PHYSICAL REVIEW B **80**, 134502 (2009)

Nonequilibrium electrons in tunnel structures under high-voltage injection

N. B. Kopnin,[1,2,3] Y. M. Galperin,[4,5] J. Bergli,[4] and V. M. Vinokur[3]
[1]*Low Temperature Laboratory, Helsinki University of Technology, P.O. Box 5100, Espoo 02015 TKK, Finland*
[2]*L. D. Landau Institute for Theoretical Physics, 117940 Moscow, Russia*
[3]*Argonne National Laboratory, Argonne, Illinois 60439, USA*
[4]*Department of Physics, University of Oslo, P.O. Box 1048, Blindern, 0316 Oslo, Norway*
[5]*A.F. Ioffe Physico-Technical Institute of Russian Academy of Sciences, 194021 St. Petersburg, Russia*
(Received 2 September 2009; published 2 October 2009)

We investigate electronic distributions in nonequilibrium mesoscopic tunnel junctions subject to a high-voltage bias V under competing electron-electron and electron-phonon relaxations. We derive conditions for reaching quasiequilibrium and show that, though the distribution can still be thermal for low energies where the rate of the electron-electron relaxation exceeds significantly the electron-phonon relaxation rate, it develops a power-law tail at energies of order of eV. In a general case of comparable electron-electron and electron-phonon relaxation rates, this tail leads to emission of high-energy phonons which carry away most of the energy pumped in by the injected current.

DOI: 10.1103/PhysRevB.80.134502 PACS number(s): 73.23.−b, 74.45.+c, 74.78.−w

I. INTRODUCTION

Under nonequilibrium conditions, the electronic system can be effectively heated such that its quasiparticle energy distribution is the Fermi function with a well-defined temperature,

$$n_F(\epsilon) = [\exp(\epsilon/T_e) + 1]^{-1}. \tag{1}$$

The *electron temperature*, T_e, (we measure the temperature in energy units) can be significantly different from the phonon bath temperature T_{ph}. This situation is referred to as the *quasiequilibrium*. Alternatively, the electronic system can be driven out of equilibrium in such a way that the quasiparticle distribution no longer has the thermal form [Eq. (1)], and the notion of temperature cannot be introduced. The correct interpretation of the data observed in the particular experiment requires the proper attribution of the specific phenomenon to either of these classes. Superconducting mesoscopic structures such as tunnel and Josephson junctions are among those that are being investigated most intensively. The common tools for inferring information about nonequilibrium states in such systems are studies of the electron-phonon interaction rates[1–6] and of the electron-phonon energy relaxation.[7–13] A standard setup is a thin-film island connected to leads (they all can be either superconducting or normal) via tunnel contacts and driven out of equilibrium by strong electron injection under bias voltages V such that eV significantly exceeds both the temperature and the superconducting gap Δ. The leads are typically much bigger than the island. Therefore one can assume that the both leads are in the equilibrium having well-defined chemical potentials and temperature and replace the contacts by effective resistances in the spirit of the tunnel approximation.

In many experiments (see, e.g., Ref. 13), a quasiequilibrium distribution is created with an electronic temperature T_e essentially higher than the bath temperature T_{ph} kept low by efficient cooling. These temperatures determine the energy transfer from electrons to the phonon bath. The electronic quasiequilibrium establishes when a big difference between fast electron-electron (e-e), $\gamma_{e\text{-}e}$, and slow electron-phonon (e-ph), $\gamma_{e\text{-}ph}$, relaxation rates exists at energies $\epsilon \sim T_e$. These conditions are usually fulfilled in aluminum samples at sub-Kelvin temperatures.

The more common is the situation, however, where, in contrast to Al samples, the ratio $\gamma_{e\text{-}e}/\gamma_{e\text{-}ph}$ at such temperatures is not very high (as illustrated, e.g., by the data in Ref. 1). In this paper we investigate formation of the electronic distribution in mesoscopic samples and show that for moderate ratios $\gamma_{e\text{-}e}/\gamma_{e\text{-}ph}$ the conditions of quasiequilibrium can be easily violated. The distribution formed under a high-voltage injection can be characterized by an effective electronic temperature $T_e > T_{ph}$ only within a low-energy region around $\epsilon \sim T_e$ provided $\gamma_{e\text{-}e}(T_e)$ is much larger than $\gamma_{e\text{-}ph}(T_e)$. We find that at energies of order of $\mathrm{eV} \gg T_e$, the distribution has a long power-law tail, which crucially changes the transport properties of the entire electronic system having much lower effective temperature T_e. Since the tail energies exceed Δ, our results are general and apply to both normal and superconducting junctions. While the derived behavior somewhat resembles the well-known electron runaway in semiconductors,[14] it is drastically different from such nonequilibrium effects as superconductivity stimulation,[15] nonequilibrium proximity effect in Josephson junction (see, e.g., Ref. 16 and references therein), etc., where the deviation from equilibrium is maximal right in the energy range near Δ.

