

# Properties of superconducting planar resonators at millikelvin temperatures

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Planar superconducting resonators are now being increasingly used at mK temperatures in a number of novel applications. We have experimentally investigated three types of niobium resonators including a lumped-element design fabricated on sapphire and SiO<sub>2</sub>/Si substrates. They all exhibit a nonmonotonic temperature dependence of their center frequency which could be accurately described by resonant absorption by two-level fluctuators (TLF). By analyzing the temperature dependence we extracted information about the distribution of TLF. Our results are consistent with there being a spread of TLF at the interface between superconductor and substrate.

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## I. INTRODUCTION

The superconducting microwave resonator is a ubiquitous device with uses ranging from the very practical such as filters for telecommunications to rather exotic such as tests of cavity quantum electrodynamics. Over recent years there has been a resurgence in the interest in on-chip resonators; the primary reason being their use as very sensitive photon detectors and much of the work to date has been motivated by the need to understand noise properties of kinetic inductance detectors.<sup>1-3</sup> Resonators are also being used as elements in circuitry for quantum-information processing.<sup>4</sup> The latter has triggered a wave of investigations into the properties of resonators when operated at mK temperatures and very low (ideally single-photon) microwave powers. Of particular interest has been the effects of two-level fluctuators (see, e.g., Refs. 5 and 6 and references therein). Superconducting resonators at microwave frequencies differ from their normal-metal counterparts primarily because of their high-quality factor ( $Q$ ) and a relatively large kinetic inductance. For example, for an ideal high- $Q$  LC resonator we can write the center frequency as  $f_0 = (2\pi\sqrt{(L+L_K)C})^{-1}$  where  $L_K$  is the contribution from the kinetic inductance.  $L_K$  depends on the order parameter and therefore varies with temperature but can also be altered, e.g., by a weak magnetic field.<sup>7</sup> As the temperature goes down the center frequency increases monotonically while the conductor losses decrease (leading to a higher quality factor) as described by the Mattis-Bardeen theory.<sup>8</sup> However, at temperatures  $\sim T_c/10$ , these mechanisms saturate (in Nb and its compounds around 1 K) and other effects become prominent,<sup>1,3,7</sup> most notably a *decrease* in the center frequency as the temperature is reduced. This “back bending” is now widely believed to be due to the presence of two-level fluctuators (TLF) in the substrate.

The aim of this study is threefold: first to verify that the theory developed for a spin solution in glass can accurately describe the temperature and power dependence of the reso-

nance frequency and losses in superconducting resonators at mK temperatures, second to compare the properties of resonators fabricated on SiO<sub>2</sub>/Si and sapphire, and finally to extract numerical values for the  $nd^2$  factor for these two materials.

## II. THEORY

A generic expression for the transmittance,  $S_{21}$ , of a transmission line shunted by a resonator (e.g., the lumped element and  $\lambda/4$  resonators considered here) can be written  $S_{21} = 2[2 + \frac{g}{1+2jQ_u\delta f}]^{-1}$  where  $Q_u$  is the unloaded quality factor ( $Q$  in the absence of coupling to the feed line),  $g$  is a coupling parameter  $\propto Q_u$  the exact expression for which will depend on the type of resonator used, and  $\delta f = (f - f_0)/f_0$  is the fractional shift from resonance. The value of  $Q_u$  is in general given by summing over all loss mechanisms  $\sum_k Q_k^{-1}$  but losses in the dielectric and the superconducting film itself<sup>9</sup> usually dominate in planar resonators, radiation losses, etc. can be neglected. Microwave losses are usually quantified by the loss tangent  $\delta = 1/Q_u$ , which in a dielectric is proportional to the sum of the various absorption mechanisms  $\sum_k \alpha_k$ .

