Vortex structures in disordered finite superconducting square networks under external magnetic field

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(Received 21 April 2009; revised manuscript received 21 June 2009; published 17 July 2009)

Magnetic-flux structures of disordered superconducting networks under an external magnetic field are studied, using the de Gennes-Alexander equation. These magnetic structures are different from those for regular network. If the wire at the edge of the network is weak superconducting, the flux enters at this wire with increasing magnetic field. And the change in equilibrium vortex structures with increasing magnetic field becomes gradual; vortices change their positions to the neighboring holes.

DOI: 10.1103/PhysRevB.80.024510 PACS number(s): 74.81.Fa, 74.25.Qt, 74.20.De

I. INTRODUCTION

Superconducting networks are composed of the thin superconducting wires, which are connected with other wires. They are examples of multiply connected superconductors.

The simplest network is a ring of superconducting wire. It shows periodic change in the transition temperature with increasing applied magnetic field perpendicular to the ring. This oscillation comes from the entrance of the quantized magnetic flux Φ_0 and is well known as the Little-Parks oscillation.[1,2] Here $\Phi_0=\frac{hc}{2e}$ is the flux quantum for superconductivity. This occurs because of the multiply connected structure of the ring. The period of the oscillation is called as a matching field, which is Φ_0 divided by the area of the hole. Increasing the connectivity of the network, it has various magnetic structures with increasing applied magnetic field and its transition temperature also shows complicated structures.[3]

Theoretical study of the superconducting networks was first done by de Gennes.[4] He solved the linearized Ginzburg-Landau equation for the superconducting lasso, which is a ring with an arm attached to it, under the magnetic field. Subsequently, Alexander generalized de Gennes' method to the general superconducting networks.[5] His theoretical formulation is summarized in coupled equations for superconducting order parameters at the vertices of the networks, which is called as the de Gennes-Alexander (dGA) equation. Several theoretical works on ladder networks,[6] networks with an external source,[7] n strips,[8] and a magnetically decorated square network[9] were followed.[10] Also other methods, Feynman path-integral approach[11] and Ginzburg-Landau approach[12,13] were used.

Previously we studied finite superconducting networks.[14] Their magnetic-field dependence of the transition temperature and magnetic structures are much different from the periodic networks. There appear the giant or doubly quantized vortices ($2\Phi_0$) and antivortices and total structures have the symmetry of the original networks. This is explained as follows. In the finite networks at the transition temperature, superconductivity is unevenly distributed at the edge of the networks. So, magnetic fluxes come together at the center of the networks. Therefore the finiteness affects the magnetic-flux structures. Such effect was also seen in mesoscopic or nanoscopic superconductors.[15–18] This is because of confinement of quantized magnetic fluxes in such superconductors. For example, in a superconducting disk, there appears a multiply quantized giant vortex, which was predicted theoretically[17] and detected by the multiple-small-tunnel-junction method.[18]

Experimental observation of these peculiar vortex structures in the finite superconducting networks by the scanning superconducting quantum interference device (SQUID) microscope was done.[19–23] But complete agreement between the theoretical predictions and experimental observations has not been obtained.

It seems that there are two main reasons for this discrepancy. First reason is that the states, which are observed by the scanning SQUID microscope, are nonequilibrium states. In order to observe the magnetic field of the vortices, the networks should be cooled down well below the transition temperature. In this cooling process, the states may become out of equilibrium because the holes in the networks act as strong pinning for the vortices. At the low temperature, the dGA equation is not applicable and the Landau-Ginzburug theory should be used. Such calculation was done by Hayashi *et al.*[12,13] Their results partly agree with experimental results but not completely.

Second reason is that there might be disorders of the superconducting wires. It is difficult to fabricate completely symmetric networks and there remain disorders in the networks. Observed magnetic-flux structures[19] sometime show disordered patterns, which are far from the symmetric flux structures.

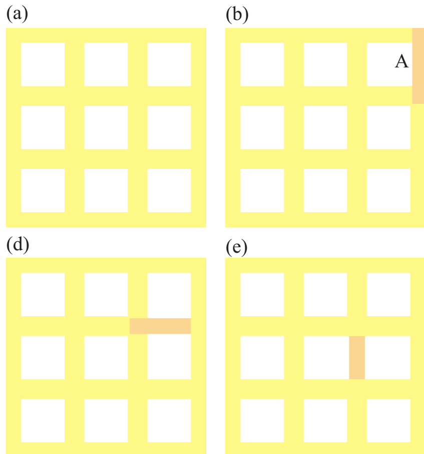

FIG. 1. (Color online) Regular and disordered superconducting networks. Dark gray bond means the transition temperatures T_{cij} are lower than bulk value T_{c0}. We denote the disordered networks (b) as #1, (c) as #2, (d) as #3 and (e) as #4.