Importance of the high-energy tail in the distribution is elucidated by considering the energy exchange between electron and phonon subsystems relevant to a rich variety of experiments.[7–13] Since $\gamma_{e\text{-}ph}(\epsilon)$ grows faster than $\gamma_{e\text{-}e}(\epsilon)$ with increasing energy, there exists certain energy, ϵ^*, at which these two rates match. One would expect that nonequilibrium effects in the energy transfer are small as long as $T_e \ll \epsilon^*$. We show that it is indeed the case if $\mathrm{eV} \ll \epsilon^*$ and the e-e interaction dominates in the entire nonequilibrium region. However, at $\mathrm{eV} \gg \epsilon^*$ a crossover from the e-e to the e-ph mechanism of relaxation takes place as a function of energy. In this case the power-law tail in the electron distribution leads to emission of high-energy phonons, which carry away most of the energy provided by the injected current. Accordingly, the energy emitted via thermal phonons becomes a much smaller fraction of the total inserted energy, implying that interpretation of the experimental databased on the quasiequilibrium distribution function with some effective temperature does not apply.

II. SETUP AND MODEL

We consider a junction consisting of a superconducting (or normal) island (i) connected via small-area high-resistance tunnel contacts with two superconducting (or normal) leads L_1 and L_2. The bias voltage V is assumed high in the scale of characteristic energies of the particular experiment. For the system mentioned above, the proper inequality is $\mathrm{eV} \gg T_\mathrm{e}, \Delta$. For the energies $T_\mathrm{e}, \Delta \ll \epsilon \lesssim \mathrm{eV}$, which we are interested in, the normal-state equations for the e-ph, e-e interactions, and the density of states can be used both for a normal and superconducting junction. To be more specific, in what follows we concentrate on the normal junctions. The electron temperature is assumed uniform along the sample. This implies that the island length L is sufficiently small such that the island resistance R satisfies the inequality $(R/R_T)(\mathrm{eV}/T_\mathrm{e})^2 \ll 1$, where R_T is the tunnel contact resistance. On the other hand, the island length is limited from below by the diffusion length, $L \gg \sqrt{D/\gamma_{\text{e-e}}}$, where D is the diffusion constant, to ensure that electrons scatter at each other many times before leaving the island.

We use the clean-limit approximations for $\gamma_{\text{e-e}}(\epsilon)$ and $\gamma_{\text{e-ph}}(\epsilon)$ because impurity scattering does not significantly renormalize the e-e or e-ph interactions for energies of interest. The influence of the impurity scattering on the e-ph relaxation is controlled by the parameter $q\ell$, where ℓ is the electronic mean-free path and $q=\epsilon/\hbar s$ is the wave vector of an emitted phonon with energy ϵ.[17–19] Taking sound velocity $s \sim 5000$ m/s and $\ell \sim 20$ nm as in Al samples of Ref. 13 we get $q\ell \sim (0.5\ \mathrm{K}^{-1})\epsilon/k_B$. Therefore, at $\epsilon/k_B \sim \mathrm{eV}/k_B \gg 1$ K the clean limit is appropriate. The impurity-induced renormalization of the e-e interaction[20] becomes important for energies $\epsilon \lesssim \hbar^3/\tau(p_F\ell)^2 \sim \epsilon_F(\hbar/p_F\ell)^3$ and can also be neglected for $\epsilon \sim \mathrm{eV}$.