At sub-Kelvin temperatures one would expect the properties of a dielectric to stay essentially constant. However, experimentally one finds a variation in both the effective dielectric constant and losses. Much of this can be explained if one assumes the presence of a bath of TLF that couple to the resonator via their electric-dipole moment. Many of the details of the corresponding theory were worked out over 30 years ago when the effects of TLF in glasses at low temperatures were investigated.<sup>10</sup> The theory is based on the application of the Bloch equations to an ensemble of spin systems and while it is simple it nevertheless captures most of the essential physics. For resonant absorption due to an ensemble of TLF  $\alpha_r$  has the form<sup>11,12</sup>

$$\alpha_r = \frac{\pi\omega d^2}{3c\epsilon_0\epsilon_r} \left(1 + \frac{P}{P_c}\right)^{-1/2} \tanh\left(\frac{\hbar\omega}{2k_B T}\right), \quad (1)$$

where  $\omega$  is the measurement frequency,  $d$  is the dipole moment,  $c$  is the speed of light, and  $n$  is the density of states of the TLF that couple to the stray field of the resonator. Note that this density should, in principle, depend on frequency but is experimentally often found to be constant. The temperature dependence of the absorption reflects the population difference in the two states of the fluctuators. Note that Eq. (1) only accounts for *resonant* absorption, nonresonant relaxation processes due the TLF bath are not taken into account.

The relation above predicts a power dependence of the absorption  $\propto P^{-1/2}$  above some critical power  $P_c$  although in practice one may expect another high-power roll off once all the TLF are saturated and other loss mechanisms start to dominate. The critical intensity is given by  $P_c = 3\hbar^2 c \epsilon_0 \epsilon_r / 2d^2 T_1 T_2$  where  $T_1$  and  $T_2$  are the relaxation and the dephasing times of the TLF, respectively.

The variation in permittivity is dominated by resonant processes and can therefore be found by applying the Kramers-Kronig relations to Eq. (1)

$$\frac{\epsilon(T) - \epsilon(T_0)}{\epsilon(T_0)} = -\frac{2nd^2}{3\epsilon} \left\{ \ln\left(\frac{T}{T_0}\right) - [g(T, \omega) - g(T_0, \omega)] \right\}, \quad (2)$$

where  $g(T, \omega) = \text{Re} \Psi\left(\frac{1}{2} + \frac{\hbar\omega}{2\pi i k_B T}\right)$ ,  $T_0$  is a reference temperature, and  $\Psi$  the complex digamma function significant only for  $kT \lesssim \hbar\omega/2$ . From cavity perturbation theory we then have that  $\Delta f_0/f_0 = -(F/2)\Delta\epsilon/\epsilon$ ,  $F$  being a filling factor which depends on the geometry and the electric field distribution. The substrate filling factor varies slightly between the three types of resonator discussed here but is over 0.9 in all cases. For samples on  $\text{SiO}_2/\text{Si}$  substrates the contribution of the  $\text{SiO}_2$  layer to  $F$  is about 0.2. Equation (1) predicts an upward turn in the temperature dependence of the center frequency at mK temperatures, where the  $g(T, \omega)$  term is important. In the experiments by Gao *et al.*<sup>13</sup> Eq. (2) was used to fit the temperature dependence of the resonance frequency but the low-temperature upturn was not clearly observed.

### III. EXPERIMENT

We have fabricated a wide variety of  $\lambda/2$ ,  $\lambda/4$ , and lumped-element (LE) niobium resonators (see Fig. 1) on  $R$ -cut sapphire and thermally oxidized high-resistivity ( $>1$  kOhm/cm<sup>2</sup>) silicon substrates with 400 nm of  $\text{SiO}_2$ . The simplest design is the inline  $\lambda/2$  resonator. Its advantages is that it can be made with loaded quality factors of  $>10^6$  and is easy to measure (due to the high signal-to-noise ratio on resonance), there is however only one resonator per chip and a calibrated measurement is needed in order to quantify its unloaded quality factor (which is difficult at mK temperatures).  $\lambda/4$  resonators have the advantage that their parameters can be extracted relatively easily (using numerical fitting) from uncalibrated measurements since the transmittance far from resonance can be used as a reference level. Several  $\lambda/4$  resonators can be coupled to a common feed

