

Photovoltaic effect in a gated two-dimensional electron gas in magnetic field

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The photovoltaic effect induced by terahertz radiation in a gated two-dimensional electron gas in magnetic field is considered theoretically. It is assumed that the incoming radiation creates an ac voltage between the source and the gate and that the gate length is long compared to the damping length of plasma waves. In the presence of pronounced Shubnikov-de Haas (SdH) oscillations, an important source of nonlinearity is the oscillating dependence of the mobility on the ac gate voltage. This results in a photoresponse oscillating as a function of magnetic field, which is enhanced in the vicinity of the cyclotron resonance, in accordance with recent experiments. Another smooth component of the photovoltage, unrelated to SdH oscillations, has a maximum at cyclotron resonance.

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The two-dimensional (2D) gated electron gas in a field effect transistor can be used for generation¹ and detection² of terahertz radiation, and both effects were demonstrated experimentally.³⁻⁷ Concerning the detection, the idea is that the nonlinear properties of the electron fluid will lead to the rectification of ac induced in the transistor channel by the incoming radiation. As a result, a photoresponse in the form of a dc voltage between source and drain appears, which is proportional to the radiation intensity (photovoltaic effect). Obviously some asymmetry between the source and the drain is needed to induce such a voltage.

There may be various reasons of such asymmetry. One of them is the difference in the source and drain boundary conditions. Another one is the asymmetry in feeding the incoming radiation, which can be achieved either by using a special antenna or by an asymmetric design of the source and drain contacts with respect to the gate contact. Thus, the radiation may predominantly create an ac voltage between the source and the gate. Finally, the asymmetry can naturally arise if a dc is passed between source and drain, creating a depletion of the electron density on the drain side of the channel.

The photoresponse can be either resonant, corresponding to the excitation of the discrete plasma oscillation modes in the channel, or nonresonant if the plasma oscillations are overdamped.² Both nonresonant⁵ and resonant^{6,7} detections were demonstrated experimentally. A practically important case is that of a long gate, such that the plasma waves excited by the incoming radiation at the source cannot reach the drain side of the channel because their damping length is smaller than the source-drain distance. Within the hydrodynamic approach the following result for the photoinduced voltage ΔU was derived for this case:²

$$\Delta U = \frac{1}{4} \frac{U_a^2}{U_0} f(\omega), \quad f(\omega) = 1 + \frac{2\omega\tau}{\sqrt{1 + (\omega\tau)^2}}, \quad (1)$$

where ω is the radiation frequency; τ is the momentum relaxation time; U_a is the amplitude of the ac modulation of the gate-to-source voltage by the incoming radiation; and U_0 is the static value of the gate-to-channel voltage swing U ,

which is related to the electron density n in the channel by the plane capacitor formula

$$en = CU. \quad (2)$$

Here, e is the elementary charge and C is the gate-to-channel capacitance per unit area. Equation (2) is applicable if the scale of the variation in the potential in the channel is large compared to the gate-to-channel separation.

Recently, the first experiments on the photovoltaic effect at terahertz frequencies in a gated high mobility two-dimensional electron gas in a magnetic field were performed.^{8,9} The main new results are (i) the photoinduced dc drain-to-source voltage exhibits strong oscillations as a function of magnetic field, similar to the Shubnikov-de Haas (SdH) resistance oscillations, and (ii) the oscillation amplitude strongly increases in the vicinity of the cyclotron resonance.

In this Rapid Communication we consider theoretically the photovoltaic effect in a gated electron gas in a magnetic field assuming, as in Ref. 2, that the incoming radiation creates an ac voltage between the source and the drain. Further, in accordance with the experimental conditions, we assume that (1) the source-drain length L (x direction) is greater than the plasma-wave damping length, so that the plasma waves excited near the source do not reach the drain, and (2) the sample width W in the y direction is much greater than L (see Fig. 1). The first assumption means that the boundary conditions at the drain are irrelevant and, as far as plasma waves are concerned, the sample can be considered to be infinite in the x direction. The second one implies a quasi-Corbino geometry (all variables depend on the x coordinate only). We explain the observed strongly oscillating photoresponse as being due to the nonlinearity originating from the oscillating dependence of the mobility on the Fermi energy, and hence on the ac part of the gate voltage.

