# Energy-Loss and Stopping-Power Measurements between 2 and 10 MeV/amu for $^3$He and $^4$He in Silicon

Bach Sellers and Frederick A. Hanser*
*Panametrics, Inc., Waltham, Massachusetts 02154*

Joseph G. Kelley
*Air Force Cambridge Research Laboratories, Bedford, Massachusetts 01730*
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The energy deposited in a 94.6-μm-thick totally depleted silicon detector by beams of $^3$He and $^4$He ions was measured at incident energies up to 10 MeV/amu. It is assumed that the stopping power of an ion of energy $E$, mass $M$, and charge $z$ can be expressed approximately by $S = A[(E/M)+r]^{1-n}$, and the parameters $A$, $n$, and $r$ are determined from the energy-loss data. Within the experimental error the results are found to agree with recent theoretical calculations which include a $z^3$ correction.

## I. INTRODUCTION

Although silicon detectors are now used widely for heavy-particle detection, few stopping-power or range measurements have been made on this material for $z = 2-8$ particles. In 1956 Gobeli[1] measured $^4$He ranges in the 1–5-MeV region, and in 1972 Eisen *et al.*[2] made $^4$He stopping-power measurements between 0.1 and 18 MeV. Extensive theoretical calculations of range and stopping power have been made by Bichsel and Tschalär[3] for *p*, *d*, *t*, $^3$He, $^4$He, and $^7$Li in Si from 1 to 200 MeV. Whaling[4] has deduced $^4$He stopping powers from Gobeli's measurements with a stated accuracy of ±10%. In the 1–5-MeV energy region these results agree to better than 4% with the Bichsel and Tschalär theoretical calculations. However, Eisen *et al.* note that their $^4$He stopping powers, which have a maximum error of ± 5%, lie above the same theoretical results in the low-energy region (~1–4 MeV from their Fig. 3).

At the time of their work (1967) Bichsel and Tschalär observed that various discrepancies existed between experiment and theory. They noted, in particular, that $\alpha$-particle range measurements in Al were 3–7% low compared to theory in the 1–2-MeV/amu region, and they suggested that the Al shell corrections did not appear to decrease rapidly enough. Recent accurate measurements by Andersen *et al.*[5] show that the relative difference between the stopping power of the He and H isotopes in Al is larger than expected on the basis of the $z^2$ charge dependence of the usual Born approximation. They believe that this difference is due to a charge-proportional correction term in the Bethe stopping-power equation and that the Al discrepancies discussed by Bichsel and Tschalär may also be due to this effect. Since the correction term produces an increase in stopping power at low energy, it is also in the proper direction to explain the type discrepancies observed by Eisen *et al.*

Ashley *et al.*[6] have carried their calculation through the $z^3$ approximation and have derived the resulting charge-proportional correction term. They adjusted a parameter in their result using the fact that the relative H-He difference measurements of Andersen *et al.* are equivalent to the $z^3$ fractional corrections for H.

Here we report measured energy losses of 3–10-MeV/amu incident energy $^3$He and $^4$He ions in Si. Absolute stopping powers derived from these measurements are then compared with the calculations of Bichsel and Tschalär as corrected by the $z^3$ term, and with the experimental results of Eisen *et al.* Results for $^{12}$C, $^{14}$N, and $^{16}$O are reported in a separate paper.[7]

## II. EXPERIMENTAL RESULTS

All measurements were made on the heavy-ion linear accelerator at Yale University. A schematic diagram of the apparatus is shown in Fig. 1. Outputs from the two totally depleted surface barrier Si detectors were linearly amplified and made available simultaneously to a single-parameter analyzer (SPA) having 256 channels and a dual-parameter analyzer (DPA) having 64×50 channels. Each analyzer was calibrated with $^{241}$Am $\alpha$ particles of 5.48 MeV and checked for linearity with a precision pulser.

Detector D2 was nominally 750 μm thick and was totally absorptive for all particles and energies of interest here. Detector D1 was measured[8] by x-ray absorption to be 94.6 ± 0.8 μm thick. A thin gold foil was used to scatter a small fraction of the incident beam into the detectors, and the transmitted beam was monitored by the Faraday cup. Collimators eliminated any possible detector edge effects.