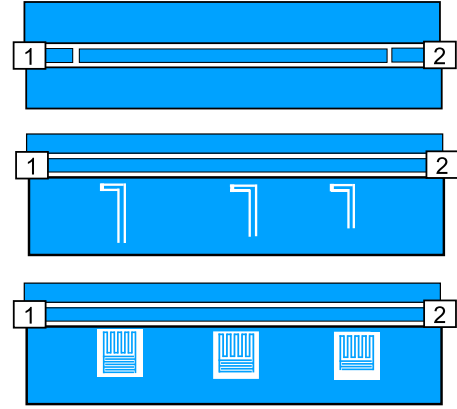


FIG. 1. (Color online) Illustration (not to scale) showing the geometry and measurement configuration (ports 2  $\rightarrow$  1) for the three types of resonators used in this work. From top to bottom:  $\lambda/2$ ,  $\lambda/4$ , and LE resonator. All coplanar lines have a 10  $\mu\text{m}$  center conductor. Typical linewidth of the LE resonators is 4  $\mu\text{m}$ .

line, making it possible to measure in a frequency-multiplexed configuration. The lumped-element resonator shares these advantages and is much more compact (typical size is about  $200 \times 200 \mu\text{m}^2$ ) than geometric resonators. Our LE resonators fabricated in Nb on sapphire exhibit loaded  $Q$  close to  $10^5$  with an unloaded  $Q$  that is about three times higher; i.e., the resonators are by design overcoupled.

The structures were patterned using standard photolithography in 200-nm-thick sputtered Nb films using argon ion-beam etching and were designed to have their center frequencies in the  $c$  band (4–8 GHz). Table I shows a summary of the samples discussed in this paper. Other resonators measured to date showed similar behavior. Each chip is  $5 \times 10 \text{ mm}^2$  in size and incorporates a central 50  $\Omega$  coplanar feed line and one or more resonators. For the measurement the chip is glued to a gold-plated alumina carrier inside a superconducting (Al,  $T_c = 1.2$  K) box, where carrier and chip are thermalized using a copper cold finger. All measurements (with the exception for the data above 1 K in Fig. 4) were carried out in a dilution refrigerator with heavily filtered mi-

TABLE I. Summary of the resonators used in these experiments presented in this paper.

	Type	Chip	Substrate	$f_0$ at 50 mK (MHz)
R1	$\lambda/2$ <sup>a</sup>	A	$\text{SiO}_2/\text{Si}$	6037
R2	$\lambda/4$	B	$\text{SiO}_2/\text{Si}$	5040
R3	$\lambda/4$	B	$\text{SiO}_2/\text{Si}$	5795
R4	$\lambda/4$	B	$\text{SiO}_2/\text{Si}$	8100
R5	LE	C	$\text{SiO}_2/\text{Si}$	6805
R6	LE	C	$\text{SiO}_2/\text{Si}$	7250
R7	LE	C	$\text{SiO}_2/\text{Si}$	7813
R8	LE	D	Sapphire	6046
R9	LE	D	Sapphire	6458
R10	LE	D	Sapphire	6959

<sup>a</sup>Single resonator on the chip, measured inline.

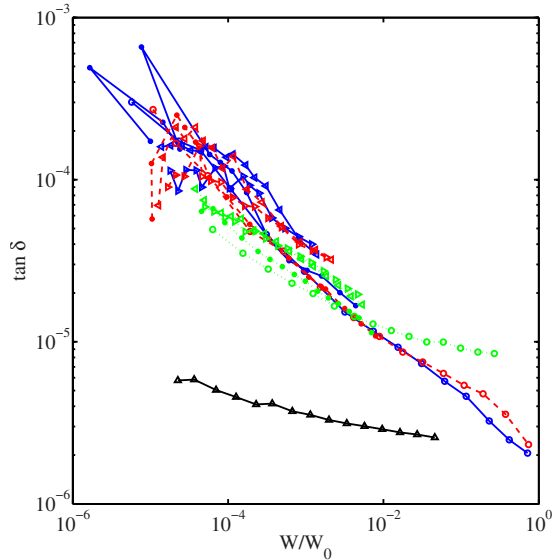


FIG. 2. (Color online) Loss tangent for three lumped-element resonators (R5–R7: dotted, dashed, and solid line, respectively) on SiO<sub>2</sub>/Si at four different temperatures: 50 mK (○), 100 mK(\*), 250 mK(◄), and 340 mK(◄). The lowest curve (▲) shows a similar measurement at 30 mK for R8, a resonator on sapphire. All curves were measured using the same *applied* power.

