Reply to "Comment on 'Interaction of a surface wave with a dislocation' "

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(Received 15 April 2009; revised manuscript received 3 July 2009; published 9 October 2009)

A subsurface moving dislocation in an elastic half space generates vertical displacements at the free surface. We compare this displacement for two different values of the dislocation viscous drag coefficient. The different resulting surface patterns suggest the free surface plays a decisive dynamical effect. We thus compare this displacement, using the dynamic Green function for an elastic half space, with the result of the calculation using the static Green function for an infinite space, as in the work of Zolotoyabko and Shilo [preceding paper, Phys. Rev. B **80**, 136101 (2009), and Shilo and Zolotoyabko, Phys. Rev. Lett. **91**, 115506 (2003)] when the dislocation dynamics is the same. Considering the static Green function of an infinite space instead of the correct dynamic Green function of the half space leads to an underestimation of the resulting displacement at the free surface by a factor up to 50 for dislocation depths smaller than one Rayleigh wavelength λ_R. We also discuss the constraints that recent ultrasound attenuation and resonant ultrasound spectroscopy experiments place on dislocation parameters, such as density and viscous drag coefficient.

DOI: 10.1103/PhysRevB.80.136102 PACS number(s): 61.72.Lk, 72.10.Fk, 11.80.La, 81.70.Cv

In Ref. 1, hereafter referred to as I, we reported results from a study of the interaction of elastic waves with subsurface dislocations in an elastic half space. One aspect of that study was the development of a scheme to efficiently take into account the presence of the free surface in the response of the elastic half space when computing the response to subsurface stimuli, such as that of a moving dislocation, a recurrent technical problem (see, for instance, Refs. 2–8 and references therein). The Comment by Zolotoyabko and Shilo criticizes our choice of value for the viscous drag coefficient parameter and suggests a different choice may lead to results in better agreement with their own study,[9] hereafter referred to as II.

I. SURFACE PATTERNS

Using the results and notation of I, we have computed the vertical displacement at the free surface of a semi-infinite elastic medium that results from the superposition of a surface Rayleigh wave incident upon a subsurface dislocation with the secondary wave emitted by the response of the dislocation. Figure 1 shows the resulting pattern for values of the dislocation viscous drag coefficient $B=10^{-5}$ Pa.s and $B=5.10^{-7}$ Pa.s. Calculations have been performed for a depth of the dislocations $h=3$ μm, well within the range $h<7.6$ μm given in the Comment by Zolotoyabko and Shilo. The two patterns are quite different and in the second one the surface vertical displacement due to the wave reemitted by the moving dislocation appears to be comparable to the incident wave. Conservation of energy suggests this effect must be confined at the free surface. In the next section we study the role of the free surface and conclude that it is essential: further modeling with two unknown parameters, depth h and viscous coefficient B, that explicitly includes the free surface, will be needed to determine which value of h and B gives the better agreement with experiment.

II. ROLE OF THE FREE SURFACE

We show below that the disagreement in the simulated patterns in our model and in the model used in II is only due to the choice of the Green function. Indeed, instead of choosing the dynamical Green function of the elastic half space, Shilo and Zolotoyabko used the static Green function for the infinite space. In order to quantitatively ascertain the role of the Green function we compute here the vertical displacement generated by an infinite, straight, dislocation placed at a depth h below the free surface of a semi-infinite, homogeneous, isotropic, medium, that oscillates along its glide plane with frequency ω (Fig. 2).

The vertical displacement $u^s_{\text{half}}(x,y,z;\omega)$ of the semi-infinite medium generated by this oscillatory dislocation is provided by formula (3.3) of I. In the case at hand, an infinitely long straight dislocation, this equation reduces to

$$u^s_{\text{half}}(y,z;\omega) = \dot{\xi}\frac{\mu b}{i\omega}\left[\frac{\partial}{\partial x}g^0_{33}(y,z;\omega) + \frac{\partial}{\partial y}g^0_{32}(y,z;\omega)\right], \tag{2.1}$$

where

$$g^0_{ij}(y,z;\omega) = \int dsG^0_{ij}(s,y,z;\omega) \tag{2.2}$$

and G^0_{ij} is the Green function for the semi-infinite space provided by formulas (2.2) and (2.3) of I. b is the Burgers vector, ω is the frequency and μ is the shear modulus of the elastic medium.

As explained in I, expression (2.1) is also valid when the medium is infinite, provided the dynamical response g^0 for the half space is replaced by the response appropriate for infinite space. Close enough to the dislocation (i.e., at distances small compared to wavelength) the response reduces to the static response, leading to the following expression for

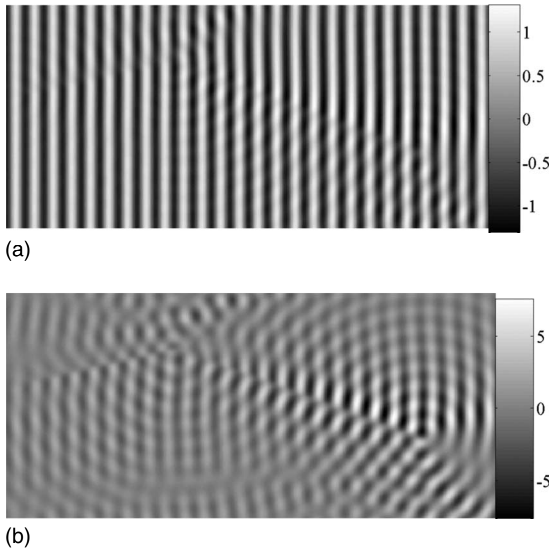

FIG. 1. Upper panel: vertical displacement at the free surface of a semi-infinite medium reproduced from I (the field is normalized to the amplitude of the incident wave, $B=10^{-5}$ Pa.s). Lower panel: same calculation, with a viscous drag coefficient $B=5.10^{-7}$ Pa.s Both simulations are performed considering the dislocation depth is 3 μm.

