Magnetic properties of RFe₂Zn₂₀ and RCo₂Zn₂₀ (R=Y, Nd, Sm, Gd–Lu)

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Magnetization, resistivity, and specific heat measurements were performed on solution-grown single crystals of RFe₂Zn₂₀ and RCo₂Zn₂₀ (R=Y,Nd,Sm,Gd-Lu). Whereas LuCo₂Zn₂₀ and YCo₂Zn₂₀ manifest unremarkable metallic behavior, LuFe₂Zn₂₀ and YFe₂Zn₂₀ display behaviors such as characteristic of nearly ferromagnetic Fermi liquids. When the well-defined 4f local moments (Gd³⁺-Tm³⁺) are embedded into this strongly polarizable host, they manifest enhanced ferromagnetic ordering and the values of $T_{\rm C}$ for $R {\rm Fe}_2 {\rm Zn}_{20}$ (R =Gd-Tm) scale with the de Gennes factor. In addition, data on the $RFe_{2}Zn_{20}$ compounds indicate a small crystal electric field (CEF) effect compared with the interaction energy scale. On the other hand, the local moment bearing members of RCo₂Zn₂₀ (R=Nd,Sm,Gd-Tm) manifest weak magnetic interactions and the magnetic properties for R=Dy-Tm members are strongly influenced by the CEF effect on the R ions. The magnetic anisotropy and specific heat data for the Co series were used to determine the CEF coefficient of Rion with its cubic point symmetry. These CEF coefficients, determined for the Co series, are consistent with the magnetic anisotropy and specific heat data for the Fe series, which indicates similar CEF effects for the Fe and Co series. Such analysis, combined with specific heat and resistivity data, indicates that for R=Tb-Ho, the CEF splitting scale is smaller than their $T_{\rm C}$ values, whereas for ErFe₂Zn₂₀ and TmFe₂Zn₂₀ the 4f electrons lose part of their full Hund's rule ground state degeneracy above $T_{\rm C}$. YbFe₂Zn₂₀ and YbCo₂Zn₂₀ manifest typical but distinct heavy fermion behaviors associated with different Kondo temperatures.

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I. INTRODUCTION

Intermetallic compounds consisting of rare-earth and transition metals, as well as metalloids, have versatile magnetic properties.^{1,2} Compounds with itinerant d electrons are of particular interest when they are in the vicinity of the Stoner transition: such systems, characterized as nearly or weakly ferromagnet, display strongly correlated electronic properties.³ On the other hand, heavy rare-earth ions manifest magnetic versatility associated with the 4f electrons: null magnetism (Y³⁺ or Lu³⁺), pure spin, local moment magnetism (Gd^{3+}), potentially anisotropic, crystal electric field (CEF) splitting, local moment magnetism (Tb³⁺-Tm³⁺), and more exotic magnetism; Yb ions may hybridize with conduction electrons and manifest socalled heavy fermion behavior. Series of compounds that combine these interesting properties has long attracted the attention of physicists. For example, the binary RCo₂ (R =rare earth) compounds, with the nearly ferromagnetic (FM) end members YCo₂ and LuCo₂, and the local moment FM members (R=Pr, Nd, Gd-Tm) have been studied for more than 35 years.4,5

Discovered by Nasch *et al.*,⁶ the RT_2Zn_{20} (R=rare earth, T=transition metal in the Fe, Co, and Ni groups) series of compounds crystallizes in the cubic CeCr₂Al₂₀ structure.⁷⁻⁹ The R and T ions occupy their own single, unique, crystallographic sites with cubic and trigonal point symmetries, respectively, whereas the Zn ions have three unique crystallographic sites [Fig. 1(a)]. Both the R and the T ions are fully surrounded by shells of Zn that consist of the nearest neighbors (NNs) and the next nearest neighbors (NNs). This means that there are no R-R, T-T, or R-T nearest neighbor and the shortest R-R spacing is ~6 Å. For the R ions, a Frank-Kasper polyhedron with coordinate number (CN) 16 is formed from 4 Zn NNs on the 16c site and 12 Zn NNNs on the 96g site [Fig. 1(b)].

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Recently, studies of the thermodynamic and transport properties of these intermetallics have revealed varied magnetic behavior.^{10–13} YFe₂Zn₂₀ and LuFe₂Zn₂₀ are archetypical examples of nearly ferromagnetic Fermi liquids (NFFLs) with Stoner enhancement factors of Z=0.88 [where $\chi_{T=0}$ $=\chi_{Pauli}/(1-Z)$]. By embedding large Heisenberg-type moments associated with Gd³⁺ ions in this highly polarizable medium, GdFe₂Zn₂₀ manifests highly enhanced FM order. On the other hand, GdCo2Zn20 manifests ordinary, low temperature, antiferromagnetic (AFM) order, correspondent to the "normal metal" behavior of the conduction electron host, YCo_2Zn_{20} .¹⁰ In addition to these interesting 3d electron and local moment properties, six related YbT2Zn20 compounds (T=Fe, Co, Ru, Rh, Os, and Ir) show heavy fermion ground states associated with different Kondo temperatures (T_K) and Yb ion degeneracies.¹¹

Given the similarities and differences between the RFe_2Zn_{20} and RCo_2Zn_{20} (R=Gd, Y, Lu) series, it becomes important to study all of the R=Y, Gd-Lu members in detail. A comparative study of the RFe_2Zn_{20} and RCo_2Zn_{20} series



FIG. 1. (Color online) (a) The cubic unit cell of RT_2Zn_{20} . (b) The CN-16 Frank-Kasper polyhedron of rare-earth ions.

will help us to further understand the magnetic interaction between the local moments by means of the strongly polarizable medium, particularly with the CEF effect associated with nonzero orbital angular momentum. Furthermore, given the virtually identical CN-16 Frank-Kasper polyhedron for *R* ions, as well as the less than 2% difference of lattice constants for the whole RT_2Zn_{20} families, the study of the CEF effect on these local moment members will also help in the understanding of the varied heavy fermion states of Yb T_2Zn_{20} , which were thought to be due to the competition between temperature scales associated with the CEF splitting and the Kondo effect.¹¹

In this paper, we present the results of magnetization, heat capacity, and resistivity measurements on RFe₂Zn₂₀ and RCo_2Zn_{20} (R=Y,Nd,Sm,Gd-Lu) compounds. Compared with the weakly correlated behaviors for YCo₂Zn₂₀ and LuCo₂Zn₂₀, YFe₂Zn₂₀ and LuFe₂Zn₂₀ manifest clear NFFL behaviors associated with the spin fluctuation of the itinerant electrons. For the RFe_2Zn_{20} compounds (R=Gd-Tm), the well-defined local moment members all manifest enhanced FM ordering with $T_{\rm C}$ values that roughly scale with the de Gennes factor. Their anomalous temperature-dependent susceptibility and resistivity can be explained as the result of local moments embedded in a NFFL host. In contrast, for the RCo₂Zn₂₀ series, only Gd and Tb members manifest AFM ordering above 2 K, and the magnetic properties for R =Dy-Tm clearly manifest features associated with single-ion CEF effects associated with the R ions in a coordination of cubic symmetry. For the R=Tb-Tm members in the Co series, the CEF parameters can be determined from the magnetic anisotropy and the specific heat data and are roughly consistent with the results of calculations using a point charge model. For the Fe series, the R=Tb-Tm members show moderate magnetic anisotropy in their ordered states mainly due to the CEF effect on the R ions, which is consistent with the magnetic anisotropy found for the Co members. These results, as well as the analysis of the heat capacity and resistivity data, indicate that the FM state emerges from the fully degenerate Hund's rule ground state for RFe_2Zn_{20} (R =Gd-Ho), whereas ErFe₂Zn₂₀ and TmFe₂Zn₂₀ manifest CEF splitting above their Curie temperatures.

II. EXPERIMENTAL METHODS

Single crystals of RFe_2Zn_{20} and RCo_2Zn_{20} (R = Y, Gd-Lu) were grown from high temperature Zn rich solutions with the initial concentration of starting elements being R:(Fe/Co):Zn=2:4:94, as described previously.^{10,14} The small amount of residual Zn on the resulting crystals was removed by submerging the crystals into an ultrasonic bath of 0.5 vol % HCl in H₂O for 0.5–1 h. Attempts were made to grow even lighter rare-earth members (R=Eu, Nd, and Ce) for the Fe series, but no ternary compounds were found to form. On the other hand, for the Co series, single crystals for the R=Nd and Sm members were grown using the same method. Room temperature powder x-ray diffraction measurements, using Cu K_{α} radiation, were performed on the samples with additional Si powder (a=5.430 88 Å) used as standard. Figure 2 shows the lattice constants for the



FIG. 2. (Color online) The lattice constants (a) for RFe_2Zn_{20} and RCo_2Zn_{20} versus the radius of the trivalent rare-earth ion with CN=9 (Ref. 15). The error bars were estimated from the standard variation in four separate measurement made on one batch of sample.

two series of compounds, obtained by using the RIETICA, Rietveld refinement program, plotted with respect to the effective radius of R^{3+} with CN=9 (Ref. 15) since the data are absent for larger CN. The variation in the lattice constant manifests the well known lanthanide contraction for R=Gd-Lu, with no evident deviation for R=Yb. However, the relatively larger lattice constants for YFe2Zn20 and YCo2Zn20 indicate that, with this large CN, the effective ionic radius of Y^{3+} is between Tb^{3+} and Dy^{3+} rather than between Dy^{3+} and Ho^{3+} . This deviation for Y^{3+} ions is not unprecedented, appearing in the isostructural compounds RRu_2Zn_{20} (Ref. 6) and $RMn_2In_xZn_{20-x}$ ¹⁶ as well as the similar structure compounds RCo_2 .¹⁷ Additional single-crystal x-ray diffraction measurements were preformed on R=Gd, Tb, Er, and Lu members of the RFe2Zn20 series and demonstrated full occupancy on all crystallographic sites (within the detection errors) and the same lattice as the powder x-ray values.¹⁸ This result is complicated by the difficulty in resolving the difference between Fe and Zn elements with very similar x-ray scattering strengths.