Measurements were made at various energies between the maximum available, 10 MeV/amu, and the range energy for D1. A nominal beam energy

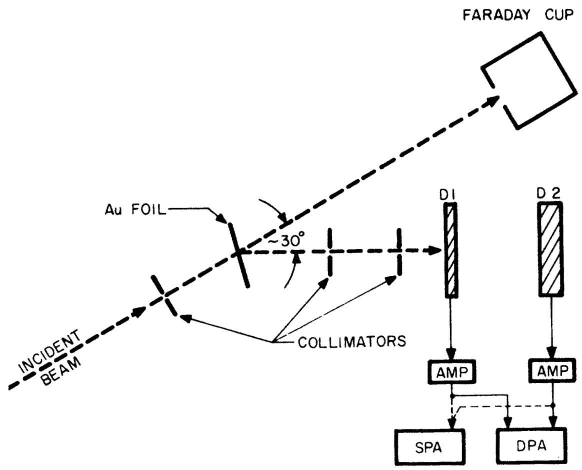


FIG. 1. Schematic diagram of experimental apparatus.

was determined by absorber foils and analyzer magnet settings. However, due to the scattering foil, approximations in magnet settings and the use of wide collimating slits, this value was not expected to be particularly accurate. D1 was remotely movable so that three measurements were obtained with the SPA at each beam setting: (i) the most probable energy loss $\Delta_p$ in D1, (ii) the incident energy $E_i$ in D2—obtained by removing D1, and (iii) the energy $E_i - \Delta_p$ in D2—obtained with D1 in place. At about half the beam settings $\Delta_p$ vs $E_i - \Delta_p$ was recorded on the DPA. As an average, the beam setting energies were found to agree with the SPA measurement (ii) within 3%.

The experimental results are given in Fig. 2. For the SPA the incident energy $E_i$ was found from measurement (ii), and the value of $\Delta_p$ is shown at that energy both as measured directly—from (i)—and as obtained from the difference of (ii) and (iii). For the DPA the value of $\Delta_p$ is plotted at the energy $E_i$ found from the sum of (i) and (iii). The uncertainties were computed from an assumed minimum $\frac{1}{4}$ channel error in the channel measurements for the energies and the $^{241}$Am calibration, combined on an rms basis. Where larger than the symbol size, the uncertainties are shown in Fig. 2 for three regions of $E_i$.

The directly measured SPA values of $\Delta_p$ and $E_i$ are thus believed to have random errors less than 1% over the entire region, whereas the difference derived values of $\Delta_p$ vary from about 2 to 5%. Scatter in the data is consistent with these estimates. A significant increase in accuracy is thus achieved when the material for which the energy deposition is measured is itself a detector. The potential accuracy of direct measurements was shown by Andersen *et al.* using a thermal technique on various metal foils.

## III. STOPPING POWER

### A. Experimental

Most of the energy losses measured are such that the usual approximation $\Delta \ll E_i$, which allows a direct calculation of stopping power, is not valid. Therefore, our approach was to assume a parametric representation of stopping power $S$ (MeV cm$^2$/g) for an ion of energy per unit mass $\mathcal{E} = E$ (MeV)/$M$ (amu):

$$S(\mathcal{E}) = -\frac{dE}{dx} = A(\mathcal{E}+r)^{1-n} \quad , \tag{1}$$

where $x$ is the path length (g/cm$^2$), and $A$, $r$, and $n$ are constants expected to be valid in the energy region in question for all isotopes of the particular ion. This expression is a trivial extension valid to lower energies of the well-known result that range is approximately proportional to $E^n$ in the 5–100-MeV/amu region. If multiple scattering effects are small, a beam of particles of energy $E_i$ incident on an absorber of thickness $x_a$ will exit with average energy $E_i$ found from

$$x_a \approx x = \int_{E_f}^{E_i} S(\mathcal{E})^{-1} dE \quad . \tag{2}$$

Use of (1) in (2) yields the average energy loss $\bar{\Delta}$, which we equate approximately with $\Delta_p$:

$$\Delta_p \approx \bar{\Delta} = E_i - E_f = (E_i + \epsilon) - [(E_i+\epsilon)^n - K]^{1/n} \quad , \tag{3}$$