We assume that phonon energy distribution in the island is equilibrium,

$$N(\epsilon) = [\exp(\epsilon/T_\mathrm{ph}) - 1]^{-1}. \tag{2}$$

Since we concentrate on the case when T_e significantly exceeds both T_ph and temperature of the leads, T_L, in what follows we assume that the leads and the phonon bath are at zero temperature, $T_L = T_\mathrm{ph} = 0$, and that all the emitted phonons are immediately removed from the sample due to ideal heat contact to the substrate. Estimates of the validity of this approximation depend on the relationship between the island thickness, d, the phonon mean-free path with respect to elastic scattering via defects, $\ell_{\text{ph-d}}$, and the mean-free path due to phonon-electron collisions, $\ell_{\text{ph-e}}$. The estimate for "clean" case, $d \ll \ell_{\text{ph-d}}$, can be obtained from the requirement that the phonon escape time d/s is shorter than the phonon-electron scattering time $\tau_{\text{ph-e}} \sim \hbar v_F/s\epsilon$. Here ϵ is the phonon energy. If the Fermi velocity is $v_F \sim 10^8$ sm/s and the bias voltage is $\mathrm{eV}/k_B \sim 100$ K as in Ref. 13, the film thickness should be $d \lesssim 10^{-5}$ cm for phonons with energy $\epsilon \sim \mathrm{eV}$.

In thick impure films, $d \gg \ell_{\text{ph-d}}$, the propagation of phonons inside the island is diffusive. The phonon kinetics in such films is rather complicated, it depends also on the interplay between the phonon-electron relaxation time, $\tau_{\text{ph-e}}(\epsilon)$, and the inelastic relaxation time due to phonon anharmonicity, $\tau_{\text{ph-ph}}(\epsilon)$. If for all relevant phonon energies $\tau_{\text{ph-ph}} \ll \tau_{\text{ph-e}}$ then the equilibrium phonon distribution [Eq. (2)] is formed. However, the phonon temperature, T_ph, can exceed the temperature T_0 of the substrate. In this case, T_ph is determined by the balance between the dissipated power and heat release into the substrate. The latter is not well known for tunneling structures. The case when $\tau_{\text{ph-ph}}$ is comparable with $\tau_{\text{ph-e}}$ is even more complicated since the phonon distribution turns out to be essentially nonequilibrium.

It is this complex character of phonon kinetics in general case that motivates us to restrict ourselves to the simplest case of thin clean films and large electron heating, $T_\mathrm{e} \gg T_\mathrm{ph} \gtrsim T_L, T_0$. Under these assumptions we can derive a linear equation for the distribution function and solve it exactly for the most relevant situations.

III. KINETIC EQUATION

We consider a symmetric structure with voltages at the leads $V_{L_1} = -V_{L_2} = V/2$ such that the chemical potential of the island $\mu_i = 0$ by symmetry while for the leads $\mu_{L_1} = -\mu_{L_2} \equiv \mu_L = -\mathrm{eV}/2$. It is convenient to write kinetic equations for odd and even components of the distribution function $n(\epsilon)$ defined as $f_1(\epsilon) = n(-\epsilon) - n(\epsilon)$ and $f_2(\epsilon) = 1 - n(\epsilon) - n(-\epsilon)$. If the leads are in thermal equilibrium at temperature T_L, we have $f_1^{(L_1)} = f_1^{(L_2)} \equiv f_1^{(L)}$ and $f_2^{(L_1)} = -f_2^{(L_2)} \equiv f_2^{(L)}$, where

$$f_{1,2}^{(L)} = \frac{1}{2}\left[\tanh\frac{\epsilon - \mathrm{eV}/2}{2T_L} \pm \tanh\frac{\epsilon + \mathrm{eV}/2}{2T_L}\right].$$

One can check that the even component of the distribution in the island vanishes by symmetry, $f_2(\epsilon) = 0$. The kinetic equation for the odd component is

$$J_1^T + J_1^{(\mathrm{e})} + J_1^{(\mathrm{ph})} = 0, \tag{3}$$

where J_1^T, $J_1^{(\mathrm{e})}$, and $J_1^{(\mathrm{ph})}$ are, respectively, the tunnel, e-e, and e-ph collision integrals in the island. The latter describe relaxation of the distribution driven out of equilibrium by the tunnel source[21]

$$J_1^T = -4\eta[f_1 - f_1^{(L)}] \tag{4}$$

that contains the distribution in the leads, $f_1^{(L)}$, and in the island, f_1. The (identical) tunneling contacts are characterized by an effective tunneling rate $\eta = (4\nu e^2 \Omega R_T)^{-1}$, where $\nu \equiv \nu(E_F)$ is the normal density of states in the island and Ω is its volume. In what follows we consider high-contact resistances, i.e., small η (the estimate will be given later).