The photovoltaic effect is due to a radiation-induced force \mathbf{G} driving the electron current. Without magnetic field, \mathbf{G} is obviously directed in the x direction and is compensated by the appearance of an electric field. In the presence of magnetic field the problem becomes more subtle, not only because in this case \mathbf{G} has a y component, but also because this

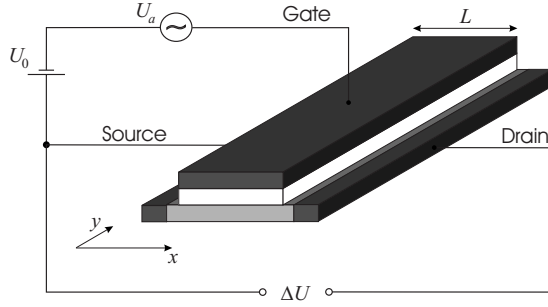


FIG. 1. Assumed design and geometry. The terahertz radiation produces an ac voltage U_a between the source and the gate inducing a dc source-drain voltage ΔU . The gate width W is much larger than the gate length L (quasi-Corbino geometry).

radiation-induced force becomes *nonpotential*: $\text{curl } \mathbf{G} \neq 0$. The nonpotential part will drive an electric current along closed loops.

The significance of the cyclotron resonance for the photovoltaic effect is related to the well-known dispersion relation for plasma waves in a magnetic field.¹⁰ For gated two-dimensional electrons it reads

$$\omega = \sqrt{\omega_c^2 + s^2 k^2}, \quad (3)$$

where ω_c is the cyclotron frequency, s is the plasma-wave velocity, and k is the wave vector. Thus, the plasma waves can propagate only if $\omega_c < \omega$. In the opposite case the wave vector becomes imaginary, so that the plasma oscillations rapidly decay away from the source. The change in regime when the magnetic field is driven through its resonant value will manifest itself in the photoresponse.

Following Refs. 1 and 2 and other theoretical work, we will use the hydrodynamical approach because, like the Drude equation, it provides a relatively simple description compared to the full kinetic theory. However it should be understood that, at low temperatures at which the experiments^{8,9} were done, this approach strictly speaking is not justified because the collisions between electrons are strongly suppressed by the Pauli principle. Nevertheless, the qualitative physical results derived from the kinetic equation and from the hydrodynamic equations are usually similar, e.g., the properties of plasma waves are identical in both approaches, provided that the plasma-wave velocity s is greater than the Fermi velocity, so that the Landau damping can be neglected.¹¹ For this reason, we leave the much more complicated approach based on the kinetic equation for future studies.

The electrons in a gated 2D channel can be described by the following equations:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{e}{m} \nabla U + \frac{e}{mc} \mathbf{B} \times \mathbf{v} - \gamma \mathbf{v}, \quad (4)$$

$$\frac{\partial U}{\partial t} + \text{div}(U\mathbf{v}) = 0, \quad (5)$$

where \mathbf{v} is the electron drift velocity, \mathbf{B} is the magnetic field along the z direction, m is the electron effective mass, and

$\gamma = 1/\tau$. The parameter γ is an oscillating function of the electron concentration (or gate voltage) and magnetic field, which results in the SdH oscillations.

Equation (4) is the Euler equation, taking account of the Lorentz force and the damping due to collisions. It differs from the conventional Drude equation only by the convective term $(\mathbf{v} \cdot \nabla) \mathbf{v}$. Equation (5) is the continuity equation rewritten with the use of Eq. (2).

The boundary condition at the source ($x=0$) is

$$U(0, t) = U_0 + U_a \cos \omega t, \quad (6)$$

where ω is the frequency of the incoming radiation and U_a is the amplitude of the radiation-induced modulation of the gate-to-source voltage. For a long sample, the boundary condition at the drain is

$$\mathbf{v} \rightarrow 0, \quad U \rightarrow \text{const} \quad \text{for } x \rightarrow \infty. \quad (7)$$

We will search for the solution of Eqs. (4) and (5) as an expansion in powers of U_a ,

$$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2, \quad U = U_0 + U_1 + U_2. \quad (8)$$

Here, \mathbf{v}_1 and U_1 are the ac components proportional to U_a , which can be found by linearizing Eqs. (4) and (5) and \mathbf{v}_2 and U_2 are the dc components, proportional to U_a^2 (we are not interested in the second harmonic terms $\sim U_a^2$). It is convenient to introduce $u = eU/m$, $u_a = eU_a/m$, and the plasma-wave velocity¹ in the absence of magnetic field $s = u_0^{1/2} = (eU_0/m)^{1/2}$.