where

$$\epsilon = rM \quad \text{and} \quad K = xnAM^{n-1} \quad . \tag{4}$$

Two possible corrections to (3) must be considered: asymmetry in the distribution of energy losses ($\bar{\Delta} > \Delta_p$) and multiple scattering effects ($x > x_a$). Asymmetry is important because $\Delta_p$ is measured, whereas $\bar{\Delta}$ is calculated, which causes an underestimate of $S$ for a given $E_i$. Thus, $(\bar{\Delta} - \Delta_p)/\bar{\Delta}$ must be small. Since all distributions were recorded, it was possible to examine each, and with a single exception no asymmetries were measurable. An asymmetry was evident for the $^4$He point at 12.4 MeV, which is expected because this point is located very near the range energy. Thus, for that point the measurement (iii) was affected by range straggling and was inaccurate. These results are consistent with the theory of Tschalär[9] for large energy losses and Vavilov[10] for small energy losses. Using Tschalär's results it can be shown that $\bar{\Delta} > 0.8\, E_i$ would be required for $(\bar{\Delta} - \Delta_p)/\bar{\Delta}$ to exceed 0.1% for $^4$He incident on Si. From approximate results[11] for the Vavilov theory it can be shown that in the $E_i \approx$ 25–40-MeV region, $(\bar{\Delta} - \Delta_p)/\bar{\Delta} < 0.2\%$ for He incident on a 95-$\mu$m Si absorber. Within the accuracy of the data in Fig. 2, therefore, we conclude that $\bar{\Delta}$ may be replaced by $\Delta_p$ as in Eq. (3).

Multiple scattering detour factors $(x - x_a)/x$ have been calculated for protons in Si by Janni,[12] by Berger and Seltzer[13] for protons in Al, and, additionally, for protons and larger mass particles in Al by Litton *et al.*[14] Results[12] for 1–10-MeV/amu protons in Si vary from 0.7 to 0.4%. Litton *et al.* note that for the same absorber and particle range, mean beam deflection and detour factor should decrease as the atomic number of the particle increases. Their results suggest a factor of 3–4 decrease for He ions compared to protons. Thus, for 1–10-MeV/amu He ions $x$ should exceed $x_a$ by a maximum of about 0.2%. This is small compared to the uncertainty in detector D1 thickness, and it can be included by assigning an over-all absolute error of 1% to $x$, including the thickness measurement, and equating it to $x_a$ as in Eq. (2).

Replacing $x$ by $x_a$ has an opposite effect to that of replacing $\bar{\Delta}$ by $\Delta_p$, in that it causes an overestimate of $S$. Therefore, these two approximations, which are highly accurate individually for the present data, tend to cancel.

The results presented here are not affected by channeling in the D1 detector. The experimental layout of Fig. 1 allowed particles to be detected within ± 1.5 deg of the detector normal. Backscattering yield measurements for 1-MeV He ions on silicon show that the important axial and planar channels have half-widths of about $\pm\frac{1}{4}$ deg.[15] Measurements of the actual channeling losses for 0.1–18-MeV $^4$He ions[2] show that channeled ions would have only about half the energy loss of a randomly directed ion. Thus channeling in the D1 detector would be observed as asymmetrically broadened, possibly double peaked, distributions in $\Delta_p$ and $E_i - \Delta_p$. Such distributions were not observed in this experiment.

The combined $^3$He, $^4$He experimental energy loss data have been fitted to obtain the last set of parameters in Table I, with results also shown in Fig. 2. Generally, this curve exceeds the data by as much as 1% at high and low energy, and is lower by as much as 1% in the central region. The fit is slight-