A. Electron-phonon relaxation

For small η, the distribution function determined by Eq. (3) is close to the thermal, $f_1(\epsilon) \approx \tanh(\epsilon/2T_\mathrm{e})$, with a certain electronic temperature T_e. For energies $\epsilon \sim T_e$ the deviation from quasiequilibrium is negligible. However, at $\epsilon \gg T_e$, where $\tanh(\epsilon/2T_e) \approx \mathrm{sign}(\epsilon)$, the injection-induced deviation becomes essential. We put $f_1(\epsilon) = \tanh(\epsilon/2T_e) + \delta f(\epsilon)$, where $|\delta f| \ll 1$. At $T_{ph}=0$, the e-ph collision integral[22] in the island vanishes for $f_1(\epsilon) = \mathrm{sign}(\epsilon)$. Therefore, neglecting exponentially small terms we get for energies $|\epsilon|, |\epsilon+\omega| \gg T_e$ and $\epsilon > 0$ (see also Ref. 23)

$$J_1^{(ph)} = -\frac{\gamma_{\text{e-ph}}(T_e)}{T_e^3}\left[\frac{\epsilon^3 \delta f(\epsilon)}{3} - \int_0^\infty d\omega \omega^2 \delta f(\epsilon+\omega)\right], \quad (5)$$

where $\gamma_{\text{e-ph}}(T_e) = \pi \lambda_{\text{e-ph}} T_e^3 / 2\hbar (s p_F)^2$ is the e-ph relaxation rate at the electronic temperature T_e while $\lambda_{\text{e-ph}}$ is the interaction constant. We use T_e simply as a convenient energy scale. In fact, the ratio $\gamma_{\text{e-ph}}(T_e)/T_e^3$ is independent of T_e.

B. Electron-electron interaction

The e-e collision integral satisfies the energy conservation law, $\int_{-\infty}^{\infty} \epsilon J_1^{(e)}(\epsilon) d\epsilon = 0$.[22] For large energies $|\epsilon|, |\epsilon_1|, |\epsilon_2|, |\epsilon_3| \gg T_e$ and $\epsilon > 0$ it has the form

$$J_1^{(e)} = -\frac{\gamma_{\text{e-e}}(T_e)}{T_e^2}\left[\frac{\epsilon^2 \delta f(\epsilon)}{2} - 3\int_0^\infty d\omega \omega \delta f(\epsilon+\omega)\right]. \quad (6)$$

Here $\gamma_{\text{e-e}}(T_e) = \pi \lambda_{\text{e-e}} T_e^2 / 8\hbar E_F$ is the e-e relaxation rate at T_e and $\lambda_{\text{e-e}}$ is the interaction constant.

C. Distribution function

The e-e relaxation dominates at very low energies, where the distribution has a thermal form with an electronic temperature T_e. At higher energies, a deviation from thermal behavior develops due to the reduced role of the e-e interaction. Consider $\epsilon > 0$. Since $f_1(\epsilon) \approx 1$ for $\epsilon \gg T_e$ the tunnel collision integral Eq. (4) becomes $J_1^T = -4\eta\Theta(\mathrm{eV}/2 - \epsilon)$. This form of the injection term suggests that at $\epsilon \gg T_e$, $\delta f(\epsilon) = -\phi(\epsilon)\Theta(\mathrm{eV}/2-\epsilon)$. Equation (3) becomes

$$\frac{1}{\epsilon_{ph}^3}\left[\frac{\epsilon^3}{3}\phi(\epsilon) - \int_0^{\mathrm{eV}/2-\epsilon} d\omega \omega^2 \phi(\epsilon+\omega)\right] + \frac{1}{\epsilon_e^2}\left[\frac{\epsilon^2}{2}\phi(\epsilon) - 3\int_0^{\mathrm{eV}/2-\epsilon} d\omega \omega \phi(\epsilon+\omega)\right] = 1. \quad (7)$$