It is worth noting that for local moment bearing members in the Fe series, single crystals obtained from different ratios of starting element concentrations manifest detectably different magnetic ordering temperatures. These differences, tentatively associated with very subtle variations in element occupancy on the crystallographic sites,¹⁹ are related to extreme sensitivity to a small amount of disorder for compounds with such strongly correlated conduction electron backgrounds. A detail discussion of this is presented in Appendix A.

Measurements of electrical resistivity were preformed by using a standard ac four-probe technique. The samples were cut into bars, with typical lengths of 2–3 mm, along their crystallographic [110] direction. These bars were measured in a Quantum Design physical property measurement system (PPMS), PPMS-14 and/or PPMS-9 instruments (T=1.85–310 K) with f=16 Hz and I=3–0.3 mA. Temperature-dependent specific heat measurements were also performed in these PPMS units using the heat capacity option. In order to study the magnetic part of specific heat, the contribution from the conduction electrons and lattice vibrations was approximated and subtracted. This was performed by subtracting the specific heat of the nonmagnetic members YT_2Zn_{20} or LuT_2Zn_{20} [C_{mag} $=C_p(RT_2Zn_{20})-C_p(Lu/YT_2Zn_{20})$]. The magnetic part of the entropy can then be estimated as $S_M = \int ([C_p(RT_2Zn_{20})]/T) dT$, in which the specific heat data below 1.8 K (0.4 K for TmCo_2Zn_{20}) were approximated by a linear extrapolation to $C_p = 0$ at 0 K.

Magnetization measurements were carried out in Quantum Design magnetic properties measurement system superconducting quantum interface device magnetometers in varied applied fields ($H \le 55$ kOe) and temperatures (T = 1.85-375 K). In the measurements of magnetization for the FM-ordered RFe_2Zn_{20} compounds, the effects of the demagnetizing field are relatively small because of the dilute nature of the magnetic moments:^{12,13} the maximum demagnetizing field is about 3000 Oe for a platelike shaped sample. Even so, in order to obtain accurate magnetization isotherms near T_C , the samples were cut and measured with the applied field along their long axis so as to minimize this already small demagnetization effect.

III. RESULTS

We start by characterizing the compounds with nonmagnetic rare-earth ions: $Y(Lu)Fe_2Zn_{20}$ and $Y(Lu)Co_2Zn_{20}$. Without any 4*f* electronic magnetism, these compounds manifest the electronic and magnetic properties associated with the conduction electron background of each series. Next, we introduce the two series of compounds with welldefined 4*f* local moments: *R*=Gd-Tm. We examine the magnetization and specific heat data for the Co series at first. An overview of the magnetic properties for the Fe series is presented next. The magnetization, specific heat, and resistivity data are discussed for each Fe member separately. Finally, similar data for the *R*=Yb heavy fermion compounds, YbFe₂Zn₂₀ and YbCo₂Zn₂₀, are shown.

A. Y(Lu)Fe₂Zn₂₀ and Y(Lu)Co₂Zn₂₀

Temperature-dependent magnetization data (divided by the applied field) for $Y(Lu)Fe_2Zn_{20}$ and $Y(Lu)Co_2Zn_{20}$ are shown in Fig. 3(a). The Fe members manifest similar, strongly enhanced, temperature-dependent paramagnetic signals, whereas the Co members manifest essentially temperature-independent Pauli paramagnetic signals. The low temperature features for the two Fe compounds are shown in the inset of Fig. 3. In an applied field of 10 kOe, the magnetization signals of both Fe members show a faint maximum below 10 K, whereas the high magnetic field (50 kOe) suppresses the lowest temperature M/H values, as well as the maximum. In our experience on the measurements of different batches of samples, these low temperature features are moderately sample dependent (different samples may show up to 20% difference in magnetization signal and 2-3 K difference in the temperature of the maximum, T_{max}). Nev-



FIG. 3. (Color online) (a) Temperature-dependent magnetization M divided by the applied field H for YFe₂Zn₂₀ and LuFe₂Zn₂₀ as well as their Co analogs for H=10 and 50 kOe. Inset: a blow-up plot at low temperature. (b) H/M for YFe₂Zn₂₀ and LuFe₂Zn₂₀. The solid lines present the modified Curie-Weiss $[\chi(T)=C/(T-\theta_{\rm C})+\chi_0]$ fit for the data above 100 K. Inset: field-dependent magnetization at 1.85 K.

ertheless, the maximum of the temperature-dependent susceptibility, $\chi(T)$, is a common feature in the NFFLs, such as Pd,²⁰ YCo₂, and LuCo₂,²¹ as well as TiBe₂,²² although quantitative calculation of $\chi(T)$ still presents a challenge even for the simplest case of Pd.^{23,24} The field suppression of the magnetization (and T_{max}) at low temperature is not attributable to the possible existence of a paramagnetic impurity contribution (which would contribute more to the value of M/H at lower temperature and lower field and therefore suppress the maximum of M/H in lower field) but, as discussed below, to the intrinsic variation in $\chi = dM/dH$ with respect to H at different temperatures.

Figure 3(b) shows that above a characteristic temperature $(T^* \sim 50 \text{ K})$, the susceptibility of YFe₂Zn₂₀ and LuFe₂Zn₂₀ can be approximately fitted by a Curie-Weiss (CW) term $[\chi(T) = C/(T - \theta_{\rm C})]$ plus a temperature-independent term (χ_0) . The values of effective moment (μ_{eff}) , θ_C , and χ_0 are extracted as $1.0\mu_B/\text{Fe}$, -16 K, 3.8×10^{-4} emu/mol and $1.1\mu_B/\text{Fe}$, -33 K, 3.4×10^{-4} emu/mol for YFe₂Zn₂₀ and LuFe₂Zn₂₀, respectively. These values of μ_{eff} are significantly larger than the estimated induced moment of Fe site in the FM ground state of $GdFe_2Zn_{20}$, $\sim 0.35 \mu_B/Fe^{.10}$ Such apparent CW-like behavior has also been observed in other NFFL systems.^{21,25} In the context of the spin fluctuation model,³ itinerant electronic systems can manifest CW-like behavior with a Curie constant related to the local amplitude of the spin fluctuation. The magnetization data at the base temperature (1.85 K) show nearly linear dependence with the applied field [inset of Fig. 3(b)], which is distinct from the Brillouin function type of magnetization curves associated with local moments.

In order to better understand the variation in the maximum in temperature-dependent M/H data for YFe₂Zn₂₀ and



FIG. 4. (Color online) M/H for LuFe₂Zn₂₀. From right to left: H=2, 5, 10, 20, and 30 kOe. Inset: $\chi(H)$ for LuFe₂Zn₂₀ at varied temperature.

LuFe₂Zn₂₀, M(T) and M(H) measurements were performed on LuFe₂Zn₂₀ for various applied fields and temperatures, respectively. Figure 4 shows that the magnetic field suppresses the values of M/H, as well as the maximum of M/H, which disappears when $H \ge 20$ kOe. The values of the fielddependent susceptibility, $\chi(H)$, were extracted as $\chi(H) = \frac{M(H+\Delta H)-M(H)}{\Delta H}$ with $\Delta H = 1000$ Oe from the M(H) data at varied temperature (inset of Fig. 4). For $T \ge 10$ K, the values of $\chi(H)$ monotonically decrease with increasing H, whereas a local maximum appears around 20 kOe in the data sets as $T \le 7$ K. This critical temperature (~7 K) is close to the T_{max} ; the maximum of $\chi(H)(H=20 \text{ kOe})$ is also close to the suppression field determined by Fig. 4. This curious, fielddependent, susceptibility at varied temperature is reminiscent to the one of TiBe2, albeit the amplitude of local maximum in $\chi(H)$ is much smaller.^{22,26} In the case of TiBe₂, the reason of anomalous field-dependent magnetization is also still not clear.27

Figure 5 presents the low temperature specific heat data for YFe₂Zn₂₀ and LuFe₂Zn₂₀, as well as for the Co analogs, plotted as C_p/T versus T^2 . All four compounds manifest clear Fermi liquid behavior ($C_p = \gamma T + \beta T^3$). The similar β values (represented as the slopes of the data sets in the plot,



FIG. 5. (Color online) Low temperature specific heat data of YFe_2Zn_{20} and $LuFe_2Zn_{20}$ (plotted as C_p/T versus T^2), as well as the Co analogs.