crowave lines. The sample is further magnetically screened using two layers of mu metal (at room temperature) and a lead shield (in the helium bath). The transmitted signal was amplified using an InP high electron-mobility transistor amplifier held at 1 K with a noise temperature of about 4 K before further amplification stages at room temperature.

#### IV. RESULTS AND DISCUSSION

Figure 2 shows how the loss tangent depends on the energy in the resonator on resonance  $W=2Q_u(1-S_{21})S_{21}P_{in}/\omega_0$  for three different lumped-element resonators on SiO<sub>2</sub>/Si coupled to the same on-chip transmission line and measured simultaneously. Absolute power metrology at mK temperatures is very problematic so only relative measurements were possible; we estimate that the maximum power reaching the chip during the measurements is about -70 dBm, meaning  $W_0$  is on the order of  $10^{-16}$  J. Shown is also R8, a sapphire resonator of the same design. For the resonators fabricated on SiO<sub>2</sub>/Si the curves have a slope of approximately -0.4 in this range of applied powers. Equation (1) predicts dielectric losses changing with a slope of -0.5 for powers above  $P_c$  and power-independent behavior below  $P_c$ , it is therefore possible that we are in the intermediate regime. However, it is also possible that losses in the superconductor not captured by Eq. (1) play a role. Indeed, the current distribution in superconducting resonators is such that its *local* value can be significant even at our lowest powers leading to higher conductor losses as the power is increased. It is also interesting to note that the power entering the resonator depends on  $Q$ , and  $Q$  due to the TLF in turn depends on the power,  $Q \propto P(Q)$ . Hence, the system is non-linear and, in principle, weakly hysteretic.

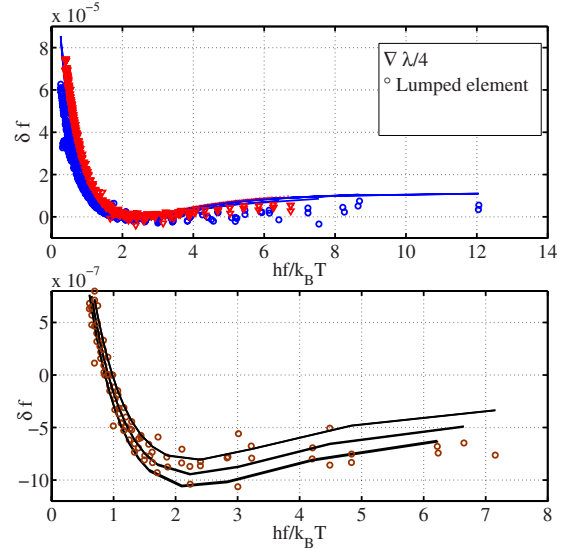


FIG. 3. (Color online) Frequency shift as a function of the normalized frequency at several different powers for LE resonators R2–R7 on SiO<sub>2</sub>/Si (*top*); and R8–R10 sapphire(*bottom*). The solid lines show fits to theory. The log term dominates at small  $\hbar\omega/k_B T$  whereas  $g(t)$  becomes important in the opposite regime. Successive curves differ in power by 10 dB.

The main conclusion from Fig. 2 is that the loss tangent is strongly power dependent for SiO<sub>2</sub>/Si whereas a weak dependence is observed for identical structures on sapphire; in the latter case  $Q$  only changes by about a factor of 2 even when the power is changed by over 40 dB.

