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Effect of the Landau A_1 Parameter on the Microwave Field Within an Electron Gas

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This is the first of a series of three papers devoted to a theoretical study of the effects of electron correlations on cyclotron phase resonance. In this paper we solve the equations governing the microwave field in a diffuse-surface semi-infinite interacting electron gas in which there is a steady magnetic field normal to the surface. The short-range interaction between electrons is assumed to be described by Landau's theory of the Fermi liquid, subject to the condition that only A_1 is nonzero. We find that the microwave field inside the gas reaches its maximum intensity near $\omega_c/\omega=1$, in spite of the fact that the nonlocal conductivity, which in many other situations seems to describe the transmitted signal, reaches its maximum at $\omega_c/\omega=(1+A_1)^{-1}$.

I. INTRODUCTION AND SUMMARY

In a recent letter,[1] Phillips, Baraff, and Dunifer (PBD) reported measurements of microwave transmission through thin (approximately one-tenth of an electron mean free path) slabs of sodium and potassium at a microwave frequency of 116 GHz. The measurements were carried out in a magnetic field placed normal to the plane of the sample, and the magnetic field strength was varied through a range such that ω_c, the cyclotron frequency of the conduction electrons, was swept through ω, the microwave frequency. They showed that the transmitted signal, plotted as a function of magnetic field strength, was remarkably similar in appearance to the nonlocal conductivity (also plotted as a function of the magnetic field) where the nonlocal conductivity is that calculated using the Landau theory of Fermi liquids[2-4] in an infinite medium. The nonlocal conductivity depends on the strength of the magnetic field and, in addition, on $\{A_n\}$, a set of parameters which describe the orbital part of the interaction function in Fermi-liquid theory. The most striking feature of the nonlocal conductivity is that at large values of $\omega\tau$ (τ is the electron mean free time) it exhibits a sharp peak at that value of the magnetic field for which $(\omega_c/\omega)=(1+A_1)^{-1}$. By fitting the position of the observed sharp peak in the transmission, PBD determined that both the sodium and potassium data required a value of about -0.01 for A_1. They were careful to point out that there exists no rigorous analysis of the transmission problem which demonstrates that the transmission amplitude should be the proportional to the nonlocal conductivity,[5,6] and therefore that this value of A_1 (which is much smaller than is to be expected on the basis of existing microscopic theory[7]) has to be considered tentative.

It is clearly of interest to know whether it is possible to determine A_1 by measuring the position of the peak in the transmission spectrum because, although some experiments have been suggested,[8,9] it has not yet been possible to observe effects of A_1 at all in metals. This paper, which is the first in a projected series of three, represents an attempt to determine, by solving the equations governing the microwave field in the interior of a semi-infinite electron gas, whether A_1 affects the position of the transmission maximum. In this present work, we shall assume the following.

(a) There is a uniform magnetic field, directed normal to the surface of the gas, of such strength that no collective modes (e.g., helicons) occur. (b) Circularly polarized microwave radiation is incident on the surface. (c) Electrons from within the gas suffer diffuse reflection on striking the surface. (d) Electrons in the gas interact with each other as described by Fermi-liquid theory, in which A_1 is the only nonzero interaction parameter.

The model we have posed here differs from the physical situation in at least two important ways. First, the physical slab has two surfaces, an emergent surface as well as an incident surface, and it is known[10] that the second surface plays as important a role in establishing the transmission peak (in the noninteracting gas) at $\omega_c/\omega=1$ as does the first surface. Thus in the one-sided problem we are considering here, the maximum in transmitted intensity can be expected to be much less pronounced than in the two-sided problem, and we should not seek to compare the results of this calculation directly with the experiment. Second, by limiting ourselves to having A_1 as the only nonzero interaction parameter, we discard all possible effects of the higher A_n parameters which, conceivably, can be more important than A_1. These two limitations will be relaxed in the second and third papers of this series. It is known that diffuse scattering is not a correct assumption for electrons striking the surface at *very* small angles. However, there are few of these and our guess is that even these few play no role in the phenomena we are interested in here.