FIG. 6. (Color online) (a) Temperature-dependent resistivity of YFe_2Zn_{20} and $LuFe_2Zn_{20}$, as well as their Co analogs. Inset: ρ versus T^2 . The solid lines present the linear fit of the data sets from 2 to 9 K. (b) Estimated spin fluctuation contribution to the resistivity for YFe_2Zn_{20} and $LuFe_2Zn_{20}$. The error bars were estimated as $\pm 10\%$ of the values of the resistivity for YCo_2Zn_{20} and $LuCo_2Zn_{20}$, respectively.

~1.2 mJ/mol K⁴) indicate similar Debye temperatures for these four compounds [~340 K (Ref. 13)], consistent with their similar molar masses, composition, and lattice parameters. On the other hand, the larger values (over 2.5 times much) of electronic specific heat (γ) of the Fe members indicate a larger density of states at the Fermi level [$N(E_F)$] compared to the Co analogs (consistent with the band structure calculation results¹³).

The values of the electronic specific heat and magnetic susceptibility can be employed to estimate the Stoner enhancement factor, *Z*, in the context of the Stoner theory: the static susceptibility is enhanced by $\frac{1}{1-Z}$, whereas the electronic specific heat is not.¹⁰ The estimated *Z* values of YFe₂Zn₂₀ and LuFe₂Zn₂₀ are 0.88 and 0.89, respectively, comparable with the estimated values of the canonical NFFL systems: Pd: 0.83 and YCo₂: 0.75.²⁸

The temperature-dependent electrical resistivity data for YFe_2Zn_{20} and $LuFe_2Zn_{20}$ are larger than that for the Co analogs over the whole temperature range [Fig. 6(a)]. This is not unexpected for a NFFL since the spin fluctuations will affect the scattering process of the conduction electrons, which leads to an additional contribution to the resistivity. In order to study the spin fluctuation contribution to the resistivity, the total electrical resistivity $\rho(T)$ is assumed to be

$$\rho(T) = \rho_0 + \rho_{ph}(T) + \rho_{sf}(T), \tag{1}$$

where the first, second, and third terms represent residual, phonon, and spin fluctuation resistivity, respectively. Assuming the phonon scattering contribution, $\rho_{ph}(T)$, is essentially the same for the Fe and Co analogs (as suggested by the similar β terms), then, the spin fluctuation scattering contribution, $\rho_{sf}(T)$, can be estimated as



FIG. 7. (Color online) Temperature-dependent magnetization of RCo_2Zn_{20} (R=Nd and Sm) compounds divided by applied field $H = 10\ 000$ Oe. Inset: applied field ($H=10\ 000$ Oe) divided by the magnetizations of RCo_2Zn_{20} (R=Nd and Sm) as a function of temperature.

$$\rho_{sf}(T) = (\rho - \rho_0)_{Y/LuFe_2Zn_{20}} - (\rho - \rho_0)_{Y/LuCo_2Zn_{20}}.$$
 (2)

Shown in Fig. 6(b), $\rho_{sf}(T)$ for these two compounds increases with temperature and is close to a saturated value (10 $\mu\Omega$ cm) at 300 K, within the accuracy of the measurements.

Analysis of the low temperature resistivity data reveals a quadratic standard Fermi liquid behavior $[\rho(T)=\rho_0+AT^2]$ for all four compounds (inset of Fig. 6). The *A* values of the Fe compounds are about seven times larger than the two Co analogs. This result is consistent with the 2.5 times larger γ values of the Fe compounds in the context of the Fermi liquid theory, meaning *A* is proportional to the square of the effective mass of the quasiparticles due to the strong correlation effect, whereas γ is proportional to the effective mass. In the point of view of spin fluctuation theory, nearly FM metals manifest Fermi liquid behaviors at low temperatures with *A* values enhanced by spin fluctuations.³

B. RCo_2Zn_{20} (R=Nd,Sm,Gd-Tm)

The results of thermodynamic measurement on NdCo₂Zn₂₀ and SmCo₂Zn₂₀ are briefly examined before discussing the heavy rare-earth compounds (R=Gd-Yb). Figure 7 shows the low temperature-dependent magnetization data (divided by the applied field H=1000 Oe) for NdCo₂Zn₂₀ and SmCo₂Zn₂₀. Neither of them displays any sign of magnetic ordering above 2 K. The temperature-dependent H/Mfor NdCo₂Zn₂₀ shows a CW behavior $[\chi(T) = C/(T - \theta_{\rm C})]$ $+\chi_0$] with $\mu_{eff}=3.7\mu_B$, $\theta_C=-2.3$ K, and $\chi_0=6.8$ $\times 10^{-4}$ emu/mol. The value of the effective moment is close to the theoretical value for Hund's rule ground state of the 4f electrons of Nd^{3+} ion $(3.6\mu_B)$. On the other hand, though the magnetization of SmCo₂Zn₂₀ drops with increasing temperature, it does so in a distinctly non-CW manner. This behavior is not unexpected in Sm containing compounds²⁹ and is most likely due to the thermal population of the first excited Hund's rule multiplet.

Specific heat data for $NdCo_2Zn_{20}$ and $SmCo_2Zn_{20}$ are shown in Fig. 8 along with data for YCo_2Zn_{20} for comparison. The low temperature upturn in the $NdCo_2Zn_{20}$ data below 2 K may be due to a lower temperature magnetic order-



FIG. 8. (Color online) Temperature-dependent specific heat for RCo_2Zn_{20} (R=Nd, Sm, and Y).

ing or a Schottky anomaly due to the CEF splitting. The specific heat data for SmCo₂Zn₂₀ manifest a broad peak around 4 K, which is most likely due to the CEF splitting of Hund's rule ground state of Sm³⁺. Both NdCo₂Zn₂₀ and SmCo₂Zn₂₀ data increase much faster above 10 K and remain more than 10 J/mol K larger above 25 K (not shown here) compared with the data for the nonmagnetic analog YCo₂Zn₂₀. On the other hand, the calculated results of the CEF splitting for Hund's rule ground state of Nd³⁺ ion in a point charge model show the splitting energy levels within 25 K (see Table II below). This large difference indicates that, at this point, the magnetic part of C_p for NdCo₂Zn₂₀ and $SmCo_2Zn_{20}$ cannot be well estimated since the C_p data of YCo₂Zn₂₀ may not an adequate approximation of the nonmagnetic background and unfortunately the LaCo₂Zn₂₀ analog does not form.

Temperature-dependent magnetization data (divided by the applied field H=1000 Oe) for RCo_2Zn_{20} (R=Gd-Yb) are presented in Fig. 9. In addition to the previously reported AFM-ordered GdCo₂Zn₂₀ with Neél temperature T_N = 5.7 ± 0.1 K, ^{10,13} TbCo₂Zn₂₀ also shows AFM ordering with $T_N=2.5\pm0.1$ K, which also clearly manifests itself in the specific heat data (shown below in Fig. 11). The rest of the members (R=Dy-Yb) do not show magnetic ordering above 2 K. Due to the relatively low $N(E_F)$ for the Y and Lu analogs^{10,13} and large R-R separation, such low temperature magnetic ordering for the 4*f* local moments coupled via the



FIG. 9. (Color online) Temperature-dependent magnetization of RCo_2Zn_{20} (R=Gd-Yb) compounds divided by applied field H = 1000 Oe.



FIG. 10. (Color online) Applied field ($H=10\ 000$ Oe) divided by the magnetizations of RCo_2Zn_{20} (R=Gd-Yb) as a function of temperature.

Ruderman-Kitter-Kasaya-Yosida (RKKY) interaction is not unexpected.

Figure 10 shows the temperature-dependent H/M for R =Gd-Tm and Yb members of the RCo_2Zn_{20} series. All the members, including YbCo_2Zn₂₀, manifest CW behavior $[\chi(T)=C/(T-\theta_C)+\chi_0]$ with negligibly small χ_0 ($\leq 2 \times 10^{-3}$ emu/mol) and the values of μ_{eff} close to the theoretical values for Hund's ground state of the 4*f* electronic configurations. All the values of θ_C are close to 0, consistent with the low magnetic ordering temperatures (Table I).