Experimentally, the temperature dependence of the resonance frequency of superconducting resonators is quite complicated since it is affected both by the properties of the superconducting film (e.g., the kinetic inductance) and changes in the dielectric and it is in general difficult to separate these two effects. However, at temperatures  $\ll T_c$ , which is the case here, we can assume that the thermal effects on the kinetic inductance are small. Figure 3 shows the relative shift of the resonance frequency as a function of  $\hbar\omega/k_B T$  together with a fit to Eq. (2). All data from the resonators on a particular substrate can be fitted using a single parameter; we find  $Fd^2n/3\epsilon = F \tan \delta_i / \pi = 6 \times 10^{-5}$  and  $2.8 \times 10^{-6}$  for the SiO<sub>2</sub>/Si and sapphire, respectively; a difference of about a factor of 20; consistent with the loss tangent measurements. Here we have introduced the *intrinsic* loss tangent  $\tan \delta_i$  which corresponds to the low power, low-temperature limit of  $\tan \delta$ . Note that the curves shown come from several resonators of two different designs (on separate chips) with each resonator measured at several different power levels but they can all be fitted using a single parameter. We note that the values for  $Fd^2n/3\epsilon$  we obtain from samples fabricated on sapphire are lower than those reported by Gao *et al.*

In order to verify that the temperature dependence does not depend on the geometry or our data analysis, we have in addition to lumped element and  $\lambda/4$  (in shunt configuration) resonators also measured undercoupled inline  $\lambda/2$  coplanar resonators. Figure 4 shows the measured temperature dependence of  $1/Q_l - Q_l$  being the loaded quality factor and  $f_0$  the center frequency. Again we can fit all the  $f_0$  curves using a

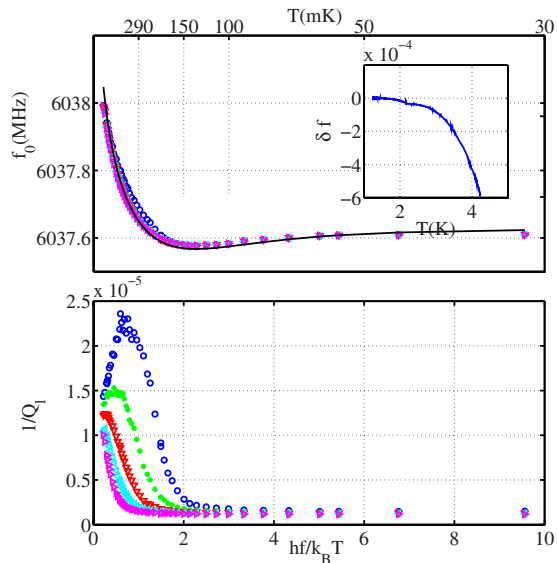


FIG. 4. (Color online) Centre frequency (top) and  $1/Q_I$  (bottom) at five different powers and as a function of temperature for sample R1, a  $\lambda/2$  resonator fabricated on  $\text{SiO}_2/\text{Si}$ . Each successive curve (top to bottom) corresponds to an increase in power by 2 dB. Solid curves show fit to theory. Inset:  $\lambda/2$  resonator measured above 1 K.

single parameter, although in this case the fitting parameter  $Fd^2n/3\epsilon$  is  $4.95 \times 10^{-5}$ , slightly lower than for the other samples. Some of this difference is likely to be due to somewhat different filling factors.

At temperatures above 1 K the losses are dominated by the conductor losses described by the Mattis-Bardeen theory but at low temperatures and low microwave power conductor losses are no longer important; resonant absorption dominates and the losses increase as the temperature is reduced as captured by Eq. (1). High power on the other hand saturates the TLS thus suppressing the resonant absorption.

Equation (1) cannot describe all phenomena which is not surprising since only resonant losses are taken into account. Figure 4 (bottom) shows a measurement where nonmonotonic losses at temperatures below about 300 mK were ob-

served in a narrow range of microwave powers. The likely explanation is that the relative significance of *relaxation* absorption<sup>14</sup> processes increases for temperatures below  $\approx \hbar\omega/k_B$  and intermediate power levels, just enough to suppress the resonant absorption. As the power is increased, the crossover to the relaxation losses is pushed to higher temperatures.