The deficiencies in the model do not affect the limited question we are going to ask: Namely, does the maximum in microwave intensity deep within the sample occur at $(\omega_c/\omega)=(1+A_1)^{-1}$, as does the maximum in the nonlocal conductivity? If it does, then it is virtually certain that A_1 is measurable via the position of the peak in the transmitted intensity. If, on the other hand, the maximum in microwave intensity deep within the slab occurs close to $(\omega_c/\omega)=1$, then it is difficult to understand how the introduction of a second surface can cause a shift to the value $(1+A_1)^{-1}$. We would then have to conclude that A_1 cannot be measured by observing the position of the transmission peak.

The importance of the restricted model we are using is that the equations describing it can be solved exactly. After solving the equations and evaluating the field deep within the slab, we find that only a very small shift in the peak position occurs. We find, in fact, that the effect of A_1 on the transmitted field is so small that it could have been calculated by first-order perturbation theory treating A_1 as the perturbation. Hence, if A_1 plays any role at all in the microwave-transmission phenomenon, it is exceedingly likely that that role is limited to a line-shape distortion. That such an effect does occur will be demonstrated in the last paper of this series.

In Sec. II we state, without derivation, the equations which have to be solved. In Secs. III–V we solve these equations, obtaining as the final result expressions for the surface impedance and the field in terms of certain specific integrals to be evaluated. The detailed description of the integrands requires a substantial amount of fairly tedious analysis, but there are no subtle features concealed here and we omit all of the details. The physical discussion resumes again in Sec. VI, where we present our final solution, evaluate it for representative cases, and comment about what it implies.

II. EQUATIONS FOR FIELD

For brevity, we adopt the notation of Ref. 11 and refer to that work for the derivation of the equations below. Briefly, in that work we took angular moments of the formal solution of the Fermi-liquid equations and found that each angular moment of the distribution function,

$$\delta n_l^m(z) \equiv \int [Y_l^m(\Omega)]^* \delta n(z,p)\, d\Omega ,$$

could be regarded as being driven by the electric field $e(z)$ and, if the Fermi-liquid parameters $\{A_n\}$ are present, by angular moments of the distribution function as well. This point of view introduced certain nonlocal transport kernels $K_{ll'}(z-z')$ which related the response of $\delta n_l^m(z)$ to each of its possible sources. Completing the set of equations was Maxwell's wave equation in which the current, which is proportional to $\delta n_1^1(z)$, becomes the source of the field. We cite Eqs. (4.4)–(4.7) of Ref. 11. Specializing to the case of $A_2=0$, we have

$$\psi_1(x)+ih_1\int_0^\infty K_{11}(|x-x'|)\psi_1(x')\,dx' + \int_0^\infty K_{11}(|x-x'|)e(x')\,dx'=0 , \tag{2.1a}$$

$$\left(\frac{d^2}{dx^2}+k_0^2 l^2\right) e(x) - ib\psi_1(x)=0 , \tag{2.1b}$$

where

$$h_1=\frac{\omega\tau A_1}{1+A_1} , \tag{2.1c}$$

$$b=\frac{\omega\omega_p^2 l^3}{v_F c^2} , \tag{2.1d}$$

$$K_{11}(|x|)\equiv\frac{3}{4}\int_0^1\left(\frac{1}{u}-u\right) du\, e^{-a|x|/u} , \tag{2.2a}$$

$$a\equiv 1-i(\omega-\omega_c)\tau . \tag{2.2b}$$

In (2.1a), we have ψ_1 (which is proportional to δn_1^1) being driven by the electric field e via the nonlocal kernel K_{11}, which is proportional to the nonlocal conductivity in the noninteracting electron gas. When the Landau parameter A_1 is not zero, the second term in (2.1a) shows that ψ_1 can be regarded as also driving itself, again via the same kernel. The quantity $\psi_1(x)$ is proportional to the current $j(x)$, which, like the microwave field $e(x)$, is transverse and circularly polarized. (The exact coefficient of proportionality can be deduced by considering the value we have taken for b.) In (2.1) and (2.2) the mean free path $l=v_F\tau$ is taken as the unit of length. We take unit incident amplitude as the boundary conditions of the field. We have

$$\left(1+\frac{1}{ik_0 l}\frac{d}{dx}\right)e(x)=2 \quad \text{at } x=0 , \tag{2.3a}$$

$$e(x)=0 \quad \text{at } x=\infty . \tag{2.3b}$$

If A_1 were zero, then we could use (2.1a) to eliminate ψ_1 from Maxwell's wave equation (2.1b) and thus obtain the integrodifferential equation for $e(x)$ derived by Reuter and Sondheimer,[12] which they solved using the Wiener-Hopf technique. A Wiener-Hopf technique can still furnish us with the exact solution[13] and with an explicit expression for the surface impedance even when A_1 is nonzero, although it cannot handle the case where any of the other A_n's are finite.