The specific heat data for RCo_2Zn_{20} (R=Gd-Tm, Y, and Lu), as well as the pseudoternary compound Tb_{0.5}Y_{0.5}Co₂Zn₂₀, are presented in Fig. 11. In addition to the previously studied GdCo₂Zn₂₀, the specific heat data for TbCo₂Zn₂₀ manifest a λ type of anomaly with a peak position at 2.5 K, the AFM ordering temperature. In addition to this peak, the C_p data also show a broad shoulder above 2.5 K, which is due to the CEF splitting above the magnetic ordering temperature. This anomaly, associated with CEF splitting of the 4f electronic configuration of Tb³⁺, displays itself more clearly in the C_p data for $Tb_{0.5}Y_{0.5}Co_2Zn_{20}$: when $T_{\rm N}$ is suppressed to well below 2 K, the C_p data show a Schottky anomaly with a peak position ~ 3 K. The magnetic contribution to the entropy for TbCo₂Zn₂₀ is shown in the inset of Fig. 11. Approximately 50% of the total magnetic entropy is recovered by T_N and by 15 K the full S_M $=R \ln 13$ is recovered (where R is the gas constant). This is consistent with a very small total CEF splitting in these compounds, associated with the highly symmetric environment of the R ions. For the rest of the members, R=Dy-Tm, the C_p data show broad Schottky-type anomalies below 10 K, as shown in Fig. 13 below. The low temperature upturn for



FIG. 11. (Color online) Temperature-dependent specific heat for RCo_2Zn_{20} (R=Gd-Tm, Y, and Lu), as well as $Tb_{0.5}Y_{0.5}Co_2Zn_{20}$. Inset: temperature-dependent magnetic entropy for $TbCo_2Zn_{20}$. The dashed line presents the entropy of the full Hund's ground state of Tb^{+3} .

 $DyCo_2Zn_{20}$ data below 2 K may indicate a magnetic ordering at lower temperature, whereas the upturn for $TmCo_2Zn_{20}$ data below 0.7 K may be associated with a magnetic ordering at very low temperature and/or a nuclear Schottky anomaly.

The magnetic part of the entropy above 2 K (above 0.4 K for the $\text{TmCo}_2\text{Zn}_{20}$) is shown in Fig. 12. For *R*=Dy-Tm, there is an obvious deficit of magnetic entropy compared with the value associated with a fully degenerated Hund's ground state, which indicates unaccounted entropy below 2 K (0.4 K for $\text{TmCo}_2\text{Zn}_{20}$) associated with low lying CEF levels and magnetic ordering.

In order to better understand the magnetic properties for R=Tb-Tm members, the CEF effect acting on the R ions is evaluated by thermodynamic measurements. The single-ion Hamiltonian for the R^{3+} is assumed to be the sum of the CEF term, an exchange interaction term, and an external field term,

$$\mathcal{H} = \mathcal{H}_{\text{CEF}} + \mathcal{H}_{exc} + \mathcal{H}_{ext}, \tag{3}$$

where $\mathcal{H}_{ext} = g_J \mu_B \vec{J} \cdot \vec{H}$ (g_J is the Landé factor, \vec{J} is the total angular momentum, and \vec{H} is the external magnetic field).

Since the rare-earth ions are located in a cubic point symmetry, the CEF term, \mathcal{H}_{CEF} , can be written as

$$\mathcal{H}_{\rm CEF} = B_4^0 (O_4^0 + 5O_4^4) + B_6^0 (O_6^0 - 21O_6^4), \tag{4}$$

where O_l^m operators are the well-known Stevens operators³⁰ and B_4^0 and B_6^0 are CEF parameters.³¹ This expression can be written as

TABLE I. Paramagnetic Curie temperature $\theta_{\rm C}$ (with ±0.1 K uncertainty), effective moment μ_{eff} [from the CW fit of $\chi(T)$ from 50 to 300 K], and Neél temperature $T_{\rm N}$ for $R\rm Co_2Zn_{20}$ compounds ($R = \rm Nd, Gd-Yb$).

	Nd	Gd	Tb	Dy	Но	Er	Tm	Yb
$\theta_{\rm C}$ (K)	-2.3	3.3	-2.6	-3.7	1.4	-2.1	-0.9	-5.2
$\mu_{eff} \left(\mu_B \right)$	3.7	8.1	9.8	10.9	10.7	9.7	7.4	4.5
$T_{\rm N}$ (K)		5.7	2.5					



FIG. 12. (Color online) Normalized magnetic part of entropy for RCo_2Zn_{20} (R=Dy-Tm) as well as for $Tb_{0.5}Y_{0.5}Co_2Zn_{20}$ (in units of per mole R^{3+}). The error bars were estimated from the $\pm 1\%$ of the total entropy.

$$\mathcal{H}_{\text{CEF}} = W \left[\frac{x}{F4} (O_4^0 + 5O_4^4) + \frac{1 - |x|}{F6} (O_6^0 - 21O_6^4) \right], \quad (5)$$

where F4 and F6 are factors introduced by Lea *et al.*³¹ and are dependent on J, W is the energy scale, and x represents the relative importance of the fourth and sixth order terms.

Noticing that the possible magnetic ordering temperatures are below 2 K for RCo_2Zn_{20} (R=Dy-Tm), as well as for Tb_{0.5}Y_{0.5}Co₂Zn₂₀, the exchange interaction term can be approximated as zero, which will be better for R=Tm than for R=Dy. Thus, the CEF parameters for different *R* ions were determined by fitting the magnetization at 2 K and the temperature-dependent specific heat data.

Figure 13 shows data and CEF fitting results of the magnetic part of specific heat with the single-ion Hamiltonian (ignoring the interaction term) for Tb_{0.5}Y_{0.5}Co₂Zn₂₀ and RCo₂Zn₂₀ (R=Dy-Tm). The specific heat data for all members are less than that of YCo₂Zn₂₀ above 30 K, which is likely due to the errors associated with resolving the difference between the sample's total C_p and the relatively large nonmagnetic contribution. Therefore, the fittings of C_M were performed below 20 K. As shown in Table II, the inferred W and x values for all five compounds are clustered in a narrow range: |W| < 0.1, |x| < 0.25. This result, indicating small energy scales of the CEF effect and relatively large B_6^0 terms, is



FIG. 13. (Color online) Magnetic part of specific heat for RCo_2Zn_{20} . The solid and dashed line present the experimental and calculated results, respectively.

roughly consistent with the calculated results based on the point charge model (see Appendix B). Furthermore, it should be noted that the signs of the B_6^0 terms for the calculated results are all consistent with the experimental ones; this is not the case for the B_4^0 terms. This behavior is not difficult to understand: as shown in Appendix B, the contributions to the CEF splitting are mainly from the CN-16 Frank-Kasper polyhedron formed by 4 NN and 12 NNN Zn neighbors. For the B_4^0 term, the contributions of the two sets of neighbors cancel each other, whereas the contribution for the B_6^0 terms is the sum. Therefore, the net B_6^0 term is relatively large and the calculated results are more reliable.

C. RFe_2Zn_{20} (R = Gd-Tm)

Before discussing each of the compounds in this series separately, an overview of their temperature-dependent magnetization data serves as a useful point of orientation. Figure 14 shows M/H versus T (the applied field H=1000 Oe) for R=Gd-Tm members. In contrast to the Co series compounds, the Fe series compounds all manifest FM ground states with

TABLE II. Comparison of the CEF parameters of RCo_2Zn_{20} compounds (R=Nd, Tb-Yb) determined from magnetization measurements to those calculated in a point charge model.

		Nd	Tb	Dy	Но	Er	Tm	Yb
Easy axis at 2 K	Expt.		[110]	[111]	[110]	[111]	[110]	[100]
W (K)	Expt.		0.084	-0.073	0.067	-0.077	0.07	
	Calc.	0.28	0.026	-0.021	0.018	-0.025	0.044	-0.28
x	Expt.		0.2	0.1	0.22	-0.1	-0.15	
	Calc.	0.26	-0.68	-0.41	0.23	-0.22	-0.41	-0.64
$B_4^0 (10^{-4} \text{ K})$	Expt.		2.8	-1.2	2.5	1.3	1.75	
	Calc.	12.2	-3.0	1.4	0.7	-0.9	-3.0	29.6
$B_6^0 (10^{-4} \text{ K})$	Expt.		8.9	-4.7	3.8	-5	7.9	
	Calc.	81.2	1.1	-0.9	1.0	-1.4	3.5	-81.4



FIG. 14. (Color online) Temperature-dependent magnetization of RFe_2Zn_{20} (R=Gd-Tm) divided by applied field H=1000 Oe.

enhanced $T_{\rm C}$ values, which systematically decrease as *R* varies from Gd to Tm. Such enhanced FM ordering has been explained as the result of local moments embedded in the NFFL host, most clearly seen in YFe₂Zn₂₀ and LuFe₂Zn₂₀.¹⁰ This systematic variation in $T_{\rm C}$ on *R* is not unexpected for such heavy rare-earth compounds when the magnetic interaction between the *R* ions is associated with the spin part of Hund's ground state of 4f electrons.

The temperature-dependent H/M data, approximately equaling inverse susceptibilities $[1/\chi(T)]$ in the paramagnetic state, for R=Gd-Tm, as well as for YbFe₂Zn₂₀, are shown in Fig. 15. Similar to GdFe₂Zn₂₀,¹⁰ the $1/\chi(T)$ data sets for R=Tb-Tm follow the CW law $[\chi(T)=C/(T-\theta_C)$ $+\chi_0]$ at high temperatures and deviate from the law when approaching their magnetic ordering. The effective moments (μ_{eff}) and the paramagnetic Curie temperatures (θ_C) for these six compounds are listed in Table III. All μ_{eff} values are close to the theoretical value for Hund's ground state of the trivalent 4*f* electronic configuration.