In this paper, we focus on these disorders in the superconducting networks. Therefore, we investigate the effect of the disordered superconducting wire in the networks to the transition temperature and magnetic-flux structures, using the de Gennes-Alexander equation.

II. METHOD

For the arbitrary networks, the de Gennes-Alexander equation becomes as[5]

$$\Delta_i \sum_j{}' \frac{\cos\,\theta_{ij}}{\xi_{ij}\,\sin\,\theta_{ij}} - \sum_j{}' \Delta_j \frac{e^{i\gamma_{ij}}}{\xi_{ij}\,\sin\,\theta_{ij}} = 0. \tag{1}$$

Here i and j are indices of the vertices and $\theta_{ij}=\frac{l_{ij}}{\xi_{ij}}$, where l_{ij} and ξ_{ij} are the length and the coherence length of the wire between the vertices i and j. Summation Σ_j' is taken over the vertex j that is connected to the vertex i. Δ_i is the order parameter at the vertex i and γ_{ij} is defined as,

$$\gamma_{ij} = \frac{2e}{\hbar c}\int_i^j \boldsymbol{A}\cdot d\boldsymbol{l}. \tag{2}$$

Here $\boldsymbol{A}$ is the vector potential for the external magnetic field. The coherence length of each wire is

$$\xi_{ij} = \frac{\xi_{0ij}}{\sqrt{1-\frac{T}{T_{cij}}}}, \tag{3}$$

where T_{cij} is the transition temperature and ξ_{0ij} is the coherence length at $T=0$ of the wire. For regular networks, Eq. (1) becomes an eigenvalue equation, where $\Sigma_j' \frac{\cos\,\theta_{ij}}{\xi_{ij}\sin\,\theta_{ij}}$ is the eigenvalue. The transition temperature is determined from the largest eigenvalue.

In Eq. (1), we can introduce disorders by changing ξ_{0ij} or T_{cij}. When ξ_{ij} or T_{cij} depends on the position of wires, Eq. (1) is no more the eigenvalue equation. But even in this case, we can determine the transition temperature, using the fact that the determinant of the matrix of coefficients for Δ_i's in this equation becomes zero at the transition temperature. We can obtain such temperature numerically. Simultaneously, the order-parameter structure and the vortex structure of the equilibrium state can be obtained.

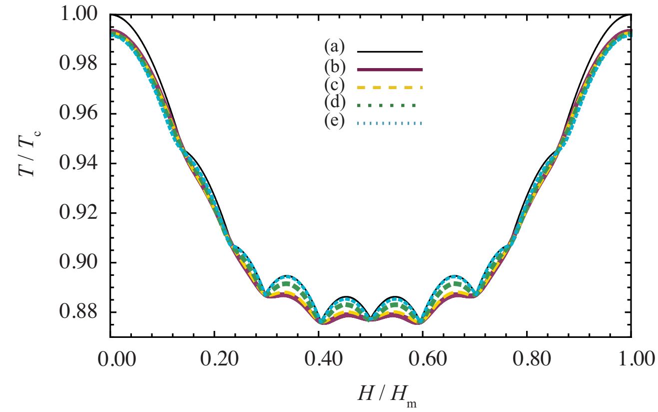

FIG. 2. (Color online) External magnetic-field dependence of transition temperature T_c for superconducting networks. Each curve [from (a) to (e)] corresponds to the network in Fig. 1

III. RESULTS

We studied 3×3 holes square lattice networks, as shown in Fig. 1. We set the length of all wires equal to $2\xi_0$, where ξ_0 is the coherence length of the pure superconductor at $T=0$. Colors of the bonds show the T_{cij} distribution. Figure 1(a) is a regular network without disorders. The magnetic-field dependence of the transition temperature of this regular network is shown in Fig. 2(a). In this figure, $H_m=0.25\frac{\Phi_0}{\xi_0^2}=\frac{\Phi_0}{4\xi_0^2}$ is a matching field, where there is a single vortex at every hole in the network. For the transition-temperature curve (a), each rounded peak of the T_c curve corresponds to each equilibrium state, which is shown in Fig. 3. For example, the first peak around $H=0$ corresponds to the Meissner state which is shown in Fig. 3(a) and the second peak corresponds to the single vortex state, which is shown in Fig. 3(b). All of the vortex structures are symmetric according to the symmetry of the original regular network. At the dip of the T_c curve, the vortex structure abruptly changes from a symmetric state to another symmetric state.