the vertical displacement $u^s_{\rm free}(x,y,z;\omega)$ of the infinite medium generated by the oscillatory dislocation:

$$u^s_{\rm free}(y,z;\omega) = \frac{-by\dot{\xi}}{i\omega 4\pi(1-\nu)}\left[\frac{(1-2\nu)}{y^2+z^2} + \frac{2z^2}{(y^2+z^2)^2}\right] \tag{2.3}$$

where ν is Poisson's ratio. This is the expression used by Shilo and Zolotoyabko.[9] Note that both Eqs. (2.1) and (2.3) are linear in the dislocation velocity $\dot{\xi}$.

We have calculated, using the parameters of II, both $u^s_{\rm half}$ and $u^s_{\rm free}$ generated by the same dislocation motion ξ at the free surface or, more precisely in the second case, at the position the free surface would have if it were present. Their ratio, which depends only on the ratio of the Green functions, is plotted in Fig. 3 as a function of position for several dislocation depths. The result is that taking into account the dynamical effect of the response and including the effect of the free surface produces an amplification of the signal by a factor of up to 50.

Our conclusion is that the simulation used in II cannot be used for a quantitative comparison with the experimental result because it does not take into account the essential role played by the free surface. Our simulation, on the contrary, does take it into account and can be used as a starting point for a systematic study of the surface displacement as a function of two unknown parameters: dislocation depth h and viscous coefficient B. A proper study would also consider the fact that experiments provide x-ray diffractograms that, while inspiring, cannot be used to infer, without elaboration, the vertical displacement at the free surface. Such a program is outside the scope of the present paper.

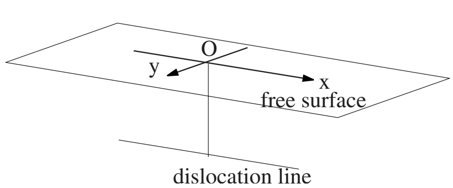

FIG. 2. An infinite dislocation lies along the x axis, parallel to the free surface at depth h.

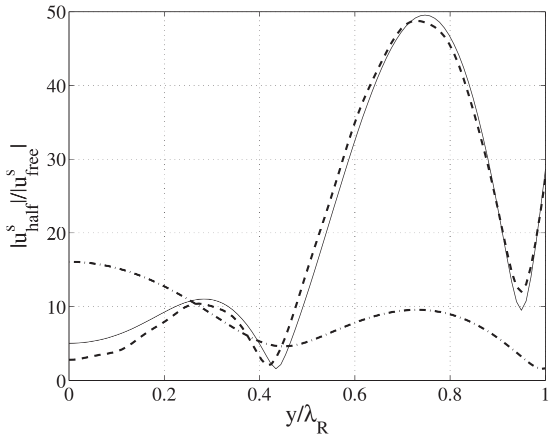

FIG. 3. The ratio $|u^s_{\rm half}|/|u^s_{\rm free}|$ along the y axis, in units of Rayleigh wavelength λ_R, on the free surface generated by an oscillating dislocation at depth h. $u^s_{\rm half}$ is the vertical displacement calculated with the dynamic Green function of the half space and $u^s_{\rm free}$ is the vertical displacement calculated with the Green function of the infinite space, as in II. The ratio only depends on the ratio of the Green functions. Choosing the erroneous Green function underestimates the vertical displacement by a factor of up to 50. dashed-dotted line $h=6$ μm, solid line $h=1$ μm, and dotted line $h=0.5$ μm.

III. DETERMINATION OF DISLOCATION PARAMETERS THROUGH ULTRASOUND ATTENUATION METHODS

When modeling the response of a subsurface dislocation to surface elastic waves, an important parameter, discussed in the previous Comment, is the phenomenological parameter B that measures energy losses due to dislocation interaction with phonons, electrons, and such. To the best of our knowledge, this coefficient is inferred from acoustic, or ultrasonic, attenuation methods. The issue of acoustic attenuation by dislocations in elastic media was studied by us in Refs. 10 and 11: when a dislocation segment is modeled as an overdamped elastic string, a common assumption, the acoustic attenuation coefficient α is proportional to the product of B and dislocation density Λ, measured in units on inverse area

$$\alpha \propto \Lambda B. \tag{3.1}$$

That is, a property of a single dislocation is inferred from a measurement that involves many such dislocations, and one of the many exciting aspects of the work of Shilo and Zolotoyabko[9] is their ability to visualize the interaction of a surface wave with a *single* dislocation. It is true, as pointed out in the previous Comment, that acoustic attenuation values have a wide variability among different materials.[12] But it is also true that dislocation densities can be very different as well: for copper, say, dislocation densities as high as 10^{10} m^{-2} and up to 10^{15} m^{-2} have been reported,[13,14] while for LiNbO3 dislocation densities as low as 10^7-10^8 m^{-2} have been reported[15–17] and it does not appear to be possible to ascertain with accuracy a value for B without an independent measurement for Λ for the same sample. We are not aware of any such measurements although recent results with aluminum[18] appear to provide a path forward.

This work has been supported by FONDAP under Grant No. 11980002 and ECOS CONICYT under Contract No. C04E01.

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