1. TbFe₂Zn₂₀

Temperature-dependent specific heat and resistivity data sets for TbFe₂Zn₂₀ are shown in Fig. 16. The magnetic phase transition manifests as a faint feature in C_p data, indicating $T_C=56\pm3$ K. This is clearer in the magnetic specific heat (C_M) data (Fig. 16, left inset) where T_C is taken as the position of the greatest upward slope. As shown in the right inset to Fig. 16(b), at T_C the magnetic entropy is close to the value for the full degeneracy of Hund's ground state of Tb³⁺,



FIG. 15. (Color online) Applied field (H=1000 Oe) divided by the magnetizations of RFe_2Zn_{20} (R=Gd-Tm) as a function of temperature.

R ln 13. For the rest of the local moment members (*R* = Dy-Er), the released magnetic entropy at $T_{\rm C}$ for the respective rare-earth ion is close to the full degeneracy value for their Hund's ground state. (There is significantly less for *R* = Tm.) The $\rho(T)$ data manifest as a change in slope, which can be seen even more clearly in the $d\rho/dT$ data, consistent with a $T_{\rm C}$ =56±1 K.

The inset of Fig. 17 presents a plot of M^2 versus H/M (an Arrott plot) isotherms near $T_{\rm C}$. The isotherm that most closely goes though the origin is the one closest to $T_{\rm C}$, giving a value of 58 K in this case, consistent with the results of the C_p and $\rho(T)$ measurements. Figure 17 shows magnetization versus external field data at 2 K along three different crystallographic directions: [100], [110], and [111]. All of these data sets are consistent with a low temperature FM ground state with moderate anisotropy. The spontaneous longitudinal magnetic moments in zero applied external field, estimated by the extrapolation of the magnetization curves to H=0, yield $M([110]) = 8.0\mu_B$, $M([111]) = 6.6\mu_B$, and M([100])=5.7 μ_B . Their ratio is very close to 1: $\sqrt{2}/3$: $\sqrt{1}/2$. Such behavior indicates that the spontaneous magnetic moments along [111] and [100] directions can be understood as the projection of the one along the easy axis ([110]). At 2 K and 55 kOe, the saturated moment along the easy axis is $8.1\mu_B$, $0.9\mu_B$ less than the value associated with Hund's ground state.

2. DyFe₂Zn₂₀

The specific heat data of $DyFe_2Zn_{20}$ show a kink associated with magnetic ordering [Fig. 18(a)], which can be seen

TABLE III. Residual resistivity ratio, RRR=R(300 K)/R(2 K), paramagnetic Curie temperature $\theta_{\rm C}$ (with $\pm 0.5 \text{ K}$ errors), effective moment μ_{eff} [from the CW fit of $\chi(T)$ from 100 to 300 K, except for GdFe₂Zn₂₀, which was fitted from 200 to 375 K (Ref. 13)], Curie temperature $T_{\rm C}$, and saturated moment at 55 kOe along the easy direction μ_{sat} for RFe_2Zn_{20} compounds (R=Gd-Yb).

	Gd	Tb	Dy	Но	Er	Tm	Yb
RRR	8.1	7.2	15.0	10.0	13.2	10.1	31.2
$\theta_{\rm C}~({\rm K})$	46	30	20	9	0	-2	-23
$\mu_{eff} \left(\mu_B \right)$	7.9	9.5	10.5	10.6	9.5	7.7	4.7
$T_{\rm C}$ (K)	86	58	46	28	17	5.5	
μ_{sat} (μ_B)	6.7	8.1	9.5	9.9	8.5	6.2	



FIG. 16. (a) C_{ρ} for TbFe₂Zn₂₀; (b) ρ and $d\rho/dT$. Left inset: magnetic part of C_{ρ} ; right inset: magnetic entropy S_M .

more clearly after the subtraction of the nonmagnetic background (left inset) and indicates $T_{\rm C}=45\pm1$ K. This FM transition temperature is further confirmed by a weak change in slope in $\rho(T)$ (associated with the low temperature loss of spin disorder scattering), indicating $T_{\rm C}=45\pm2$ K [Fig. 18(b)]. Given that the loss of spin disorder scattering in intermetallics often scales with de Gennes parameter,³² the feature we find in $\rho(T)$ below $T_{\rm C}$ becomes fainter as R progresses from Gd to Tm. The value of $T_{\rm C}$ is consistent with the result of the Arrott plot analysis, from which a value of $T_{\rm C}=45\pm1$ K can be inferred (inset of Fig. 19).

It is worth noting that the specific heat data show a faint shoulder near 10 K, which appears to be a broad peak after the background subtraction and is coincident with a slope change feature in $\rho(T)$ data. As seen below, such an anomaly below $T_{\rm C}$ in C_p and $\rho(T)$ data also appears for the R=Ho, Er, and Tm members. Those anomalies are likely due to the



FIG. 17. (Color online) Field-dependent magnetization of TbFe₂Zn₂₀ along the three principle axes at 2 K. The three lines represent the calculated results based on molecular field approximation (all being clustered near $9\mu_B$ and appearing as a single line). Inset: Arrott plot of magnetic isotherms for TbFe₂Zn₂₀.



FIG. 18. (a) C_p for DyFe₂Zn₂₀; (b) ρ and $d\rho/dT$. Left inset: magnetic part of C_p ; right inset: magnetic entropy S_M .

magnetic excitation energy spectrum associated with Hund's rule multiplet of R^{3+} ions in their FM states. (Further discussion will be presented below.)

The 2 K field-dependent magnetization isotherms for $DyFe_2Zn_{20}$ are shown in Fig. 19. Compared to $TbFe_2Zn_{20}$, the magnetization curves for DyFe₂Zn₂₀ reveal a slightly more complicated anisotropic behavior. The magnetization along the [100] direction manifests a metamagnetic phase transition near 12 kOe. Above this transition, the magnetization along the [100] direction is essentially the same as that for the field along the easy [111] axis. The spontaneous longitudinal magnetization along the three directions, $M([111]) = 9.1\mu_B$, $M([110]) = 7.4\mu_B$, and $M([100]) = 5.3\mu_B$, has a ratio very close to $1:\sqrt{1/2}:\sqrt{1/3}$. These results indicate that M([110]) and M([100]) can be seen as projections of M([111]). The metamagnetic phase transition along [100] can be understood as a classical spin reorientation in a cubic symmetry coordination. As in the case of GdFe₂Zn₂₀ and TbFe₂Zn₂₀, the saturated moment of DyFe₂Zn₂₀ at 55 kOe



FIG. 19. (Color online) Field-dependent magnetization of $DyFe_2Zn_{20}$ at 2 K along the three principle axes. The solid lines represent the calculated result. Inset: Arrott plot of magnetic isotherms for $DyFe_2Zn_{20}$.



FIG. 20. (a) C_{ρ} for HoFe₂Zn₂₀; (b) ρ and $d\rho/dT$. Left inset: magnetic part of C_{ρ} ; right inset: magnetic entropy S_M .

 $(9.5\mu_B)$ is slightly less than the value of Hund's ground state value $(10\mu_B)$.

3. HoFe₂Zn₂₀

The anomalies associated with the FM transition in HoFe₂Zn₂₀ in the specific heat and resistivity data are relatively weak (Fig. 20). The specific heat anomaly can be associated with $T_{\rm C} \sim 28$ K, and the $d\rho/dT$ data show a faint anomaly at this temperature (Fig. 20). The $T_{\rm C}$ value is determined as 28 ± 1 K from C_p data, as well as 29 ± 1 K from $\rho(T)$ data. This $T_{\rm C}$ value is consistent with the result of the Arrott plot analysis (inset of Fig. 21), which gives $T_{\rm C} = 28 \pm 1$ K.

The low temperature magnetic isotherms for $HoFe_2Zn_{20}$ (Fig. 21) display similar but larger anisotropy to the ones for $TbFe_2Zn_{20}$. The ratio of the spontaneous magnetization



FIG. 21. (Color online) Field-dependent magnetization of $HoFe_2Zn_{20}$ at 2 K along the three principle axes. The solid lines represent the calculated result. Inset: Arrott plot of magnetic isotherms for $HoFe_2Zn_{20}$.



FIG. 22. (a) C_p for ErFe₂Zn₂₀; (b) ρ and $d\rho/dT$. Left inset: magnetic part of C_p ; right inset: magnetic entropy S_M .

 $[M([110]):M([111]):M([100])=9.1\mu_B:7.0\mu_B:6.1\mu_B]$ is close to the ratio of $1:\sqrt{2/3}:\sqrt{1/2}$. This ratio is consistent with the projection of the local moment from the easy [110] axis onto the [111] and [100] axes. In an external field of 55 kOe, the magnetization along the easy axis, ([110]), reaches a value of $9.9\mu_B$, very close to the value of Hund's ground state $(10\mu_B)$.