From Fig. 2(a), we notice that the magnetic-field dependence of the T_c has a mirror symmetry about the $H=0.5H_m$ line. And the state Fig. 3(b) at $H=0.20H_m$ can be converted to the state Fig. 3(i) at $H=0.08H_m=H_m-0.20H_m$ by changing a vortex to a vortex hole and vice versa. In Fig. 3, there are such correspondences for (c) and (h), (d) and (g), and (e) and (f). Ishida *et al.*[19] claimed such symmetry experimentally. We call this symmetry vortex-vortex hole symmetry. This symmetry for regular square networks is proved analytically in Ref. 24.

Next, we show the results for a disordered network #1, which is shown in Fig. 1(b). In this network, we decrease the transition temperature of the wire at the corner (A) to $0.8T_{c0}$, where T_{c0} is the transition temperature of the pure supercon-

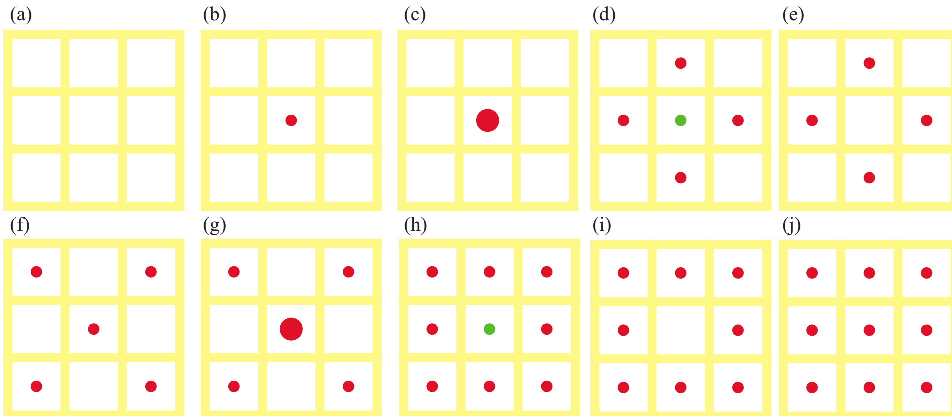

FIG. 3. (Color online) Vortex structures in a pure network. Magnitude of the external fields are (a) $H/H_m=0.004$, (b) 0.20, (c) 0.260, (d) 0.360, (e) 0.460, (f) 0.540, (g) 0.640, (h) 0.740, (i) 0.800, and (j) 1.00. Thick solid lines are superconducting wires and dark gray (red) and light gray (green) disks are up and down magnetic fluxes. A large (small) disk is a doubly (single) quantized vortex, respectively.

ductor. The transition-temperature curve is shown in Fig. 2(b). The curve becomes rather smooth compared to that of the regular network (a) but it contacts with the regular network curve (a) at its dips. Also, at zero external magnetic field, the transition temperature decreases compared to the pure network case (a). This is explained as that the whole superconductivity is weakened by the disordered wire because it costs the energy to support the superconductivity of the weakened wire.

The vortex structures for the disordered network #1 are shown in Fig. 4 for $0<H<0.5H_m$. Increasing magnetic field from zero, first single vortex appears at the right-upper hole (b). Next, it moves downward (c) and moves to center hole (d), and the second single vortex appears at the upper-right hole (e), moves downward (f), and moves to the center hole to form a doubly quantized vortex ($2\Phi_0$) (g). Then the doubly quantized vortex separates to two vortices (h) and the center vortex moves downward (i), and the third single vortex appears right-upper hole (j), moves downward (k). Then a pair of single vortex and antivortex appears at the upper-center and center holes (l), and a pair of vortex and antivortex at the right-center and center holes disappears (m). Finally, the forth single vortex appears at the upper-right hole (n) and moves downward (o). In this sequence of equilibrium states, there are symmetric states (d), (g), (l), and (o), which appear in the regular network (Fig. 3). The effect of the disordered wire on this sequence is the appearance of the intermediate states between the symmetric states. And in these states, vortices change their positions one by one across the wire perpendicularly, not diagonally. Especially, the vortices enter across the weak wire but they are not pinned at the weak wire. The weak wire acts only as the gate for the vortex entry, not the pinning site.

For the external field $0.5H_m<H<H_m$, the equilibrium states are shown in Fig. 5. Just after $H=0.5H_m$, five vortices state become stable (a) and the change from the state Fig. 4(o) to this state is abrupt. Then upper-right vortex moves downward (c), sixth vortex enters at upper-right hole, center-right vortex moves left and forms a doubly-quantized vortex with the center vortex, then the doubly-quantized vortex separates into center and upper-center vortices, and so on. This sequence of the equilibrium states is similar to that in Fig. 4. There are symmetric states [(a), (d), (i), (l), and (o)], which appear for the regular network (Fig. 3). And, there are the intermediate states between the symmetric states, where vortices change their positions one by one across the wire perpendicularly, not diagonally.