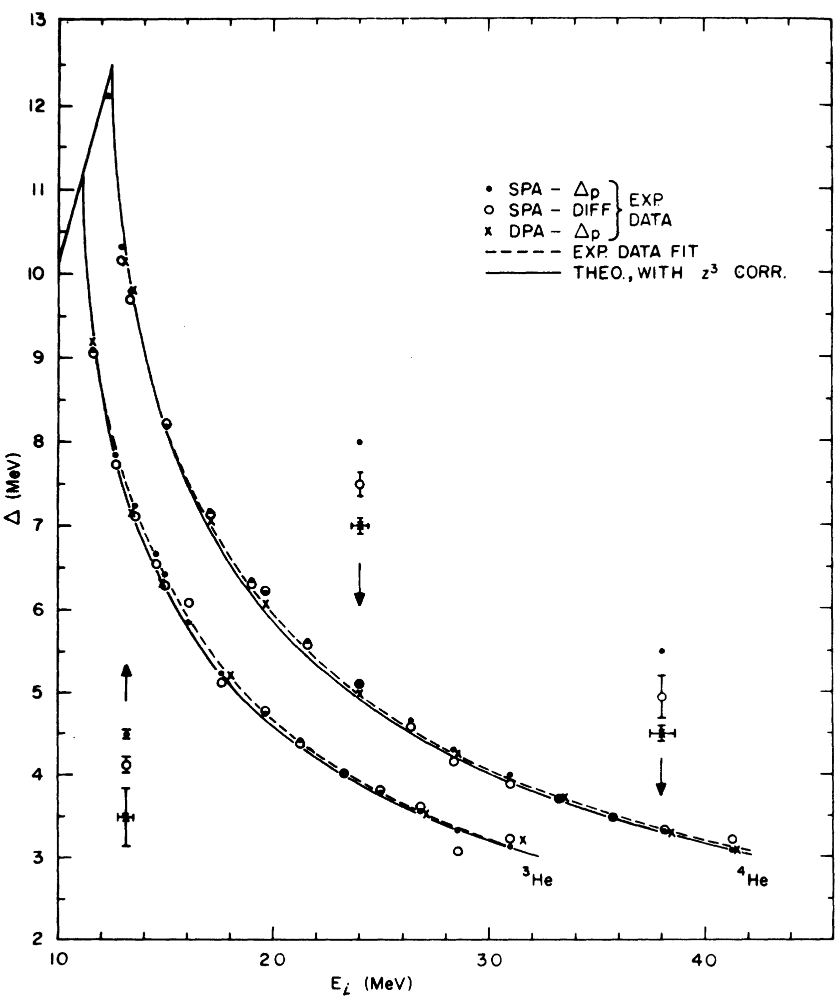


FIG. 2. Plot of energy-loss data for $^3$He and $^4$He ions incident on 94.6-μm-thick Si detector. See text for discussion of experimental data, the fit to the data (dashed curve), and the theoretical result (solid curve).

TABLE I. Stopping-power parameters for use in Eq. (1).

| Stopping power | $A$ $\frac{\text{MeV}^n\ \text{cm}^2}{\text{g amu}^{n-1}}$ | $n$ | $r$ $\frac{\text{MeV}}{\text{amu}}$ |
|---|---|---|---|
| Bichsel-Tschalär, $S_b(\mathcal{E})$ | 864.95 | 1.7876 | 0.3163 |
| With $z^3$ correction, $S_c(\mathcal{E})$ | 854.43 | 1.7840 | 0.2020 |
| Expt. data fit | 905.43 | 1.8007 | 0.3080 |

ly better for $^3$He than for $^4$He. Better results can be obtained by fitting the data for each ion separately, although this does not appear justified theoretically. Values of stopping power calculated from (1) using this set of parameters are given in Table II labeled "Expt. fit."

### B. Theoretical

Ashley[16] has suggested that the corrected stopping power for an ion of atomic number $z$, and energy per unit mass $\mathcal{E}$ (MeV/amu) incident on a material of atomic number $Z$, may be written in a form equivalent to the following:

$$S_c(z, \mathcal{E}, Z) = S_b(z, \mathcal{E}, Z)[1 + zu(\mathcal{E}, Z)] \ , \qquad (5)$$

where $S_b(\mathcal{E})$ is the "best" previous theoretical value, $zu$ is the fractional correction due to the $z^3$ effect, given by Eq. (15) of Ref. 6, with

$$u(\mathcal{E}, Z) = F(w)/[Z^{1/2}x^{3/2}L(x)] \ , \qquad (6)$$

$$w = 1.8/x^{1/2} \ , \qquad (7)$$

$$x = 40.2\,\mathcal{E}/Z \ . \qquad (8)$$

The use of tables of the functions $F(w)$ and $L(x)$[16] in (6) yields the values of $zu$ listed in Table II for He ($z=2$) ions incident on Si ($Z=14$). In the energy region in question the correction varies between about 6 and 0.5%. Also shown are results of the "best" previous theoretical calculation for He, $S_b(\mathcal{E})$, taken from Bichsel-Tschalär tables,[3] the resulting values of $S_c(\mathcal{E})$ from (5), and the percentage differences between $S_c(\mathcal{E})$ and the experimental results.

In the energy region 1–10 MeV/amu, $S_b(\mathcal{E})$ and $S_c(\mathcal{E})$ are given by Eq. (1) with an accuracy of better than ±0.1% using the sets of parameters of Table I. We have calculated the energy loss $\bar{\Delta}$ from (3), with the results from the $S_c(\mathcal{E})$ parameters being given in Fig. 2. The theoretical energy losses are as much as 2.5% below some of the experimental data in the central region. Although not shown, the energy-loss curve for the $S_b(\mathcal{E})$ parameters is lower than the $z^3$ corrected curve by an amount expected from the values of Table II, i.e., ~2–0.5% between 3 and 10 MeV/amu.

## IV. DISCUSSION

Although the minimum ion exit energy for the data of Fig. 2 is near 1 MeV/amu, the average ion energy contributing to $\Delta_p$ has a minimum of about 2 MeV/amu. The experimental data fit parameters are thus based principally on ion energies above 2 MeV/amu, which is consistent with the crossover of experimental and theoretical results near this energy in Table II. We believe, therefore, that the lower limit to our results should be taken as 2 MeV/amu. The upper limit is more easily established as approximately the maximum used, 10 MeV/amu. Within this energy region our measured stopping power is given by use of the "Expt. data fit" parameters of Table I in Eq. (1), as tabulated partially in Table II. This equation should not be extrapolated outside of the above energy limits with these parameters. As noted above, our path length measurement contains an uncertainty of about 1%, which is systematic, and the individual energy-loss measurements have random uncertainties of similar magnitude. Including all errors, we believe our measured stopping powers should be assigned an uncertainty of about ±1.5% on a one sigma basis.