To the first order in U_a , we obtain

$$\frac{\partial v_{1x}}{\partial t} + \frac{\partial u_1}{\partial x} + \omega_c v_{1y} + \gamma v_{1x} = 0, \quad (9)$$

$$\frac{\partial v_{1y}}{\partial t} - \omega_c v_{1x} + \gamma v_{1y} = 0, \quad (10)$$

$$\frac{\partial u_1}{\partial t} + s^2 \frac{\partial v_{1x}}{\partial x} = 0, \quad (11)$$

where $\omega_c = eB/mc$ is the cyclotron frequency. The boundary conditions follow from Eqs. (6) and (7): $u_1(0, t) = u_a \cos(\omega t)$ and $u_1(\infty, t) = 0$, $\mathbf{v}_1(\infty, t) = 0$.

Searching for the solutions $\sim \exp(ikx - i\omega t)$, we obtain the dispersion equation for the plasma waves as

$$\frac{s^2}{\omega^2} k^2 = 1 + i\alpha - \frac{\beta^2}{1 + i\alpha}, \quad (12)$$

where $\alpha = (\omega\tau)^{-1}$ and $\beta = \omega_c/\omega$ is the magnetic field in units of its resonant value for a given ω . To ensure the boundary condition at $x \rightarrow \infty$ the root with a positive imaginary part of k should be chosen. If damping is neglected ($\alpha = 0$), this equation reduces to Eq. (3). The explicit expressions for u_1 , v_{1x} , and v_{1y} are easily obtained from Eqs. (9)–(11).

In the second order in U_a , we find

$$\frac{du_2}{dx} + \omega_c v_{2y} + \gamma v_{2x} + \left\langle v_{1x} \frac{\partial v_{1x}}{\partial x} \right\rangle + \gamma' \langle u_1 v_{1x} \rangle = 0, \quad (13)$$

$$-\omega_c v_{2x} + \gamma v_{2y} + \left\langle v_{1x} \frac{\partial v_{1y}}{\partial x} \right\rangle + \gamma' \langle u_1 v_{1y} \rangle = 0, \quad (14)$$

$$\frac{dj_x}{dx} = 0, \quad j_x = v_{2x} + \frac{1}{u_0} \langle u_1 v_{1x} \rangle, \quad (15)$$

where the angular brackets denote the time averaging over the period $2\pi/\omega$. Here, we have expanded the function $\gamma(u)$ to the first order in u_1 . The quantities γ and $\gamma' = d\gamma/du$ should be taken at $u = u_0$. The boundary conditions for Eqs. (13)–(15) are $u_2(0) = 0$, $v_{2x}(\infty) = v_{2y}(\infty) = 0$.

From Eq. (15) we derive the obvious fact that $j_x = 0$ (j_x differs from the x component of the true current density only by a factor en). Using this, and introducing the y component of the current j_y , by a relation similar to Eq. (15), we can rewrite Eqs. (13) and (14) as follows:

$$\omega_c j_y = G_x(x) - \frac{du_2}{dx}, \quad \gamma j_y = G_y(x), \quad (16)$$

where the additional driving force $\mathbf{G}(x)$ induced by the incoming radiation is given by

$$G_x = \left(\frac{\gamma}{u_0} - \gamma' \right) \langle u_1 v_{1x} \rangle + \frac{\omega_c}{u_0} \langle u_1 v_{1y} \rangle - \left\langle v_{1x} \frac{\partial v_{1x}}{\partial x} \right\rangle, \quad (17)$$

$$G_y = \left(\frac{\gamma}{u_0} - \gamma' \right) \langle u_1 v_{1y} \rangle - \frac{\omega_c}{u_0} \langle u_1 v_{1x} \rangle - \left\langle v_{1x} \frac{\partial v_{1y}}{\partial x} \right\rangle. \quad (18)$$

Both G_x and G_y depend on x as $\exp(-2k''x)$, where k'' is the imaginary part of the wave vector defined by Eq. (12), reflecting the decay of the plasma-wave intensity away from the source. Thus, $\text{curl } \mathbf{G} \neq 0$.