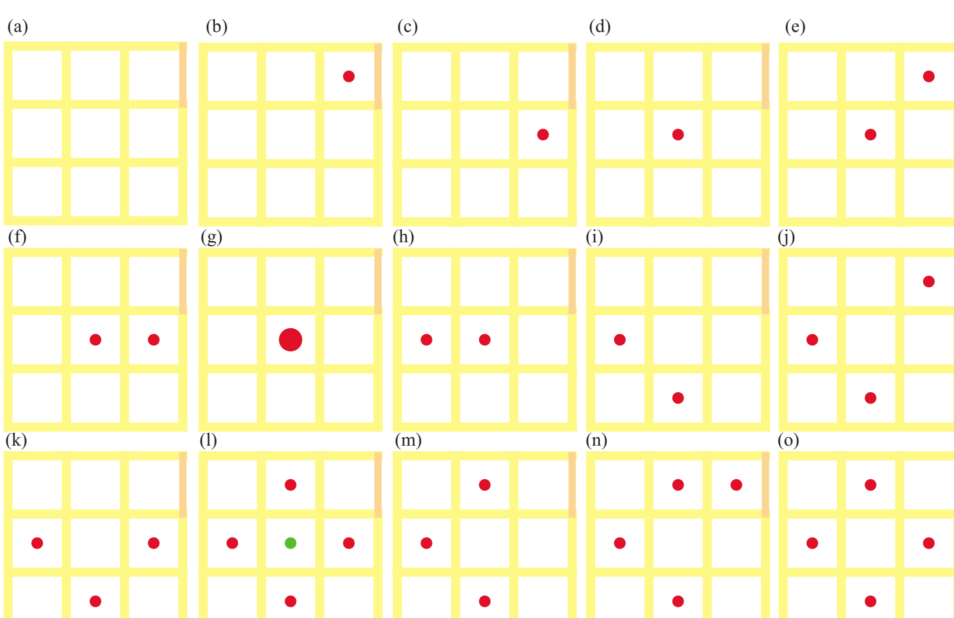

FIG. 4. (Color online) Vortex structures in the # 1 disordered network. Magnitude of the external fields are (a) $H/H_m=0.004$, (b) 0.140, (c) 0.160, (d) 0.180, (e) 0.240, (f) 0.260, (g) 0.288, (h) 0.2952, (i) 0.2960, (j) 0.2980, (k) 0.312, (l) 0.320, (m) 0.400, (n) 0.408, and (o) 0.412. The symbols are same as those in Fig. 3.

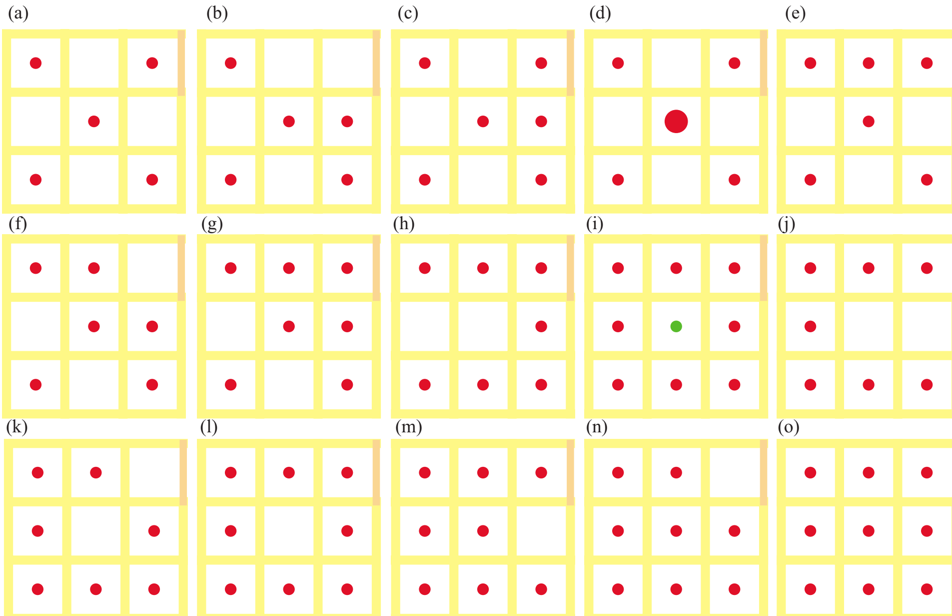

FIG. 5. (Color online) Vortex structures in the # 1 disordered network. Magnitude of the external fields are (a) H/H_m=0.580, (b) 0.592, (c) 0.600, (d) 0.680, (e) 0.688, (f) 0.702, (g) 0.704, (h) 0.7048, (i) 0.712, (j) 0.740, (k) 0.760, (l) 0.820, (m) 0.840, (n) 0.860, and (o) 1.00. The symbols are same as those in Fig. 3.