4. ErFe₂Zn₂₀

The specific heat and transport properties of $\text{ErFe}_2\text{Zn}_{20}$ are shown in Fig. 22. The specific heat data show a kink near 18 K [Fig. 22(a)], which can be seen more clearly after the background subtraction (left inset) and indicates $T_{\rm C}$ =18±1 K. The resistivity data show no clear anomaly at this temperature (Fig. 22). The released magnetic entropy reaches 21 J/mol K at $T_{\rm C}$, 90% of that associated with Hund's ground state of Er^{3+} , $R \ln 16$ (Fig. 22, right inset). Although $\rho(T)$ data show no anomaly at $T_{\rm C}$, we will see that the weak anomaly associated with magnetic ordering can be blown up after the background [$\rho(T)$ for LuFe₂Zn₂₀] is subtracted. The Arrott plot for $\text{ErFe}_2\text{Zn}_{20}$ (inset of Fig. 23), although showing nonlinear isothermal curves, demonstrates $T_{\rm C}$ =17±0.5 K with little ambiguity. The nonlinear feature is not unexpected for the 4*f* local moment systems associated with the CEF induced anisotropy.³³

The magnetic anisotropy of $\text{ErFe}_2\text{Zn}_{20}$ is reminiscent of that of $\text{DyFe}_2\text{Zn}_{20}$: both have the same easy and hard magnetization orientations, [111] and [110], respectively, as well as a metamagnetic transition along the [100] direction (Fig. 23). The ratio of the spontaneous longitudinal magnetic moments [M([111]):M([110]):M([100]))=7.4 μ_B :5.9 μ_B :4.2 μ_B] is also close to the ratio of 1: $\sqrt{2}/3$: $\sqrt{1}/3$.

5. TmFe₂Zn₂₀

The specific heat data for $\text{TmFe}_2\text{Zn}_{20}$ have only one broad peak at 4.5 K [Fig. 24(a)], which is less like the anomalies



FIG. 23. (Color online) Field-dependent magnetization of $ErFe_2Zn_{20}$ at 2 K along the three principle axes. The solid lines represent the calculated result. Inset: Arrott plot of magnetic isotherms for $ErFe_2Zn_{20}$.

associated with $T_{\rm C}$ for R=Gd-Er and more like a Schottky anomaly associated with a CEF splitting. The resistivity data also show an anomaly below 5 K [Fig. 24(b)], but at this point it is difficult to determine whether this anomaly is associated with the magnetic ordering or the CEF splitting of the 4*f* electrons of the Tm³⁺ ions. As we can see below, after the subtraction of the nonmagnetic background, the anomaly associated with the loss of the spin disorder scattering can be seen more clearly.

For TmFe₂Zn₂₀, the Arrott plot analysis provides the most reliable criterion for $T_{\rm C}$ determination. The inset of Fig. 25 shows that $T_{\rm C}$ can be determined as 5.5 ± 0.5 K without any ambiguity. At this temperature, the magnetic entropy is 15 J/mol K, only 70% of the value of the fully released entropy of Hund's ground state of Tm³⁺, $R \ln 13$ (Fig. 24, right inset).

The low temperature magnetic isotherms for $TmFe_2Zn_{20}$ manifest the same easy and hard axes as Tb and Ho members



FIG. 24. (a) C_p for TmFe₂Zn₂₀; (b) ρ and $d\rho/dT$. Left inset: magnetic part of C_p ; right inset: magnetic entropy S_M .



FIG. 25. (Color online) Field-dependent magnetization of $TmFe_2Zn_{20}$ at 2 K. The solid lines represent the calculated result. Inset: Arrott plot of magnetic isotherms for $TmFe_2Zn_{20}$.

([110] and [111], respectively) (Fig. 25). The spontaneous longitudinal magnetic moments along the three principle axes are all close to $4\mu_B$. This result may be due to the relatively low value of T_C , which makes the spontaneous magnetic moment less anisotropic at 2 K. The saturated moment along the easy axis reaches $6.2\mu_B$ at 55 kOe, $0.8\mu_B$ less than the value of Hund's ground state, $7\mu_B$.

D. YbFe₂Zn₂₀ and YbCo₂Zn₂₀

The physical properties of YbFe₂Zn₂₀ and YbCo₂Zn₂₀ have been discussed in Ref. 11. They manifest typical heavy Fermion behavior with large electronic specific heat, 520 and 7900 mJ/mol K², respectively. Figure 26 shows temperature-dependent susceptibility and resistivity data for these two Yb compounds. The susceptibility data for YbFe₂Zn₂₀ manifest a broad Kondo-type peak about 20 K, indicating a clear loss of local moment behavior, whereas the susceptibility for YbCo₂Zn₂₀ shows CW behavior down to 1.8 K (Fig. 10), associated with the effective moment value μ_{eff} =4.5 μ_B . Above ~50 K, $\chi(T)$ for YbFe₂Zn₂₀ displays a



FIG. 26. (Color online) (a) Temperature-dependent M/H and (b) resistivity for YbFe₂Zn₂₀ and YbCo₂Zn₂₀ (H=1000 Oe).



FIG. 27. (Color online) (a) $T_{\rm C}$ and $\theta_{\rm C}$, the maximum value on $\Delta \rho$ (b) with respect to the de Gennes factor for $R {\rm Fe}_2 {\rm Zn}_{20}$ ($R = {\rm Gd-Tm}$).

CW behavior with an effective moment of $4.7\mu_B$, close to the value of Hund's ground state of Yb³⁺, $4.5\mu_B$ (see Fig. 15). The resistivity data for YbFe₂Zn₂₀ show a broad shoulder about 30 K, whereas for YbCo₂Zn₂₀, the resistivity data show a Kondo resistance minimum about 50 K and a clear coherent peak around 2 K. These apparently different behaviors for these two Yb-based heavy fermion compounds with virtually identical structures have been explained as the result of significantly different Kondo temperatures: $T_{\rm K}$ =33 and 1.5 K for Fe and Co compounds, respectively.

IV. DATA ANALYSIS AND DISCUSSION

As shown in Fig. 27(a), the $T_{\rm C}$ values of $R {\rm Fe}_2 {\rm Zn}_{20}$ compounds ($R = {\rm Gd-Tm}$) scale fairly well with the de Gennes factor, ${\rm dG} = (g_J - 1)^2 J(J+1)$, consistent with an RKKY interaction. The $\theta_{\rm C}$ values for the compounds are smaller than their respective $T_{\rm C}$ values (for $R = {\rm Er}$ and Tm, the values of $\theta_{\rm C}$ are even negative). These small $\theta_{\rm C}$ values are consistent with the deviation of $\chi(T)$ from the CW law (Fig. 15). As observed in the case of pseudoternary compounds ${\rm Gd}_x {\rm Y}_{1-x} {\rm Fe}_2 {\rm Zn}_{20}$, such deviation can be explained as a result of increasing coupling between the local moments embedded in the strongly temperature-dependent, polarizable matrix, ${\rm YFe}_2 {\rm Zn}_{20}$ or LuFe₂Zn₂₀.¹⁰

Previous studies show that the magnetization of $GdFe_2Zn_{20}$ at base temperature is nearly isotropic, with a slightly reduced saturated moment (~0.5 μ_B less than the value of Hund's rule ground state of Gd^{3+}). For R=Tb-Tm, the magnetization anisotropy at base temperature is significant and correlates with the easy and hard axes of the respective RCo_2Zn_{20} analogs. This behavior indicates that the anisotropy of the RFe_2Zn_{20} (R=Tb-Tm) compounds may mainly be due to the CEF effect on the R^{3+} ions. The M(H) curves at 2 K manifest divided behavior with non-Kramer ion (R=Tb, Ho, and Tm) on one hand and Kramer ion (R =Dy and Er) on the other: for R=Tb, Ho, and Tm, the magnetization processes are gradual along all three principal

axes; for R=Dy and Er, the magnetization data along the [100] direction shows a clear and sharp metamagnetic transition. Both types of magnetization processes (gradual increase and metamagnetic transition) are common for the FMordered 4*f* local moments with CEF anisotropy associated with the *R* in a cubic point symmetry and can be understood in terms of the purification of the CEF split 4*f* electronic wave function due to the Zeeman effect of the external field and the rotation of the local moment.³⁴ Given that Tb³⁺ and Tm³⁺, as well as Dy³⁺ and Er³⁺ ions have same total 4*f* electronic Hund's rule ground state quantum number (J=6 and 15/2, respectively), the similar magnetic anisotropy indicates similar CEF effect for the two sets of rare-earth ions, respectively.

In order to better understand the magnetic anisotropy of RFe_2Zn_{20} compounds (R=Tb-Tm), the CEF effect acting on the R ions must be considered. However, multiple difficulties associated with the strongly polarizable background [Y(Lu)Fe₂Zn₂₀], as well as the strong magnetic interaction, make the determination of the CEF parameters hard. For example, in order to reduce the magnetic interaction, the magnetic R^{3+} ions were placed into a dilute coordination, $R_xY_{1-x}Fe_2Zn_{20}$ or $R_xLu_{1-x}Fe_2Zn_{20}$. A FM ground state has been found even for very dilute magnetic Rconcentration: Tb_{0.05}Y_{0.95}Fe₂Zn₂₀, Dy_{0.05}Y_{0.95}Fe₂Zn₂₀, and Ho_{0.1}Y_{0.9}Fe₂Zn₂₀ manifest FM ordering above 2 K. For such small x, the background subtraction (magnetization and/or specific heat of YFe₂Zn₂₀ or LuFe₂Zn₂₀), as well as the uncertainty of x, makes the fitting process unreliable.