One could solve Eqs. (16) to obtain the photoinduced voltage $\Delta u = \int_0^\infty [G_x - (\omega_c/\gamma)G_y] dx$ and this would be the correct result for the *true* Corbino geometry, where the current j_y can freely circulate around the ring. However, we believe that this is not correct for a finite strip, even if $W \gg L$, because in this case the current j_y induced by the nonpotential part of the driving force, $G_y(x)$, obviously must return back somewhere, forming closed loops.¹² How exactly this will happen is not quite clear. In our model, the current loops are likely to close through the source contact; however, in reality the oppositely directed y current will probably flow in the ungated part of the channel adjacent to this contact. Anyway, since the current j_y integrated over x must be zero (except near the extremities), we believe that the correct way is to integrate the first of Eqs. (16), taking this into account, and to ignore the second one, which is not applicable beyond the gated part of the channel. The integration interval should be expanded to include the region where the current lines return backward.

We have no rigorous proof that this idea is correct; however, we have checked that both methods give similar qualitative results (but differ in the exact form of the magnetic field dependence of the photovoltage).

As described above, we obtain $\Delta u = u_2(\infty)$,

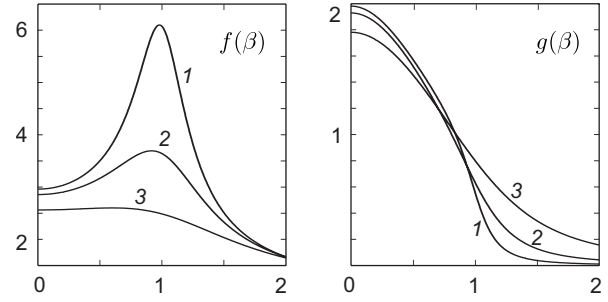


FIG. 2. The functions $f(\beta)$ (left) and $g(\beta)$ (right) describing, respectively, the smooth part and the envelope for the oscillating part of the photovoltage. The values of the parameter $\alpha = (\omega\tau)^{-1}$ are (1) 0.2, (2) 0.4, and (3) 0.8

$$\Delta u = \int_0^\infty G_x(x) dx. \quad (19)$$

Using Eqs. (17) and (19) we finally calculate the dc photovoltage $\Delta U = m\Delta u/e$, between drain and source induced by the incoming radiation,

$$\Delta U = \frac{1}{4} \frac{U_a^2}{U_0} \left[f(\beta) - \frac{d\gamma n}{dn \gamma} g(\beta) \right]. \quad (20)$$

Here, we have separated the photoresponse in a smooth part and in an oscillating part. The second one, proportional to $d\gamma/dn$, is an oscillating function of gate voltage or magnetic field $\sim d\rho_{xx}/dn$, where ρ_{xx} is the longitudinal resistivity of the gated electron gas.

Note that, even if the amplitude of the SdH oscillations is small, the parameter $|d\rho_{xx}/dn|(n/\rho_{xx})$ can be large, so that the oscillating contribution may dominate.

The frequency and the magnetic field dependences of the photovoltage are described by the functions $f(\beta)$ and $g(\beta)$, which are given by the following formulas:¹³

$$f(\beta) = 1 + \frac{1+F}{\sqrt{\alpha^2 + F^2}}, \quad (21)$$

$$g(\beta) = \frac{1+F}{2} \left(1 + \frac{F}{\sqrt{\alpha^2 + F^2}} \right), \quad (22)$$

where F depends only on the ratio $\beta = \omega_c/\omega$ and the dimensionless parameter $\alpha = (\omega\tau)^{-1}$,

$$F = \frac{1 + \alpha^2 - \beta^2}{1 + \alpha^2 + \beta^2}. \quad (23)$$

In the absence of magnetic field, $\beta = 0$, $F = 1$, and Eq. (21) reduces to Eq. (1).