Here $\epsilon_{ph}^3 = 4\eta T_e^3/\gamma_{\text{e-ph}}(T_e)$ and $\epsilon_e^2 = 4\eta T_e^2/\gamma_{\text{e-e}}(T_e)$. Equation (7) has a characteristic energy scale

$$\epsilon^* = \epsilon_{ph}^3/\epsilon_e^2 \sim T_e[\gamma_{\text{e-e}}(T_e)/\gamma_{\text{e-ph}}(T_e)], \quad (8)$$

such that $\gamma_{\text{e-ph}}(\epsilon^*) = \gamma_{\text{e-e}}(\epsilon^*)$. Therefore, the e-e interaction dominates for $\epsilon \ll \epsilon^*$ while the e-ph interaction takes over for $\epsilon \gg \epsilon^*$. As explained earlier, we consider here the situation when $\gamma_{\text{e-e}}(T_e) \gg \gamma_{\text{e-ph}}(T_e)$, thus $\epsilon^* \gg T_e$.

Calculating triple derivative over energy the integral Eq. (7) can be transformed into a differential one, which can then be easily analyzed and solved numerically. The boundary conditions are obtained by putting $\epsilon = \mathrm{eV}/2$ at each step. In the situations of dominant e-e relaxation, $\gamma_{\text{e-ph}} \to 0$ (i.e., $\epsilon_{ph} \to \infty$), it can be reduced to the second-order differential equation for a function ϕ_e determined entirely by the e-e interaction

$$\frac{d^2}{d\epsilon^2}[\epsilon^2 \phi_e(\epsilon)] = 6\phi_e(\epsilon), \quad (9)$$

with the boundary conditions $\epsilon^2 \phi_e(\epsilon) = 2\epsilon_e^2$ and $(d/d\epsilon)[\epsilon^2 \phi_e(\epsilon)] = 0$ at $\epsilon = \mathrm{eV}/2$. Solution of Eq. (9) with these boundary conditions is

$$\phi_e = \frac{4\epsilon_e^2}{5}\left[\frac{3}{2}\frac{(\mathrm{eV}/2)^2}{\epsilon^4} + \frac{\epsilon}{(\mathrm{eV}/2)^3}\right]. \quad (10)$$

Fast e-ph relaxation, $\gamma_{\text{e-ph}} \to \infty$, leads to

$$\frac{d^3}{d\epsilon^3}[\epsilon^3 \phi_{ph}(\epsilon)] + 6\phi_{ph}(\epsilon) = 0 \quad (11)$$

with the conditions $\epsilon^3 \phi_{ph}(\epsilon) = 3\epsilon_{ph}^3$ and $(d/d\epsilon)[\epsilon^3 \phi_{ph}(\epsilon)] = (d^2/d\epsilon^2)[\epsilon^3 \phi_{ph}(\epsilon)] = 0$ at $\epsilon = \mathrm{eV}/2$. The solution is

$$\phi_{ph}(\epsilon) = \frac{18}{11}\left(\frac{\epsilon_{ph}}{\epsilon}\right)^3 \mathcal{F}\left(\frac{\mathrm{eV}}{2\epsilon}\right), \quad (12)$$

where $\mathcal{F}(x) \equiv x + \frac{1}{x^2}[\frac{5}{6}\cos(\sqrt{2}\ln x) + \frac{\sqrt{2}}{3}\sin(\sqrt{2}\ln x)]$. It was obtained in Ref. 23 (see also Ref. 24) and used in Ref. 25 for analysis of electron distribution created by absorption of a high-energy photon.

At comparatively low voltages, $\mathrm{eV} \ll \epsilon^*$, the e-e interaction dominates in the entire energy range $0 < \epsilon < \mathrm{eV}/2$ and the distribution function obeys Eq. (10). At high voltages, $\mathrm{eV} \gg \epsilon^*$, one can discriminate between two regions with different relaxation mechanisms with a crossover between them at $\epsilon \sim \epsilon^*$. For $0 < \epsilon \ll \epsilon^*$ the e-e interaction dominates. For $\epsilon^* \ll \epsilon < \mathrm{eV}/2$ the e-ph interaction wins and the distribution function approaches Eq. (12). In both cases the distribution has a long power-law nonequilibrium tail $\phi(\epsilon) \sim (\tilde{\epsilon}/\epsilon)^4$ at energies $T_e \ll \epsilon \ll \mathrm{eV}$. For $\mathrm{eV} \ll \epsilon^*$, one has $\tilde{\epsilon} = (\epsilon_e \mathrm{eV})^{1/2}$ while $\tilde{\epsilon} = (\epsilon_{ph}^3 \mathrm{eV})^{1/4}$ for $\mathrm{eV} \gg \epsilon^*$. The deviation from equilibrium becomes of the order unity for $\epsilon \lesssim \tilde{\epsilon}$. Thus the low-energy distribution can be thermal only if $\tilde{\epsilon} \ll T_e$. This requires

$$\mathrm{eV}/\epsilon^* \ll \xi \cdot \max\{\xi, 1\}, \quad \xi \equiv T_e^2/\epsilon_e \epsilon^*. \quad (13)$$