III. FORMULATION OF WIENER-HOPF SOLUTION

Equations (2.1)–(2.3) define e and ψ_1 only for $x>0$, and so we are free to define

$$e(x)\equiv 0 , \quad x<0 \tag{3.1a}$$

$$\psi_1(x)\equiv 0 , \quad x<0 . \tag{3.1b}$$

These definitions are incompatible with (2.1a) for $x<0$. However, by defining

$$g(x)\equiv 0 \quad (x>0) \tag{3.2a}$$

$$= -\int_{-\infty}^{\infty} K_{11}(x-x')[ih_1\psi_1(x')+e(x')]\,dx' \quad (x<0)\ , \tag{3.2b}$$

we obtain

$$\psi_1(x)+\int_{-\infty}^{\infty} K_{11}(x-x')[ih_1\psi_1(x')+e(x')]\,dx' + g(x)=0 \tag{3.3}$$

as an equation which, because of (3.1) and (3.2), is valid for all x and implies (2.1a) for $x>0$.

We introduce the Fourier transforms (FT)

$$E(k)\equiv \int_{-\infty}^{\infty} dx\, e(x)\, e^{-ikx}\ , \tag{3.4}$$

and, similarly, $J(k)$, $G(k)$, and $K(k)$ are the FT's of $\psi_1(x)$, $g(x)$, and $K_{11}(x)$, respectively. Then, taking the FT of (2.1b) and using that to eliminate $J(k)$ from the FT of (3.3), we obtain

$$Q(k)\,E(k)=ibG(k)-[1+ih_1K(k)][ike(0)+e'(0)]\ , \tag{3.5}$$

where

$$e'(0)\equiv\left(\frac{de}{dx}\right)_{x=0} \tag{3.6}$$

and

$$Q(k)\equiv(k^2-k_0^2 l^2)[1+ih_1K(k)]-ibK(k)\ . \tag{3.7}$$

Let $Q(k)$ be factorized according to the Wiener-Hopf scheme, so that

$$Q(k)=f^-(k)/f^+(k)\ , \tag{3.8a}$$

where

$f^-(k)$ is analytic for $\mathrm{Im}k<\mu$, $0<\mu<1$ (3.8b)

$f^+(k)$ is analytic for $-\mu<\mathrm{Im}k$, (3.8c)

$f^+(k)$ and $f^-(k)$ are algebraic, not exponential, as $k\to\infty$. (3.8d)

Using (3.8a) in (3.5), we have

$$f^-(k)E(k)=ibf^+(k)G(k)-f^+(k)[1+ih_1K(k)] \times[ike(0)+e'(0)]\ . \tag{3.9}$$

We decompose $f^+(k)K(k)$ into a sum of separately analytic parts. That is, let

$$b^+(k)-b^-(k)=if^+(k)K(k)\ , \tag{3.10a}$$

where

$b^+(k)$ is analytic for $-\mu<\mathrm{Im}k$, (3.10b)

$b^-(k)$ is analytic for $\mathrm{Im}k<\mu$. (3.10c)

Then (3.9) is

$$f^-(k)E(k)-h_1b^-(k)[ike(0)+e'(0)]=ibf^+(k)G(k)-[f^+(k)+h_1b^+(k)][ike(0)+e'(0)]\ . \tag{3.11}$$

What we have done here is to use the standard Wiener-Hopf technique for an inhomogeneous equation. The standard analyticity arguments apply and we conclude that each side of (3.11) is a polynomial (of arbitrary degree as yet and with arbitrary coefficients). Setting the left-hand side of (3.11) equal to this polynomial and solving for $E(k)$, we have

$$E(k)=\left(\sum_{j=0}^{J} a_j k^j+h_1b^-(k)[ike(0)+e'(0)]\right)\Big/ f^-(k)\ . \tag{3.12}$$