On the other hand, due to the very similar *R* coordination and the lattice parameters for Fe and Co series, the CEF parameters determined from RCo_2Zn_{20} compounds should be close to those for the RFe_2Zn_{20} compounds for respective *R* members. This assumption is consistent with the magnetic measurements for the pseudoternary Fe compounds, $Er_{0.1}Y_{0.9}Fe_2Zn_{20}$ and $Tm_{0.1}Y_{0.9}Fe_2Zn_{20}$. Manifesting a paramagnetic state at 2 K, their anisotropic magnetization isotherms show the same easy and hard axes as the calculated results from the determined CEF parameters of the Co compounds.

The magnetizations along the three axes for all Fe compounds were calculated based on the molecular field approximation in a self-consistent manner. In the single-ion Hamiltonian for the R^{3+} ions [Eq. (3)], with the molecular field approximation, the magnetic interaction term is written as

$$\mathcal{H}_{exc} = g_J \mu_B \tilde{J} \cdot \tilde{H}_M,\tag{6}$$

where H_M is the molecular field. It obeys the self-consistent condition

$$H_M = \lambda g \,\mu_B \langle \vec{J} \rangle, \tag{7}$$

$$\langle \vec{J} \rangle = \frac{\sum_{0} J_n \exp(-E_n/k_B T)}{\sum_{0} \exp(-E_n/k_B T)},$$
(8)

where J_n and E_n are the eigenvalues and eigenenergies of the *n*th eigenfunction and λ is the molecular field constant



FIG. 28. (Color online) Magnetic part of specific heat versus $T/T_{\rm C}$ for $R{\rm Fe_2Zn_{20}}$ ($R={\rm Gd-Tm}$).

which can be obtained from the ordering temperature: $\lambda = 3k_B T_C / \mu_{eff}^2$.

The calculated magnetizations were compared with the experimental results as shown in Figs. 17, 19, 21, 23, and 25. All these calculated magnetization values are obviously larger than the experimental results. This difference is probably due to (i) the molecular field approximation overestimating the molecular field constant as well as the internal field and (ii) the induced moments from the Fe site aligning in an antiparallel manner with respect to the R^{3+} local moments [as in the case of GdFe₂Zn₂₀ (Ref. 10)].

Figure 28 shows the magnetic part of specific heat as a function of $T/T_{\rm C}$ for $R \text{Fe}_2 \text{Zn}_{20}$ (R = Gd-Tm). The magnetic ordering temperature (T_c) of R=Gd-Er members manifests itself as the position of maximum slope, with a decreasing sharpness as R varies from Gd to Er. TmFe₂Zn₂₀ does not appear to have any sharp anomaly in the C_M data at T_C . Below $T_{\rm C}$, the data sets for R=Dy-Tm show a broad peak, which shifts closer to its $T_{\rm C}$ as R varies from Dy to Tm, whereas the data for GdFe₂Zn₂₀ show no explicit peak. If the broad peaks correspond to the magnetic excitation energy spectrum associated with CEF effect, then the relative positions of these peaks to $T_{\rm C}$ to some extent indicate the ratio of the energy scales of the CEF splitting (for a single ion) to the magnetic interaction. The shift of the peak position as Rvaries from Dy to Tm indicates that the energy scale of the magnetic order decreases relative to the CEF splitting. Such phenomena are consistent with the analysis on the magnetic part of entropy: as shown in the insets of Figs. 16, 18, 20, 22, and 24, Tb, Dy, and Ho compounds manifest fully released S_M at their T_C , whereas Er and Tm compounds still release part of S_M above their T_C , which indicates that unlike R =Gd-Ho members, the CEF splitting for the 4f electronic configuration of the Tm³⁺ and Er³⁺ may extend above magnetic ordering temperature.

Based on the assumption that the Fe and Co series have similar CEF splitting (for similar R ions), the comparison between the magnetic ordering temperature and the CEF splitting for different R ions is qualitatively diagrammatized in Fig. 29. The levels represent the single-ion CEF splitting



FIG. 29. (Color online) Single-ion CEF splitting energy levels for RCo_2Zn_{20} (R=Tb-Tm). The arrows present the T_C values for RFe_2Zn_{20} with respective R.

of Hund's ground state of 4f electronic configuration of R^{3+} , determined from RCo_2Zn_{20} , and the arrows represent the T_C values of RFe_2Zn_{20} . The T_C value is comparable with the highest energy level of CEF splitting for R=Ho. For R=Er and Tm, the T_C values are about $\frac{1}{2}$ and $\frac{1}{5}$ of the highest CEF levels, respectively. This diagram, though it cannot be used to determine the precise energy splitting of the RFe_2Zn_{20} compounds (the CEF levels have been strongly modulated and mixed by the interaction energy), is qualitatively consistent with the specific heat measurements and indicates that at least for TmFe_2Zn_{20}, the CEF energy splitting already happens well above its T_C . In summary, it appears plausible that, due to extremely similar liganal environments, equivalent members of the RFe_2Zn_{20} and RCo_2Zn_{20} series have similar CEF splitting schemes.

Further insight can also be gained from a careful revisiting of the transport data. The total resistivity of RFe_2Zn_{20} (R=Gd-Yb) can be written as

$$\rho(T) = \rho_0 + \rho_{ph}(T) + \rho_{mag}(T),$$
(9)

where ρ_{mag} is associated with scattering from the 4*f* moments and the spin fluctuation of itinerant electrons. As seen in Fig. 30(a), for the whole series above 250 K, the resistivity data sets show essentially linear behavior with slopes differing by less than 12% within the estimated dimension error $(\pm 10\%)$ of these bar-like-shape samples. These similar high temperature behaviors indicate that, in the high temperature limit, the magnetic scattering is saturated and the phonon scattering is essentially similar for the whole series (due to the very dilute nature of the *R* ions). Therefore, the magnetic contribution to the resistivity can be estimated by (1) subtracting residue resistivity, ρ_0 , (2) normalizing the high temperature slope of all $\rho(T)$ to that of LuFe₂Zn₂₀, and then (3) subtracting the $\rho_{Lu}(T) - \rho_{Lu0}$ data from the normalized data. The result is written as

$$\Delta \rho(T) = (\rho_{\rm R} - \rho_{\rm R0}) \frac{\frac{d\rho_{\rm R}}{dT}|_{275 \rm K}}{\frac{d\rho_{\rm Lu}}{dT}|_{275 \rm K}} - (\rho_{\rm Lu} - \rho_{\rm Lu0}).$$
(10)

As shown previously, the subtraction background $\rho_{Lu}(T)$ already includes the scattering associated with the spin fluctuation of itinerant electrons. Thus $\Delta \rho$ will not only include



FIG. 30. (Color online) (a) ρ versus *T*, (b) $\Delta\rho$ versus *T*, and (c) $\Delta\rho$ versus $T/T_{\rm C}$ for $R{\rm Fe}_2{\rm Zn}_{20}$ (*R*=Gd-Tm). Inset: the blow-up $\Delta\rho$ data for TmFe₂Zn₂₀. The arrow presents the FM ordering temperature.

the scattering from the 4f moments but also include scattering associated with the interaction between the 4f moments and itinerant electrons. Figures 30(b) and 30(c) show $\Delta \rho$ versus temperature, as well as normalized temperature $(T/T_{\rm C})$ for R=Gd-Tm. For R=Gd-Er, a pronounced upward cusp, whose height decreases from Gd to Er, is centered about $T_{\rm C}$, whereas TmFe₂Zn₂₀ manifests a broad feature and only very weak anomaly around its $T_{\rm C}$ (see the blow-up inset of Fig. 30). As shown in Fig. 27(b), the maximum values of the cusps for different R scale with the de Gennes factor, which indicates that the decrease in $\Delta \rho$ with T below T_C is the result of a loss of spin disorder scattering of conduction electrons, associated with the 4f local moment. However, as found in the pseudoternary compounds $Gd_xY_{1-x}Fe_2Zn_{20}$,¹² the decreasing behavior of $\Delta \rho$ with increasing T above $T_{\rm C}$ is more conspicuous and must come from a different conduction electron scattering process [simple models of $\rho(T)$ due to magnetic scattering cannot explain this anomaly^{35,36}]. Given that RT_2Zn_{20} compounds only manifest this behavior when the local moments are embedded in the highly polarizable background (GdCo₂Zn₂₀ does not show this behavior¹⁰), this anomaly is thought to be associated with the spin fluctuation of the 3d electrons. Also appearing in the resistivity of RCo_2 (R=Gd-Tm),³⁷ this decreasing behavior of $\Delta \rho$ with increasing T above $T_{\rm C}$ has been explained as the result of the increase in the spin fluctuation of 3d electrons, which is provided by the increasing nonuniform fluctuating 4f-d electron exchange interaction, as the temperature approaches $T_{\rm C}$ in the paramagnetic state. Since both Y(Lu)Co₂ and Y(Lu)Fe₂Zn₂₀ are classical examples of NFFLs, such an anomaly could be associated with these strongly correlated electron systems. On the other hand, considering that Hund's

ground state of Tm^{3+} has been significantly split above the T_{C} for $\text{TmFe}_2\text{Zn}_{20}$, it is not unexpected that the conduction electron scattering process manifests a different behavior associated with the CEF effect.