As noted by Eisen *et al.*,[2] their results, which have an estimated maximum error of ±5%, agree at higher energies (~1–5 MeV/amu from their Fig. 3) with the Bichsel-Tschalär theoretical calculations, $S_b(\mathcal{E})$ in Table II. Our "Expt. fit" values exceed $S_b(\mathcal{E})$ by 2–4% in the 2–5 MeV/amu region. Thus, they also exceed the Eisen *et al.* results in this region, although the difference is within the experimental uncertainties.

Table II shows an rms deviation of approximately 1.25% between the experimental results and $S_c(\mathcal{E})$ within the 2–10-MeV/amu region. This difference cannot be eliminated by a minor change in the $z^3$ theory, because the disagreement exists up to 10 MeV/amu where the $z^3$ contribution is only 0.5%. Subsequent to completion of the calculations in Table II a new theoretical $z^3$ correction was published by Jackson and McCarthy.[17] The functional

TABLE II. Stopping-power data for $^3$He and $^4$He in silicon.

| $\mathcal{E}$[a] $\frac{\text{MeV}}{\text{amu}}$ | $z^3$ correction $u$ | $zu(z=2)$ | Stopping power $S_b(\mathcal{E})$[b] | $S_c(\mathcal{E})$[c] | $\frac{\text{MeV cm}^2}{\text{g}}$ (Expt. fit) | $\frac{\text{Expt.} - S_c(\mathcal{E})}{\text{Expt.}}$ (%) |
|---|---|---|---|---|---|---|
| 1 | 0.0309 | 0.0618 | 696.6 | 739.6 | 730.3 | −1.3 |
| 2 | 0.0158 | 0.0316 | 446.0 | 460.1 | 463.5 | +0.7 |
| 3 | 0.0100 | 0.0200 | 336.3 | 343.0 | 347.4 | +1.3 |
| 4 | 0.0070 | 0.0140 | 273.4 | 277.2 | 281.2 | +1.4 |
| 5 | 0.0054 | 0.0108 | 232.2 | 234.7 | 237.9 | +1.4 |
| 6 | 0.0042 | 0.0084 | 202.7 | 204.4 | 207.2 | +1.4 |
| 7 | 0.0035 | 0.0070 | 180.6 | 181.9 | 184.2 | +1.2 |
| 8 | 0.0029 | 0.0058 | 163.2 | 164.1 | 166.2 | +1.3 |
| 9 | 0.0025 | 0.0050 | 149.2 | 149.9 | 151.8 | +1.3 |
| 10 | 0.0022 | 0.0044 | 137.7 | 138.3 | 139.8 | +1.1 |

[a] $^4$He, $M = 4.0015$ amu; $^3$He, $M = 3.0149$ amu.

[b] Theoretical calculation of Ref. 3 for He, $I = 173.5$ eV.

[c] Theoretical calculation including $z^3$ effect of Ref. 6, i.e., $S_b(\mathcal{E})$ as corrected in Eq. (5).

form of the quantity $u$ in Eq. (5) is different than that of Eq. (6), and for He the $z^3$ fractional correction $zu$ is about 15–20% smaller than the theoretical values given in Table II. The rms deviation in this case is about 1.4% so that the agreement with experiment is not as good. The agreement can be made almost the same as that for the Ashley *et al.* theory by using the Lindhard-Scharff parameter $\chi \approx \sqrt{3}$ in the Jackson and McCarthy theory.

From the tables of Barkas and Berger[18] we find that a 5-eV reduction in $I$ produces about 1% increase in stopping power for few MeV/amu He in Si. Hence, near exact agreement between theory and experiment could be obtained by reducing the value $I = 173.5$ eV used by Bichsel and Tschalär to 168.5 eV, which is close to the value 170 eV used by Janni.[12]

Since our experimental uncertainty is about 1.5%, it is not possible for us to choose between the above values of $I$, or between the two $z^3$ theories. However, within this uncertainty our results do agree with—and in the few-MeV/amu region require—the $z^3$ contribution to the stopping power. This suggests that only small changes, if any, are required in the $I$ value and shell corrections used by Bichsel and Tschalär[3] to obtain $S_b(\mathcal{E})$.

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