To fix the unknown constants $e(0)$, $e'(0)$, $a_0,\ldots,a_J$, we use the theorem about the large-k behavior of Fourier transforms to write, at large k,

$$E(k)=[e(0)/ik]+e'(0)/(ik)^2+\ldots\ . \tag{3.13}$$

In Sec. IV, when we construct $b^-(k)$ and $f^-(k)$, we find that the large-k behavior of these functions is $f^-(k)\sim k$ and $b^-(k)\sim 1/k$. Hence, if we substitute (3.13) into (3.12), multiply by ik and pass to the large-k limit, we find that $a_j\equiv 0$ for $j>0$. We also find that

$$e(0)=[ia_0-h_1b_1e(0)]/f_1\ , \tag{3.14}$$

where

$$b_1\equiv\lim_{k\to\infty} kb^-(k)\ , \tag{3.15a}$$

$$f_1\equiv\lim_{k\to\infty} f^-(k)/k\ . \tag{3.15b}$$

Again, we substitute (3.13) into (3.12), divide (3.14) by ik and subtract this from (3.12) to obtain

$$\frac{e'(0)}{(ik)^2}+\ldots=\frac{h_1b^-(k)e'(0)}{f^-(k)} +a_0\left(\frac{1}{f^-(k)}-\frac{1}{kf_1}\right)+e(0)h_1\left(\frac{ikb^-(k)}{f^-(k)}+\frac{b_1}{ikf_1}\right)\ . \tag{3.16}$$

From (3.14) we have

$$a_0=-i(f_1+h_1b_1)e(0)\ . \tag{3.17}$$

After eliminating a_0 from (3.16), we multiply by $(ik)^2$ and, on taking the large-k limit, we have

$$(1+h_1b_1/f_1)e'(0)=i\mathfrak{F}e(0)\ , \tag{3.18}$$

where

$$\mathfrak{F}\equiv\lim_{k\to\infty} k^2\left[(f_1+h_1b_1)\left(\frac{1}{f^-(k)}-\frac{1}{kf_1}\right) -h_1\left(\frac{kb^-(k)}{f^-(k)}-\frac{b_1}{kf_1}\right)\right]. \tag{3.19}$$

This provides us with an expression for the dimensionless surface impedance, since

$$\frac{e'(0)}{ie(0)}=\frac{\mathfrak{F}}{1+h_1b_1/f_1}\ . \tag{3.20}$$

Furthermore, (2.3a), (3.17), and (3.18) provide us with the three conditions needed to fix the three constants in (3.12), and we obtain

$$E(k) = -ie(0)\{(f_1 + h_1 b) - h_1 b^-(k) \times [k + \mathfrak{F}/(1 + h_1 b_1/f_1)]\}/f^-(k) \quad (3.21)$$

with

$$e(0) = \frac{2k_0 l}{k_0 l + \mathfrak{F}/(1 + h_1 b_1/f_1)} \quad . \quad (3.22)$$

These expressions can be shown to agree, in the limit $A_1 = 0$, with those obtained by Reuter and Sondheimer. Note, however, that they certainly do *not* correspond to replacing the free-electron conductivity, which appears in Reuter and Sondheimer's final formulas, with the correlated conductivity. However, such a replacement *is* correct under conditions of specular reflection. Having obtained the transform $E(k)$, we have the field $e(x)$:

$$e(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(k) e^{ikx} dk \ . \quad (3.23)$$

Hence we have an explicit expression for the field and surface impedance, expressed, however, in terms of the still-to-be-calculated functions $f^-(k)$ and $b^-(k)$.

IV. PERFORMING WIENER-HOPF FACTORIZATION

From the definition (3.7) and (2.2), it follows that $Q(k)$ is an even function of k which goes as k^2 as k goes to infinity and which is analytic in the entire k plane, except for a pair of branch points at $k = \pm\beta$, where

$$\beta = ia = i + (\omega - \omega_c)\tau \ . \quad (4.1)$$

Because $Q(k)$ is even, its roots occur in pairs at $k = \pm k_n$, and if there are N of these roots in the upper half-plane, then[14]

$$Q(k_n) = 0 \ , \quad \mathrm{Im} k_n = 0 \ , \quad n = 1, \ldots, N \ . \quad (4.2)$$