Therefore, one can interpret the data inferred from the experiment in terms of a quasiequilibrium electronic temperature T_e only if the condition (13) is fulfilled. Otherwise, the distribution is not thermal even at small ϵ; instead, the scale of its variation, and thus the apparent "temperature," is determined by $\tilde{\epsilon}$.

IV. ENERGY BALANCE

Even if the quasiequilibrium condition (13) holds, the long power-law tail in the distribution can strongly influence electronic processes at a much lower temperature T_e. An important example is the e-ph energy relaxation. Using Eq. (3) one finds the energy balance per unit volume of the island

$$2\nu \int_0^{\mathrm{eV}/2} \epsilon J_1^T d\epsilon + 2\nu \int_0^{\mathrm{eV}/2} \epsilon J_1^{(ph)} d\epsilon = 0. \quad (14)$$

The e-e collision integral vanishes due to the energy conservation. With Eq. (5) this gives

$$P_{\rm eq}(T_{\rm e}) + P_{\rm neq} = P_V. \tag{15}$$

Equation (15) determines $T_{\rm e}$ as a function of the injected power $P_V = V^2/4R_T\Omega$. The latter is obtained from the first integral in Eq. (14) under the assumption that $eV \gg T_{\rm e}$. This is the power (per unit volume) deposited into the island in a setup with two contacts of total resistance $2R_T$ under the total voltage bias V. It is half of the total input energy, the other half of which goes into the leads. The left-hand side of Eq. (15) comes from the second integral in Eq. (14) and consists of the energy transfers, $P_{\rm eq}$ and $P_{\rm neq}$, to zero-temperature phonon bath by thermalized and by nonequilibrium electrons, respectively. The energy transferred by thermal electrons, $P_{\rm eq}(T_{\rm e})$, is determined by the e-ph interaction integrated over the low-energy domain $\epsilon \gtrsim T_{\rm e}$ with $f_1 = \tanh(\epsilon/2T_{\rm e})$ neglecting the nonequilibrium correction. For a clean normal metal, $P_{\rm eq}(T_{\rm e}) = \Sigma T_{\rm e}^5$, where $\Sigma \approx 78\nu\lambda/(sp_F)^2\hbar^9$. For clean superconductors, $P_{\rm eq}(T_{\rm e})$ was calculated in Ref. 13.

The energy transferred by nonthermal electrons is

$$P_{\rm neq} = \frac{8\nu\eta}{\epsilon_{\rm ph}^3}\int_0^{eV/2} \epsilon d\epsilon \left[\frac{\epsilon^3\phi(\epsilon)}{3} - \int_0^{eV/2-\epsilon} d\omega\omega^2\phi(\epsilon+\omega)\right]. \tag{16}$$

The main contribution here comes from $\epsilon \sim eV$.

For voltages $eV \ll \epsilon^*$ when the e-e interaction always dominates, the nonthermal contribution can be calculated using Eq. (10). The ϵ^{-4} term in Eq. (16) is cancelled out leading to $P_{\rm neq} = (eV/3\epsilon^*)P_V$, and Eq. (15) yields

$$P_{\rm eq}(T_{\rm e}) = P_V[1 - eV/3\epsilon^*]. \tag{17}$$

The second term in the brackets is a small correction. Almost all the injected energy is absorbed and then transmitted to the phonon bath by thermal electrons.