Furthermore, there is the vortex-vortex hole symmetry for this disordered network. The transition-temperature curve Fig. 2(b) has a mirror symmetry about the $H=0.5H_m$ line. And the state Fig. 4(b) at $H=0.14H_m$ corresponds to Fig. 5(n) at $H=0.86H_m=H_m-0.14H_m$. Also, there are correspondences between other states, Fig. 4(c) and Fig. 5(m), Fig. 4(d) and Fig. 5(l), Fig. 4(e) and Fig. 5(k), Fig. 4(f) and Fig. 5(j), Fig. 4(g) and Fig. 5(i), and so on. This vortex-vortex hole symmetry for disordered networks is also proved analytically as the case of regular networks.[25] And, the state Fig. 4(o) and the state Fig. 5(a) are under such correspondence until $H=0.5H_m$ but they cannot be transformed to each other by a single-vortex movement. Therefore, the change at $H=0.5H_m$ becomes abrupt.

For the disordered network #2 in Fig. 1(c), the equilibrium states for $0<H<0.5H_m$ are shown in Fig. 6. Because of the vortex-vortex hole symmetry, the equilibrium states for $H_m<H<H_m$ can be construct from these figures. Unlike to the network #1, there still symmetry about the center horizontal line. Therefore the vortex configurations also have this symmetry. The abrupt change between states (f) and (g) comes from this symmetry. But except such restriction, changes between adjacent equilibrium states are gradual.

For the disordered network #3 and #4 in Figs. 1(d) and 1(e), the transition temperature is shown in Figs. 2(d) and 2(e). They are not much decreased from the regular network compared to the network #1 and #2. But the sequence of equilibrium states for #3 is same as that for the disordered network #1. Also, the sequence of the equilibrium states for the disordered network #4 is same as that for the disordered network #2. But the transition fields take different values between two networks.

IV. SUMMARY

We have investigated the vortex structures for disordered superconducting networks under the external magnetic field. We found that only one single disordered superconducting wire changes vortex structures totally. In the sequence of the equilibrium states with increasing magnetic field, the vortex configuration changes gradually. The disordered wire at the edge of the network acts as the gate for vortex entry, not the pinning site.

This result means the experimentally observed vortex configurations may correspond to the one of the intermediate states. But there are still symmetric vortex configurations and

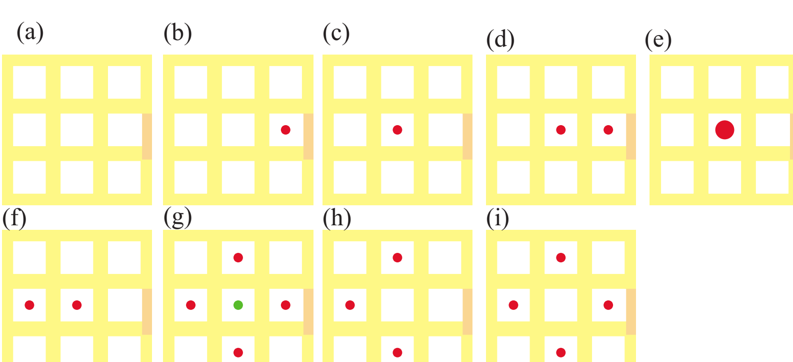

FIG. 6. (Color online) Vortex structures in the # 2 disordered network. Magnitude of the external fields are (a) H/H_m=0.004, (b) 0.140, (c) 0.180, (d) 0.240, (e) 0.288, (f) 0.2952, (g) 0.298, (h) 0.400, and (i) 0.408. The symbols are same as those in Fig. 3.

therefore to observe such theoretical predictions for the regular networks, it is needed to vary the magnetic field carefully, we think.

In this study we only considered single disordered wire. The effect of a number of disordered wires to the vortex structures in the superconducting network is a future problem.

ACKNOWLEDGMENTS

This work was supported by "The Faculty Innovation Research Project" of The Graduate School of Engineering of Osaka Prefecture University. We thank T. Ishida, M. Hayashi, H. Ebisawa, T. Koyama, T. Tamegai, and K. Hirata for fruitful discussions.

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