These particular properties of Q ensure that the function $q(k)$, defined as

$$q(k) = \frac{Q(k)}{k^2 - \beta^2} \prod_{n=1}^{N} \left(\frac{k^2 - \beta^2}{k^2 - k_n^2} \right) , \quad (4.3)$$

can be factorized by applying the Cauchy integral theorem to $\ln q$:

$$\ln q(k) = \frac{1}{2\pi i} \oint \frac{dz \ln q(z)}{z - k} = \frac{1}{2\pi i} \int_{-\infty - i\mu}^{\infty - i\mu} \frac{dz \ln q(z)}{z - k} - \frac{1}{2\pi i} \int_{-\infty + i\mu}^{\infty + i\mu} \frac{dz \ln q(z)}{z - k} \quad (0 < \mu < 1)$$
$$\equiv S^+(k) - S^-(k) \ . \quad (4.4)$$

We combine (4.4) and (4.3) to give

$$Q(k) = \frac{\prod_n^N (k^2 - k_n^2) e^{S^+(k) - S^-(k)}}{(k^2 - \beta^2)^{N-1}} \quad ,$$

from which the factorization (3.8) gives

$$f^-(k) = \frac{\prod^N (k - k_n) e^{-S^-(k)}}{(k - \beta)^{N-1}} \quad , \quad (4.5a)$$

$$f^+(k) = \frac{(k + \beta)^{N-1} e^{-S^+(k)}}{\prod^N (k + k_n)} \quad . \quad (4.5b)$$

It is useful to note that the property $\ln q(z) = \ln q(-z)$ gives

$$f^+(k) = -1/f^-(-k) \ . \quad (4.6)$$

The transform $K(k)$ goes as $1/k$ as $k \to \infty$ and, since $f^+(k)$ also has $1/k$ behavior at infinity, we can again use Cauchy's theorem to write

$$if^+(k)K(k) = \frac{1}{2\pi i} \oint \frac{dz\, if^+(z)K(z)}{z - k} = \frac{1}{2\pi i} \int_{-\infty - i\mu}^{\infty - i\mu} \frac{dz\, if^+(z)K(z)}{z - k} - \frac{1}{2\pi i} \int_{-\infty + i\mu}^{\infty + i\mu} \frac{dz\, if^+(z)K(z)}{z - k}$$
$$\equiv b^+(k) - b^-(k) \ . \quad (4.7)$$

The function $b^{\pm}(k)$ defined here clearly satisfies the conditions for the decomposition (3.10).

The quantities b_1 and f_1 defined by (3.15) can be evaluated using (4.4), (4.5a), and (4.7):

$$b_1 = -\frac{1}{2\pi} \int_{-\infty + i\mu}^{\infty + i\mu} f^+(z) K(z) dz \ , \quad (4.8)$$

$$f_1 = 1 \ . \quad (4.9)$$

Finally, we consider the evaluation of $\mathfrak{F}$: At large k, we can expand (4.5a),

$$\frac{1}{f^-(k)} = \frac{1}{k} \left(1 + \frac{1}{k} \sum k_n - (N - 1)\frac{\beta}{k} + S^-(k) \right) . \quad (4.10)$$

Using (4.9) and (4.10) in (3.19) and using the definition (3.15) gives

$$\mathfrak{F} = \lim_{k \to \infty} \left(\sum_n k_n - (N-1)\beta + kS^-(k) + h_1 k [b_1 - kb^-(k)][1 + S^-(k)] \right).$$

It will turn out that at large k both $S^-(k)$ and $b_1 - kb^-(k)$ behave as $k^{-1} \ln k$ and so, on passing to the large-k limit, we have

$$\mathfrak{F} = \sum_n k_n - (N - 1)\beta + \lim_{k \to \infty} k \{ S^-(k) + h_1 [b_1 - kb^-(k)] \} \ . \quad (4.11)$$

In evaluating $\mathfrak{F}$, the terms in the braces must be kept together because they individually diverge. At this stage, the task of evaluating the field and surface impedance has been reduced to carrying out certain specified integrations.