Figure 2 shows the behavior of the functions $f(\beta)$ and $g(\beta)$ for several values of the parameter α . One can see that for small values of α (or large $\omega\tau$) the smooth part displays the cyclotron resonance with the unusual line shape $f(\beta) \sim [(1-\beta)^2 + \alpha^2]^{-1/2}$. The envelope for the oscillating part, $g(\beta)$, exhibits a fast decay beyond the cyclotron resonance ($\beta > 1$), confining the oscillations of the photovoltage to the region $\beta \sim 1$.

To display the oscillating contribution, we take the param-

eter γ in the conventional form,¹⁴ which is valid when the SdH oscillations are small,

$$\gamma = \gamma_0 \left[1 - 4 \frac{\chi}{\sinh \chi} \exp\left(-\frac{\pi}{\omega_c \tau_q}\right) \cos\left(\frac{2\pi E_F}{\hbar \omega_c}\right) \right], \quad (24)$$

where $\chi = 2\pi^2 kT / \hbar \omega_c$, τ_q is the “quantum” relaxation time, and E_F is the Fermi energy, which is proportional to the electron concentration n , and hence to the gate voltage swing U .

We introduce the parameter $N = E_F / \hbar \omega$, which is the number of Landau levels below the Fermi level at cyclotron resonance. Figure 3 presents the oscillating part of the photovoltage [the function $-(d\gamma/dn)(n/\gamma)g(\beta)$] for $\alpha = 0.1$, $\chi = 0.7$ (corresponding to $T = 4$ K, $\omega = 2\pi \times 2.5$ THz), and $\omega\tau_q = 0.5$ for two values of N .

In spite of the crudeness of our model, which does not account for various features of the experimental situation (the unavoidable presence of ungated parts of the channel, etc.), our results show a good agreement with the recent experimental findings.⁹

In summary, we have calculated the photovoltage induced in a gated electron gas by terahertz radiation in the presence of the magnetic field. As a function of magnetic field, the photoresponse contains a smoothly varying part and an oscillating part proportional to the derivative of the SdH oscillations with respect to the gate voltage. The smooth part shows an enhancement in the vicinity of the cyclotron resonance.

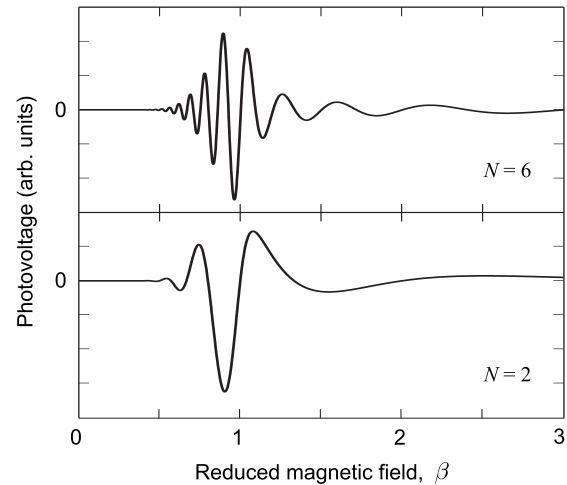


FIG. 3. Magnetic field dependence of the oscillating part of the photovoltage for $\alpha = 0.1$, $\omega\tau_q = 0.5$ for two values of $N = E_F / \hbar \omega$. The vertical scale for the lower trace is expanded four times with respect to the upper trace; $\beta = \omega_c / \omega$.

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¹²In a Hall transport experiment there is no significant difference between the true Corbino geometry and the quasi-Corbino case of a finite strip with $W \gg L$. The current j_y exists everywhere, except the extremities of the sample at $y = \pm W/2$, where the current lines exit and enter the left and the right contacts, respectively. In our case, the current lines must form closed loops, which most probably will pass through the source contact, or the adjacent to this contact ungated part of the channel.

¹³Similar results can be obtained within the Drude theory [neglecting the convective term $(\mathbf{v} \cdot \nabla)\mathbf{v}$]. The oscillating part remains the same, while Eq. (21) acquires an additional factor of 1/2 in the second term, which does not modify the qualitative behavior of $f(\beta)$.

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