Finally, the nearly FM compounds YFe₂Zn₂₀ and LuFe₂Zn₂₀ also merit some further discussion. Shown in Fig. 3, the low field susceptibility (H=10 kOe) manifests a maximum near 6 and 8 K for YFe2Zn20 and LuFe2Zn20, respectively. Such a maximum in the temperature-dependent susceptibility also appears for other examples of nearly FM compounds. For example, Pd manifests $T_{\text{max}} \sim 70 \text{ K}$,²⁰ YCo₂ and LuCo₂ manifests $T_{\text{max}} \sim 100$ K,²¹ and TiBe₂ manifests $T_{\rm max} \sim 10$ K.²² Another interesting phenomena in nearly FM materials are the so-called itinerant electron metamagnetism (IEM), which is an applied magnetic field induced first-order phase transition between a paramagnetic state and spin polarized state.³⁸ Experimentally, IEM has been observed for YCo₂ and LuCo₂ around 70 T.^{39,40} Within the framework of Landau theory, the maximum in temperature-dependent susceptibility is thought to be related to IEM.⁴¹ The magnetic part of the free energy ΔF can be written as the function of the magnetic moment M,

$$\Delta F = \frac{1}{2}aM^2 + \frac{1}{4}bM^4 + \frac{1}{6}cM^6, \tag{11}$$

where a, b, and c are the Landau expansion coefficients.

As shown by Shimizu,⁴¹ the condition for the existence of IEM is a > 0, b < 0, c > 0, and $3/16 < ac/b^2 < 9/20$. Within the framework of the spin fluctuation theory, Yamada⁴² generalized this work by introducing a temperature-dependent function of the mean square amplitude of spin fluctuations. These theoretical works demonstrated that the existence of IEM is associated with the maximum in $\chi(T)$ by means of the factor of ac/b^2 , which can be estimated as

$$\frac{ac}{b^2} = \left[1 - \frac{\chi(0)}{\chi(T_{\text{max}})}\right]^{-1}.$$
 (12)

Furthermore, the IEM can only happen below T_{max} . These results seem to be consistent with the experimental results in various itinerant electronic systems.⁴³

According to Eq. (12), the values of ac/b^2 can be estimated as 310 and 72 for YFe₂Zn₂₀ and LuFe₂Zn₂₀, respectively $[M/H \sim \chi(T) \text{ at 10 kOe}]$, which are much larger than the region of the existence of IEM, indicating that IEM may not exist. Indeed, recent measurements on a part of the LuFe₂Zn₂₀ sample used for the magnetization data in Fig. 3 in a pulse magnetic field up to 55 T at 0.3 K show no evidence of IEM appears for TiBe₂,⁴⁴ which also manifests a relatively low value of T_{max} . From these points of view, Y(Lu)Fe₂Zn₂₀ and TiBe₂ may represent the examples of NF-FLs different from YCo₂ and LuCo₂.

This lack of an IEM sheds further light on the magnetic properties of the local moment bearing RFe_2Zn_{20} (R = Gd-Tm) compounds. As shown before, all the members

manifest second-order paramagnetic to ferromagnetic phase transitions. This behavior is different from that seen in the RCo_2 (R=Gd-Tm) system: the magnetic phase transitions of R=Dy-Tm members for RCo_2 are first order whereas R =Gd and Tb members have second-order transitions.⁵ This difference is not difficult to explain in Landau theory: unlike Y(Lu)Co₂, the host of Y(Lu)Fe₂Zn₂₀ lacks of ability to show IEM and therefore cannot be induced to show metamagnetic transition by any molecular field associated with the 4f local moments.

V. SUMMARY

RFe₂Zn₂₀ and RCo₂Zn₂₀ (R=Gd-Lu, Y) demonstrate diverse magnetic properties. The conspicuous differences between these two related series are mainly associated with the conduction electron polarizability of the host (nonmagnetic) compounds. YFe₂Zn₂₀ and LuFe₂Zn₂₀ manifest similar nearly ferromagnetic properties. When 4f local moments are embedded in this highly polarizable medium, the RFe_2Zn_{20} (R=Gd-Tm) series shows highly enhanced FM ordering temperatures. In contrast, YCo₂Zn₂₀ and LuCo₂Zn₂₀ manifest normal Pauli paramagnetic behaviors. In a related manner, GdCo₂Zn₂₀ and TbCo₂Zn₂₀ show low temperature AFM ordering, and the magnetic properties for RCo_2Zn_{20} (R =Dy-Tm) are more strongly influenced by the CEF splitting of the R ions (the dominant energy scale). The CEF coefficients determined for the Co series are consistent with the observed anisotropies of the Fe series, indicating that the CEF splitting of the R ions is similar for the all Zn Frank-Kasper polyhedra that fully encompass the R site. On the other hand, YbFe₂Zn₂₀ and YbCo₂Zn₂₀ manifest different heavy Fermion behaviors.

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APPENDIX A

Figure 31 shows the magnetization (at H=1000 Oe) and zero applied field resistivity for three batches of TbFe₂Zn₂₀, which were synthesized from different initial molar ratios of starting elements, Tb:Fe:Zn=2:3:95, 2:4:94, and 2:5:93. The $T_{\rm C}$ (determined as 52 ± 2 , 56 ± 1 , and 67 ± 2 K by resistivity measurements for the three samples) increases as the initial concentration of Fe in the solution increases. We also found similar features for R=Gd and Er, but the variation in their $T_{\rm C}$ values is less than in the Tb case (Fig. 32). Comparative single-crystal x-ray diffraction measurements performed on the samples, albeit inconclusive, indicated that the crystallographic differences were perhaps due to subtle variations in occupancy of the Fe site.¹⁸ The main difficulty



FIG. 31. (Color online) (a) Temperature-dependent M/H for TbFe₂Zn₂₀(H=1000 Oe) from different initial growth molar ratio of starting elements; (b) temperature-dependent ρ in zero applied field.

associated with the x-ray diffraction measurement was completely resolving the mixed site occupancies for Zn and Fe which have similar atomic number values. Recently, carefully prepared pieces of TbFe₂Zn₂₀ samples with the same geometric dimensions, from the starting elements, Tb:Fe:Zn=2:3:95 and 2:5:93, were measured using singlecrystal neutron diffraction.¹⁹ This measurement showed that the Fe site has ~1% deficiency for the 2:3:95 sample. All these crystallographic measurements indicate the sensitivity of the magnetic properties to the small disorder for RFe_2Zn_{20} in this family which has local moments submerged in a highly polarizable conduction electron matrix.

APPENDIX B

As shown in Fig. 1, the distance between the rare-earth ion and the Zn NNs, as well as NNNs, is close to 3 Å, whereas the distance with the next nearest neighbors (six Zn in 48f site) is larger than 5 Å. Due to this isolated cagelike configuration of rare-earth ions, the effect of ions other than this CN-16 Frank-Kasper polyhedron can be neglected in the



FIG. 32. $T_{\rm C}$ values for different growth of $R \text{Fe}_2 \text{Zn}_{20}$ (R = Gd-Tm) samples with respect to the de Gennes factor.

calculation of the CEF coefficients based on the point charge model.

Appreciably, the ligands of the rare-earth ions in the C-15 Laves compounds (RNi_2) form a same polyhedron, whose CEF coefficients have been calculated by Bleaney⁴⁵ based on the point charge model. Therefore, one can directly cite the following results:

$$B_{4}^{0} = -\frac{3}{2} \left(\frac{91e^{2}Z_{1}}{726R_{1}^{5}} - \frac{7e^{2}Z_{2}}{54R_{2}^{5}} \right) \langle r^{4} \rangle \langle J \| \beta \| J \rangle, \tag{B1}$$

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$$B_6^0 = \frac{9}{16} \left(-\frac{8e^2 Z_1}{363R_1^7} - \frac{8e^2 Z_2}{81R_2^7} \right) \langle r^6 \rangle \langle J \| \gamma \| J \rangle, \qquad (B2)$$

where Z_1e and Z_2e are the charges of the NN and NNN ions $(Z_1=Z_2=2 \text{ for } Zn^{2+})$, R_1 and R_2 is the distance between the R ion and the two sets of ions, $\langle r^4 \rangle$ and $\langle r^6 \rangle$ are the mean fourth and sixth powers of the electronic radius for the 4*f* electrons, and β and γ are the Steven multiplicative factors. Extracting the values of $\langle r^4 \rangle$ and $\langle r^6 \rangle$ from Ref. 46, β and γ values from Ref. 31, and R_1 and R_2 values from the results of single-crystal x-ray diffraction,¹⁸ one can calculate the B_4^0 and B_6^0 values (shown in Table II).

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