V. CONVERSION OF VARIOUS INTEGRALS TO BRANCH-CUT INTEGRATIONS

Although the problem of calculating the electric field and surface impedance has been reduced to one of evaluating certain specific integrals in the complex k plane, the integrals themselves can be greatly simplified by taking advantage of the analytic properties of the integrands. We convert the contour integrations defined in (4.4), (4.7), (4.8), and (3.23) to integrations of the discontinuity of the integrand along the branch cut. Examples follow.

Consider (4.7). We need $b^-(k)$ for $\mathrm{Im}k < \mu$. Since $f^+(z)$ is analytic in the upper half-plane, we sweep the contour upwards to surround the branch cut along which $K(z)$ is discontinuous. That cut runs from $z=\beta$ to $z=\infty$. There are no other singularities of the integrand. Denoting the values of z along the cut as

$$z_c=\beta+te^{i\varphi} \tag{5.1a}$$

$$=\beta+\theta t \quad 0\leq t<\infty , \tag{5.1b}$$

where φ specifies how we want the cut to go off to infinity, we let $K^\pm(z_c)$ denote values of $K(z)$ along the incoming (K^-) and outgoing (K^+) sides of the cut. The discontinuity in K,

$$\Delta K(z_c)\equiv K^+(z_c)-K^-(z_c) , \tag{5.2}$$

goes to zero at the branch point $z=\beta$. Thus, (4.7) becomes

$$b^-(k)=\frac{1}{2\pi i}\int_0^\infty \theta\, dt\, \frac{f^+(z_c)\,\Delta K(z_c)}{z_c-k} . \tag{5.3}$$

Similarly, in the evaluation of $S^-(k)$ in (4.4), the singularity in the integrand is

$$\Delta \ln q(z_c)\equiv \ln q^+(z_c)-\ln q^-(z_c)=\ln[q^+(z_c)/q^-(z_c)] .$$

Since the only factor in $q(z)$ which is discontinuous is $Q(z)$, we have

$$S^-(k)=\frac{1}{2\pi i}\int_0^\infty \theta\, dt\, \frac{\ln G(z_c)}{z_c-k} , \tag{5.4a}$$

where

$$G(z_c)\equiv Q^+(z_c)/Q^-(z_c) . \tag{5.4b}$$

Here again, Q^+ and Q^- refer to values of Q on the outgoing and incoming sides of the cut.

Finally, let us consider the inversion of the transform as in (3.23). It is clear that $E(k)$ has both poles (at $k=k_n$) and a branch-cut singularity in the upper half-plane. We sweep the k integration contour upwards to surround the branch cut and, in doing so, we pick up contributions at the poles. These pole contributions have a spatial dependence e^{ik_nx} and, because k_n^{-1} is of the order of anomalous skin depth, these pole contributions can be ignored completely at depths x which are well beyond the skin depth. For the branch-cut contribution, it is useful to denote the boundary values of $b^-(k)$ at the cut as $B^\pm(k_c)$ along the outgoing (+) and incoming (−) sides of the cut, i.e.,

$$[b^-(k_c)]^\pm\equiv B^\pm(k_c) , \tag{5.5a}$$

$$k_c=\beta+\theta t . \tag{5.5b}$$

Also, we denote the boundary values of $1/f^-(k)$ along the cut as

$$[1/f^-(k_c)]^\pm = T(k_c)e^{\pm i\varphi(k_c)} . \tag{5.6}$$

Since we wish to calculate the microwave field well beyond the skin depth, the branch-cut contribution is all that we shall need. Thus combining (3.21) and (3.22) and making use of the notation (5.5) and (5.6), we have

$$e(x)=\frac{e(0)}{2\pi i}\int_0^\infty \theta\, dt\, T(k_c)e^{ik_cx}\left[(1+h_1b_1)(e^{i\varphi(k_c)}-e^{-i\varphi(k_c)})-h_1\left(k_c+\frac{\mathfrak{F}}{1+h_1b_1}\right)[B^+(k_c)e^{i\varphi(k_c)}-B^-(k_c)e^{-i\varphi(k_c)}]\right]. \tag{5.7}$$

VI. DISCUSSION

The exact evaluation of the integrals we have derived here is quite tedious, even if one works to lowest order in $b^{-1/3}$. This parameter is a tiny number, being essentially the ratio of the anomalous skin depth to the mean free path. In the experiments of PBD, this number was typically 10^{-3}. If, in these same experiments, A_1 were as large as 0.15, which is Rice's estimate,[7] then $h_1=\omega\tau A_1/(1+A_1)$ would be approximately 50.

