$P = \frac{j_\uparrow - j_\downarrow}{j_\uparrow + j_\downarrow}; \quad P_n = \frac{n_e^\uparrow - n_e^\downarrow}{n_e^\uparrow + n_e^\downarrow}$$

$$v_e = -j/n_e e; \quad \omega_D = g\mu_0\mu_B M_s/\hbar. \tag{12}$$

Here, μ_B is the Bohr magneton, and all carriers are assumed electronlike. And v_e is the average electron drift speed. Also, $n_e = n_e^\uparrow + n_e^\downarrow$ and $j = j_\uparrow + j_\downarrow$. Note that θ has dropped out of the expression for ψ, thus justifying our assumption of a constant ψ.

The demagnetizing-field torque τ_y of Eq. (10) and, depending on the frame, the $-divj_s$ term are the only external torques along y acting on the $3d$ spins **S** of the wall. The $-divj_s$ term plays the same role in our $3d$ model as the so-called adiabatic torque in the s-d exchange model.[15,17] In the latter theory, that torque had the nature of an s-d exchange torque.

By Eq. (12), the maximum stable value of ψ is $\pi/4$, and the corresponding critical value of the current density is[15]

$$j_\psi = \pm \frac{\mu_0 M_s^2 e\Delta}{P\hbar}. \tag{13}$$

Field $\mathbf{H}_D$ also has a component in the plane P, which has the same effect on **S** as an additional anisotropy with easy axis along X. This tends to reduce the wall width below the value Δ holding at $\psi=0$. This effect varies like ψ^2 at small ψ, and we will ignore it.

V. NONADIABATIC TORQUE

As before [Eq. (7)], each $3d$ spin $\mathbf{s}_n$ relaxes toward the instantaneous direction of the total field acting on it. Here, this field is $\mathbf{H}_{dd}+\mathbf{H}_D$ [Fig. 2(b)]. After summing over n and assuming $|\psi| \ll 1$ rad and $H_D \ll H_{dd}$, **S** is found to relax toward $\mathbf{S}_0 = -S_z(\mathbf{H}_{dd}+\mathbf{H}_D)/H_{dd}$. The spin-relaxation torque acting on **S** is

$$\tau_x = \hbar\frac{(S_0)_x}{\tau_{sr}^d} = -\hbar S\frac{H_D^x}{H_{dd}\tau_{sr}^d} = \hbar\frac{S\omega_D\psi\sin\theta}{\omega_d\tau_{sr}^d}. \tag{14}$$

This spin-relaxation torque plays the same role in the present $3d$ theory as the so-called nonadiabatic torque in theories[16,17] based on the s-d exchange model. Contributions to τ_x from interatomic-exchange and anisotropy torques cancel each other as long as the wall has the structure discussed in the last section. We substitute ψ from Eq. (12) into Eq. (14). Also, torque τ_x is equivalent to the torque $\mu_0 M_s H_{nad}^X \sin\theta$ of a fictitious field $\mathbf{H}_{nad}$ along the easy axis X. From all this, we obtain finally

$$H_{nad}^X = -\frac{\hbar n_e(P_n v_w - P v_e)}{2\mu_0 M_s \Delta(\tau_{sr}^d \omega_d)}, \tag{15}$$

where θ has dropped out. The term in v_w represents Gilbert damping. The positive sign of its coefficient P_n [Eq. (12)] is required by the second law.

In real magnetic materials, it is important to take into account domain-wall pinning, caused by lattice defects. It is characterized[13] by the coercivity H_c. The wall will move whenever $H_{nad}^X = \pm H_c$. Combining this condition with Eq. (15), we obtain

$$v_w = \frac{P}{P_n}(v_e \mp v_{ec}); \quad v_{ec} = \frac{2\mu_0 M_s \Delta(\tau_{sr}^d \omega_d)}{P_n \hbar n_e} H_c. \tag{16}$$

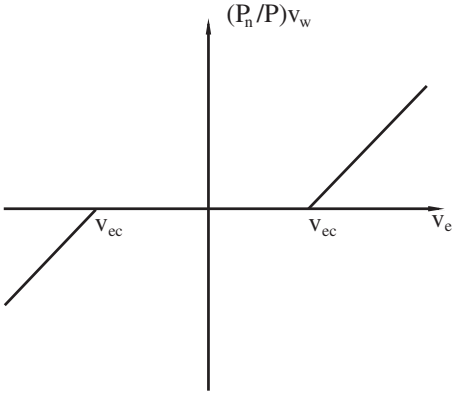

FIG. 3. Normalized wall speed v_w versus average electron drift speed v_e, according to Eqs. (16). Here, P and P_n are the current-polarization and electron-density polarization factors. These are defined in Eqs. (12).