The situation is totally different at higher bias voltages, $eV \gg \epsilon^*$. In this case almost all the injected power is absorbed by high-energy electrons and $T_{\rm e}$ is less sensitive to V. Using $\phi_{\rm ph}(\epsilon)$ from Eq. (12), which satisfies the kinetic equation without the e-e collision integral, the energy balance Eq. (15) becomes

$$P_{\rm eq}(T_{\rm e}) = \frac{8\nu\eta}{\epsilon_{\rm ph}^3}\int_0^{eV/2} \epsilon d\epsilon \left(\frac{\epsilon^3}{3}[\phi_{\rm ph}(\epsilon) - \phi(\epsilon)] - \int_0^{eV/2-\epsilon} \omega^2 d\omega[\phi_{\rm ph}(\epsilon+\omega) - \phi(\epsilon+\omega)]\right).$$

The function $\phi(\epsilon)$ satisfies the full kinetic Eq. (7). The main contribution to the integral comes from $\epsilon \sim eV \gg \epsilon^*$, where Eq. (7) coincides with that for small e-e interaction within the accuracy ϵ^*/eV. Therefore $(\phi_{\rm ph} - \phi)/\phi_{\rm ph} \sim \epsilon^*/eV$. This results in a slower dependence of $P_{\rm eq}$ on the bias voltage,

$$P_{\rm eq}(T_{\rm e}) \sim P_V(\epsilon^*/eV) = \nu\eta\epsilon^* eV. \tag{18}$$

Expressions (17) and (18) are the main result of this paper. They reveal a crossover from $\propto V^2$ to $\propto V$ behavior that occurs at $eV \approx \epsilon^*$. This is confirmed by numerical solution of the kinetic Eq. (7).

V. DISCUSSION

The data for Al samples[26] suggest that the inelastic relaxation at $T \sim 1$ K is determined by e-e interaction, $\gamma_{\text{e-e}}(T_c) \sim 10^8$ s^{-1} while the e-ph rate is much slower.[1] According to Refs. 5 and 26, $\gamma_{\text{e-ph}}(T_c) \sim 10^6 - 10^7$ s^{-1}. Therefore, the crossover frequency ϵ^*/T_c appears to be rather high, $\sim 10 - 10^2$. In experiments of Ref. 13 the quasiequilibrium conditions were found to be well satisfied even for voltages $eV \lesssim 10^2 T_c$, which suggests that the e-e interaction in Al still dominates at energies as high as $10^2 T_c$ and seems to support the above estimate of ϵ^*. We should note, however, that the mentioned experimental data are obtained for dirty Al samples. Since the crossover energy in dirty materials is increased as compared to that in clean samples, our value for ϵ^* may be overestimated.

However, the above situation is rather an exception to the rule—it is in a sense unique to aluminum samples. In other materials the e-ph relaxation rates for $T \sim 1$ K are normally considerably larger;[1,27] they are higher or at least of the same order as the e-e rates making the ratios $\gamma_{\text{e-e}}/\gamma_{\text{e-ph}}$ not as high as in Al. For example, in copper at 1 K the inelastic rate $\sim 10^7$ s^{-1} is mostly due to the e-ph interaction.[28] Therefore, the crossover from e-e to e-ph dominated relaxation in Cu should occur already at much lower energies, $\epsilon^*/k_B < 1$ K. It is exactly the situation most favorable for observation of the predicted crossover in the energy exchange between the electron and phonon subsystems as a function of bias voltage in the range $eV > \epsilon^*$. Although the deviation from the thermal distribution could be still small within the low-energy range, the high-energy tail of the distribution will be essential for the energy relaxation between the electron and phonon subsystems since the injected power is mostly absorbed by high-energy phonons.

To summarize, we have established the quasiequilibrium condition Eq. (13), and shown that the energy exchange rate $P_{\rm eq}(T_{\rm e})$ between thermal electrons and the phonon bath experiences a crossover from Eq. (17) to Eq. (18) as a function of the bias voltage. The crossover energy provides additional information on the relative strength of the e-e and e-ph interactions. Combined with other measurements, for example, of the dephasing rates, it will help to identify the two contributions to the inelastic processes in various materials.

ACKNOWLEDGMENTS

We thank J. Pekola for many stimulating discussions. This work was supported by the Academy of Finland Centers of Excellence program, by the Russian Foundation for Basic Research under Grant No. 09-02-00573-a, by Deutsche Forschungsgemeinschaft within GK 638, by the U.S. Department of Energy Office of Science under Contract No. DE-AC02-06CH11357, and by Norwegian Research Council through STORFORSK program. Y. G. is thankful to the Ben Gurion University of Negev and to the Weizmann Institute of Science for hospitality.

[1] S. B. Kaplan, C. C. Chi, D. N. Langenberg, J. J. Chang, S. Jafarey, and D. J. Scalapino, Phys. Rev. B **14**, 4854 (1976).