While the presence of TLF is known to affect resonators it is nevertheless surprising how well our data can be fitted using a single parameter independent of both the frequency (over a range of several GHz) and the location on the chip. While our data do not give us a reliable way to identify the nature of these TLF, one can with certainty conclude that presence of TLF is intrinsic to the materials used. The bath of TLF appears to have a very wide distribution of both the tunnel splittings and the symmetry parameter of the (effective) double well. The large  $nd^2$  factor in  $\text{SiO}_2/\text{Si}$ , consistent with previous measurements,<sup>15</sup> comes as no surprise given the large density of TLF one would expect in an amorphous oxide material. The  $nd^2$  factor in sapphire is significantly smaller but still measurable. This is in itself quite interesting as it is well known that the only two-levels systems seen in very high-quality dielectric sapphire resonators are known to be due to relatively narrow electron-spin resonances on incidental paramagnetic ions, such as Fe or Cr; if these were the origin of the TLF in our samples one would not expect the data to agree with the theory used here. This suggests that the TLF seen here are located in the interface between the film and the substrate, as opposed to the bulk material.

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<sup>1</sup>J. Gao, J. Zmuidzinas, B. Mazin, H. LeDuc, and P. Day, *Appl. Phys. Lett.* **90**, 102507 (2007).

<sup>2</sup>S. Kumar, J. Gao, J. Zmuidzinas, B. Mazin, H. LeDuc, and P. Day, *Appl. Phys. Lett.* **92**, 123503 (2008).

<sup>3</sup>R. Barends, H. L. Hortensius, T. Zijlstra, J. J. A. Baselmans, S. J. C. Yates, J. R. Gao, and T. M. Klapwijk, *Appl. Phys. Lett.* **92**, 223502 (2008).

<sup>4</sup>A. Wallraff, D. Schuster, A. Blais, L. Frunzio, R. Huang, J. Majer, S. Kumar, S. Girvin, and R. Schoelkopf, *Nature (London)* **431**, 162 (2004).

<sup>5</sup>J. Clarke and F. K. Wilhelm, *Nature (London)* **453**, 1031 (2008).

<sup>6</sup>L. Faoro, A. Kitaev, and L. B. Ioffe, *Phys. Rev. Lett.* **101**, 247002 (2008).

<sup>7</sup>J. E. Healey, T. Lindstrom, M. S. Colclough, C. M. Muirhead, and A. Y. Tzalenchuk, *Appl. Phys. Lett.* **93**, 043513 (2008).

<sup>8</sup>D. Mattis and J. Bardeen, *Phys. Rev.* **111**, 412 (1958).

<sup>9</sup>C. Song, T. W. Heitmann, M. P. DeFeo, K. Yu, R. McDermott, M. Neeley, J. M. Martinis, and B. L. T. Plourde, *Phys. Rev. B* **79**, 174512 (2009).

<sup>10</sup>W. A. Phillips, *Rep. Prog. Phys.* **50**, 1657 (1987).

<sup>11</sup>U. Strom, M. von Schickfus, and S. Hunklinger, *Phys. Rev. Lett.* **41**, 910 (1978).

<sup>12</sup>M. von Schickfus and S. Hunklinger, *Phys. Lett.* **64A**, 144 (1977).

<sup>13</sup>J. Gao, M. Daal, A. Vayonakis, S. Kumar, J. Zmuidzinas, B. Sadoulet, B. A. Mazin, P. K. Day, and H. G. Leduc, *Appl. Phys. Lett.* **92**, 152505 (2008).

<sup>14</sup>Y. Galperin, V. Gurevich, and D. Parshin, in *Hopping Transport in Solids*, edited by B. Shklovskii and M. Pollak (Elsevier, New York, 1991).

<sup>15</sup>A. O'Connell, M. Ansmann, R. Bialczak, M. Hofheinz, N. Katz, E. Lucero, C. McKenney, M. Neeley, H. Wang, and E. Weig, *Appl. Phys. Lett.* **92**, 112903 (2008).