Because of the existence of the coercivity, a minimum electron drift speed v_{ec} is needed before wall motion can start [Eq. (16) and Fig. 3]. For $3d$ electrons, P/P_n is on the order of unity. Then, Eq. (16) shows that v_w is on the order of the electron drift speed v_e, whenever $|v_e|$ exceeds the critical value v_{ec} [Fig. 3].

VI. APPLICABILITY OF 3*d* MODEL

The equilibrium physical and magnetic properties of Ni, Co, and Fe depend primarily[10] on the $3d$ band. By themselves, $3d$ electrons are already itinerant, with a bandwidth[18] of several electron volts. As shown by Hodges *et al.*[19] for Ni, the addition of the $4s$ band causes only minor changes in the structure and bandwidth of that $3d$ band. Despite significant hybridization of $3d$ and $4s$ states, $3d$ electrons retain distinct physical properties, such as high density of states and low velocity. These electrons are the basis of the present d-d model.

This model applies best to the problem of Gilbert damping (Sec. III) in transition-metal materials. The best example is that of Permalloy thin films, studied experimentally[20] by Ingvarsson. For Ni and Co, it has to be complemented by the Kambersky Fermi surface breathing mechanism[21] of damping, which depends in opposite fashion on electron relaxation time.

On the other hand, band-structure calculations for ferromagnetic Ni[19,22] all show that the spin-up Fermi level is located above the top of the $3d$ band, in a region with the low density of states and high electron velocity characteristic of $4s$ electrons. The spin-up Fermi surface of Ni even has[23] necks similar to those of Cu. This is confirmed by ordinary Hall effect data[24] for Ni, Ni-Fe, Ni-Fe-Cu, and Ni-Co, which show that a small number $\simeq 0.3$ el./at. of carriers carry most of the current. Also by deviations from Matthiessen rule,[25]

which indicate a large ratio 3–20 of spin-up to spin-down conductivities. Again, despite *s-d* hybridization, it is these distinct properties which justify giving the name 4*s* to these spin-up electrons at the Fermi level. They are responsible for most of the electrical conductivity.

It appears, therefore, that the *s-d* exchange model (Sec. II) would be more reasonable[15,17] for the problem of current-induced torques on domain walls, in many materials. One exception is iron-rich Fe-Mn, Fe-Cr, Fe-V, and Fe-Ti, where deviations from Matthiessen rule[25] show conduction by spin-down 3*d* carriers to be dominant. Hall effect data for Fe-Cr (Ref. 26) show these carriers to be holelike. There, our purely 3*d* model may apply even for current-induced torques (Sec. IV).

VII. CONCLUSIONS AND FINAL REMARKS

The model based on 3*d* itinerant electrons only, used by Tserkovnyak *et al.*[7] for their original derivation of Eq. (2) is conceptually simpler than the *s-d* exchange model, which uses two different kinds of electrons. Also, it is less plagued by uncertainties arising from *s-d* hybridization.

Our present treatment of Gilbert damping in this model achieves maximum mathematical simplicity, as well as maximum physical clarity and insight [Fig. 1(b)], through the use of a semiclassical equation [our Eq. (6)] for the precession of a 3*d* spin $\mathbf{s}_n$. This method was pioneered by Turov[5] in connection with the *s-d* exchange model, but has almost been forgotten since.

Further simplification happens because we do not try, like Tserkovnyak *et al.*, to rederive known results about spin relaxation (see Refs. 3 and 8). Instead, we just focus on the Gilbert damping part of the problem.

The most important and least trivial ingredient for our calculation is the choice[8] of the direction $\mathbf{s}_0$ toward which the spins relax [Eq. (4) and (7)], also made by Turov for the *s-d* exchange model.

In the case of current-induced torques on a domain wall, the formulas obtained for the angle ψ [Eq. (12)] and for the fictitious field H_{nad} [Eq. (15)] are the same as they would be in a similar theory[14,15,17] based on *s-d* exchange, even though exchange plays a much less explicit role in the equations. Of course, the values of parameters such as P, P_n and n_e may be somewhat different. Our results are consistent with those of Tserkovnyak *et al.*;[7] for example, the dimensionless coefficient β, used by these authors to describe the intensity of the nonadiabatic torque, can be shown in the 3*d* model to be equal to the Gilbert constant α, itself given by our Eq. (2). On the other hand, $\beta > \alpha$ holds in the *s-d* exchange model.

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