The reader who wishes to work through the form of the various integrands will undoubtedly find it useful to have Ref. 11 at hand. One can consistently work to order $b^{-1/3}$ and then can consistently drop terms of order $h_1/b^{-1/3}=0.05$ compared to unity. When one does this, one finds that A_1 almost drops completely out of (5.7) which, to this order, becomes

$$e(x)=\frac{e_0(0)}{\pi}\int_0^\infty \theta\, dt\, T_0(k_c)e^{ik_cx}\Big(\sin\varphi_0(k_c) - \frac{1}{2i}\, h_1(k_c+\mathfrak{F}_0)[B_0^+(k_c)-B_0^-(k_c)]\cos\varphi_0(k_c)\Big) . \tag{6.1}$$

A subscript zero means that the quantity is to be

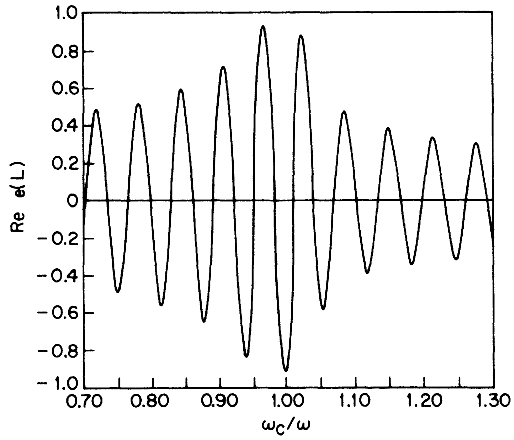

FIG. 1. Real part of the microwave field at a depth $L = 0.3\times$ (mean free path) within a semi-infinite gas of interacting electrons vs the strength of the magnetic field normal to the surface, where $\omega\tau = 300$, $A_1 = 0.15$, and $b = 0.95\times 10^{10}$.

calculated as though A_1 were zero. The relative size of the second (the A_1-dependent) term to the first (the A_1-independent term) is $h_1/(\beta b^{1/3})^{1/2}$.

There are several interesting points to be made here. The first is that the relative change in surface impedance is of order $h_1/\beta b^{1/3}$, a much smaller quantity. Thus, in the search for an electromagnetic effect caused by A_1, the cyclotron-phase-resonance experiments are much more likely candidates than are the surface-impedance observations in the same geometry. The same comment also applies to those experiments in which an A_1-induced change in the infinite-medium dispersion relation is sought[8]: The cyclotron-phase-transmission experiments are going to be more sensitive to A_1.

The second interesting point to be made here is that (6.1) is exactly the same form that one would have obtained by treating h_1 as a first-order perturbation in (2.1), that is, by solving (2.1) as though h_1 were zero, using the resulting zero-order ψ_1 to evaluate the term proportional to h_1 in (2.1a), and then using the Green's function for the coupled zero-order equations (2.1a) and (2.1b)[15] to evaluate the change in field. The significance of this observation is that the shift in the position of the resonant peak in the nonlocal conductivity arises from a resonant term in the denominator, a term of the form $[(\omega_c-\omega)\tau + h_1]^{-1}$. Such a form requires that the first-order term in a perturbation expansion be $h_1/(\omega-\omega_c)\tau$, which becomes infinite at cyclotron phase resonance, rather than the much smaller term $h_1/(\beta b^{1/3})^{1/2}$ which we have found. Hence there is no way in which the expression (6.1) can exhibit an amplitude maximum at the magnetic field value $\omega_c/\omega = (1+A_1)^{-1}$. The only effect that A_1 has here is to augment the amplitude on one side of $\omega_c/\omega = 1$ and depress it on the other side. This does shift the field at which maximum occurs, but the shift is far less than that required to put the maximum at $\omega_c/\omega = (1+A_1)^{-1}$. Which side is augmented and which is depressed depends on the sign of A_1. In Fig. 1 we have evaluated (6.1) for the values of the parameters indicated, and it is clear that a positive value of A_1 augments the amplitude on the low-field side. This does indeed shift the weight of the curve in the same direction as would the formula $\omega_c/\omega = (1+A_1)^{-1}$, but it is quite clear that there is no $(1+A_1)^{-1}$ dependence of the peak position.