[2] M. Yu. Reizer, Phys. Rev. B **40**, 5411 (1989).

[3] P. K. Day, H. G. LeDuc, B. A. Mazin, A. Vayonakis, and J. Zmuidzinas, Nature (London) **425**, 817 (2003).

[4] A. G. Kozorezov, J. K. Wigmore, A. Peacock, A. Poelaert, P. Verhoeve, R. den Hartog, and G. Brammertz, Appl. Phys. Lett. **78**, 3654 (2001).

[5] R. Barends, J. J. A. Baselmans, S. J. C. Yates, J. R. Gao, J. N. Hovenier, and T. M. Klapwijk, Phys. Rev. Lett. **100**, 257002 (2008).

[6] A. Rothwarf and B. N. Taylor, Phys. Rev. Lett. **19**, 27 (1967).

[7] V. F. Gantmakher, Rep. Prog. Phys. **37**, 317 (1974).

[8] M. L. Roukes, M. R. Freeman, R. S. Germain, R. C. Richardson, and M. B. Ketchen, Phys. Rev. Lett. **55**, 422 (1985).

[9] F. C. Wellstood, C. Urbina, and J. Clarke, Phys. Rev. B **49**, 5942 (1994).

[10] E. Chow, H. P. Wei, S. M. Girvin, and M. Shayegan, Phys. Rev. Lett. **77**, 1143 (1996).

[11] F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, Rev. Mod. Phys. **78**, 217 (2006).

[12] J. T. Karvonen and I. J. Maasilta, Phys. Rev. Lett. **99**, 145503 (2007).

[13] A. V. Timofeev, C. P. García, N. B. Kopnin, A. M. Savin, M. Meschke, F. Giazotto, and J. P. Pekola, Phys. Rev. Lett. **102**, 017003 (2009).

[14] H. Fröhlich, Phys. Rev. **56**, 349 (1939); B. I. Davydov and I. M. Shmuskevich, Zh. Eksp. Teor. Fiz. **10**, 1043 (1940).

[15] G. M. Eliashberg and B. I. Ivlev, in *Nonequilibrium Superconductivity*, edited by D. N. Langenberg and A. I. Larkin (Elsevier, New York, 1986), p. 211.

[16] A. G. Kozorezov, R. A. Hijmering, G. Brammertz, J. K. Wigmore, A. Peacock, D. Martin, P. Verhoeve, A. A. Golubov, and H. Rogalla, Phys. Rev. B **77**, 014501 (2008).

[17] B. L. Altshuler, Zh. Eksp. Teor. Fiz. **75**, 1330 (1978) [Sov. Phys. JETP **48**, 670 (1978)].

[18] M. Yu. Reizer and A. V. Sergeev, Zh. Eksp. Teor. Fiz. **90**, 1056 (1986) [Sov. Phys. JETP **63**, 616 (1986)].

[19] A. Sergeev and V. Mitin, Phys. Rev. B **61**, 6041 (2000).

[20] B. L. Altshuler and A. G. Aronov, Zh. Eksp Toer. Fiz. Pis'ma Red **30**, 514 (1979) [JETP Lett. **30**, 482 (1979)]; B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (Elsevier, New York, 1985), p. 1.

[21] J. Voutilainen, T. T. Heikkilä, and N. B. Kopnin, Phys. Rev. B **72**, 054505 (2005).

[22] N. B. Kopnin, *Theory of Nonequilibrium Superconductivity* (Clarendon, Oxford, 2001).

[23] A. F. Volkov and A. V. Zaitsev, Sov. Phys. JETP **42**, 1130 (1975).

[24] R. A. Vardanyan and B. I. Ivlev, Zh. Eksp. Teor. Fiz. **65**, 2315 (1973) [Sov. Phys. JETP **38**, 1156 (1974)].

[25] A. G. Kozorezov, A. F. Volkov, J. K. Wigmore, A. Peacock, A. Poelaert, and R. den Hartog, Phys. Rev. B **61**, 11807 (2000).

[26] P. Santhanam and D. E. Prober, Phys. Rev. B **29**, 3733 (1984).

[27] J. J. Lin and J. P. Bird, J. Phys.: Condens. Matter **14**, R501 (2002).

[28] J. Kivioja, Ph. D. thesis, Helsinki University of Technology, 2005.