There is a third interesting point raised here, and that concerns the validity of the heuristic model used to interpret the data. The heuristic model was based on the three following rather simple ideas.

(a) The effect of Maxwell's equations is to confine the microwave field to a narrow skin-depth region at the incident face of the slab.

(b) The current in the slab is given by

$$j(x) = \int_0^L \sigma(x, x')\, e(x')\, dx' ,$$

where σ is the nonlocal conductivity. If idea (a) is valid, then

$$j(x) \sim \sigma(x, x'=0) .$$

(c) The field which one observes is proportional to the current at the emergent face of the slab, or at best arises from currents which are within an anomalous skin depth of that face.

These three ideas, taken together, yield a transmission amplitude which is proportional to $\sigma(L, 0)$. This model has been very successful in accounting for the existence and field dependence of the Gantmakher-Kaner oscillations,[16] as well as a wide variety of more detailed effects observed in single-particle-transmission experiments.[6,17–20] However, the logic of this model would also imply that the field at a depth L from the incident surface of a semi-infinite medium would be $\sigma(L, 0)$, because the emergent surface plays no role in the model except to define the position of the plane on which to evaluate the current. Yet our calculation here shows almost no resemblance at all between the field at a depth L in the semi-infinite medium and the nonlocal conductivity. This is evident in Fig. 2, where both quantities have been calculated and exhibited for $A_1 = 0$, i.e., where there are no correlation effects to confuse the issue. The peak in the conductivity [Fig. 2(b)] is rather well defined, while the amplitude of the field exhibits only a gentle maximum at $\omega_c = \omega$.

To return to the question we posed at the outset of the present work, we can surmise that a calcula-

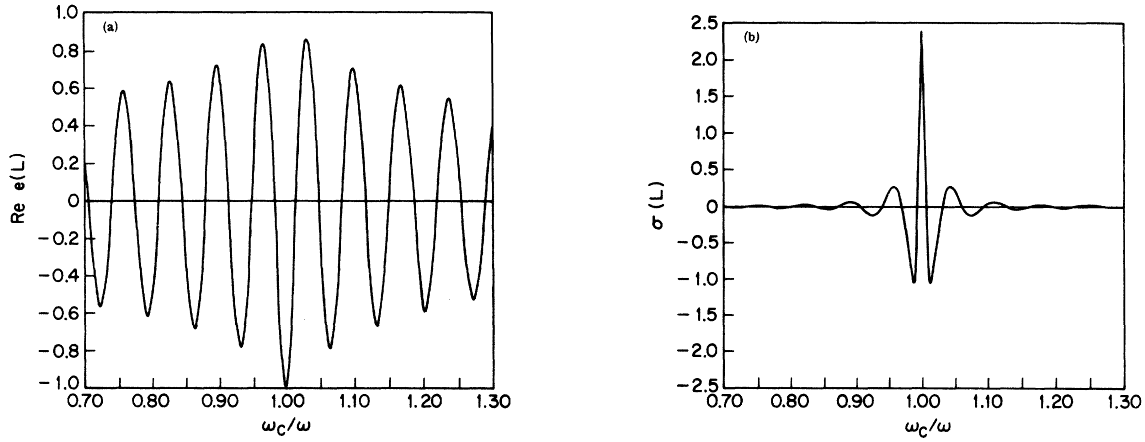

FIG. 2. (a) Real part of the microwave field at a depth $L = 0.3 \times$ (mean free path) within a semi-infinite gas of free electrons vs the strength of the magnetic field normal to the surface. (b) Real part of the nonlocal conductivity of the free-electron gas evaluated as a function of magnetic field for a distance L between the field plane and the current plane.

tion of the transmission through a thin two-sided slab is not likely to yield a result looking like the conductivity: The conductivity has a peak at $\omega_c/\omega = (1+A_1)^{-1}$ and it is exceedingly likely that the transmission through the slab, like the field in the semi-infinite medium, will have its maximum close to $\omega_c/\omega = 1$. The remarkable resemblance between the transmission measured by PBD and the conductivity they calculated is, at this stage, still not explained.

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