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Measurements of the Shear Modulus of the Superconducting Mixed State of Thin Films

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Existence of a shear modulus for the vortex-lattice structure in the superconducting mixed state of several oxygen-doped aluminum films is deduced from observations of the quantum-interference effect. From a detailed study of the widths of the interference transition, it is shown that at low magnetic fields the shear modulus c_{66} is in agreement with a recent theoretical consideration of the thin-film case. In addition, it is found that c_{66} vanishes at H_{c2} in accordance with a theoretical result for the bulk case. Enhanced influence of the pinning effect upon local lattice parameter and velocity fluctuations is observed at small transport velocities, in qualitative agreement with the recent theory of Schmid and Hauger.

I. INTRODUCTION

The elastic properties of the vortex lattice in a thin film, for perpendicularly applied magnetic fields, has an interesting history of contradictory opinion in the theory literature.[1-3] We are concerned with the thin-film limit, where the thickness d is much smaller than the bulk penetration depth λ. This defines a regime where the length that characterizes the spatial variation of the local magnetic induction and supercurrents becomes an effective penetration depth $\Lambda = 2\lambda^2/d$. A distinctive feature of the film is that the vortex-vortex interaction is long-range, falling off as r^{-1} at large distances,[1] unlike the finite range in the bulk superconductor. As we conclude below, a modulus of compression for the vortex lattice may very well not exist, but the shear modulus remains finite.

II. THEORY

Finding that the free energy of the vortex structure in a film is insensitive to its structure, yet sensitive to its density, Pearl concluded that the shear modulus vanishes and that the modulus of compressibility must be very high.[1] The large cost in energy for density-changing distortions is a consequence of the large demagnetization effect for a thin film in a perpendicular field. The work of Fetter and Hohenberg demonstrated the stability of the triangular lattice for their special model of vortex dynamics that neglected damping.[2] Since the dispersion relations they obtained were not analytic at long wavelength, being of the form $\omega \propto q^{3/2}$, where ω is the angular frequency and q the magnitude of the wave vector for modes traveling in the plane of the film, they stated that an elastic theory is inapplicable.

Recently, Schmid pointed out that one should not expect unusual behavior of the shear modulus, even if the interaction is long range.[4] His argument is based upon the independence of the long-range contribution to the lattice energy with respect to changes in vortex configuration where density is conserved. He has also pointed out that in their comparison of vortex and Newtonian dynamics, Fetter and Hohenberg had essentially derived expressions for the lattice restoring force in terms of parameters which are functions of the wave vector of the lattice distortion. Consequently, their results at small q (q is the magnitude of the wave vector of the distortion) for the restoring force per unit displacement per vortex, which we denote by D_l and D_t for longitudinal and transverse (or shear) deformations, respectively, can be expressed as the following[5]:

$$D_l = (B\varphi_0/2\pi d)\,q, \tag{1}$$

$$D_t = \tfrac{1}{4}(\varphi_0^2/16\pi^2\lambda^2)\,q^2 \qquad (b \ll \Lambda), \tag{2}$$

$$D_t = \tfrac{1}{4}(\varphi_0^2/16\pi^2\lambda^2)(\Lambda/b)\,q^2 \qquad (b \gg \Lambda), \tag{3}$$

where b is a measure of the lattice constant $b=(\pi\varphi_0/B)^{1/2}$, and $B \ll H_{c2}$. In a medium that macroscopically obeys Hooke's law, the force on a constituent particle is proportional to the second derivative of the displacement–leading to $D \propto q^2$. Thus, a modulus of compressibility, as it is usually defined, does not exist, whereas, on the basis of the quadratic power law, a shear modulus does exist. The coefficient of q^2 is denoted by K_t, which is related to the shear modulus c_{66} by

$$K_t = \varphi_0 c_{66}/B. \tag{4}$$

In terms of experimentally measurable quantities, the shear constant may also be written

$$K_t = \tfrac{1}{8}(\varphi_0 H_{c2}/4\pi\kappa^2) \qquad (b \ll \Lambda), \tag{5}$$

$$K_t = \tfrac{1}{8}(\varphi_0 H_{c2}/4\pi\kappa^2)\,\Lambda/b \qquad (b \gg \Lambda), \tag{6}$$

where κ is the Ginzburg-Landau parameter.

In comparing the above results with the bulk theory, one finds that Eq. (5) is the same as the bulk-theory result.[6,7] At high magnetic field, the only theory available is for the bulk, which was worked out by Labusch, with the result[8]

$$c_{66}=\frac{0.24(2\kappa^2-1)}{4\pi[1+(2\kappa^2-1)\beta]^2}(H_{c2}-B)^2. \qquad (7)$$

It was shown in a previous paper that the interference effect method can be applied in a study of the properties of the vortex structure of the mixed state.[9] A theory of the interference effect is detailed in a recent paper by Schmid and Hauger.[10] Dissipative excitation of dynamic fluctuations of the vortex lattice as it is driven through an inhomogeneous superconductor causes interference phenomena that couple rf and dc motions and currents. As a key ingredient of the phenomenological equation of motion, the lattice restoring force for shear deformations D_t is taken to be of the form $K_t q^2$ at small q. Excitation of the other modes of deformation require much larger forces and need not be considered. A mathematical consequence of the quadratic power law is the prediction of steplike transitions in the rf voltage as a function of the dc voltage. We will be concerned with the width of the fundamental, or $n=1$, transition for weak applied rf currents, which follows from the general expression given by the theory

$$\delta x_n \equiv \frac{\Delta E_{n1}}{E_{n1}}=\frac{\varphi_0^2 f z_n}{2\sqrt{3}\pi n^3 c^2 \rho_f K_t}, \qquad (8)$$

where f is the frequency of the rf current, z_n is a constant of order unity, ρ_f is the resistivity in the mixed state, and E_{n1} is the dc electric field at the nth transition point

$$E_{n1}=(f/nc)(2\varphi_0 B/\sqrt{3})^{1/2}. \qquad (9)$$

Interpretation of these formulas are discussed in Sec. III.

It was also shown that at small dc velocity, local-velocity, and lattice-parameter fluctuations can make an important contribution to the observed transition width. This new effect had not been considered in the earlier work of either Schmid or this author. Its estimated contribution is given as

$$\delta x_n \approx (2-3)n\epsilon\rho_f J_p/E_{11}, \qquad (10)$$

where ϵ is on the order of unity and J_p is the pinning current.

Therefore, the theoretical prediction is that at large velocities and frequencies the widths are proportional to f and Eq. (8) applies, while at small velocities and frequencies the widths are proportional to f^{-1} and Eq. (10) applies. Between those limits, a combination of the two effects is expected. In Sec. IV, after demonstrating the existence of the two limiting regions, data taken at large velocity are analyzed to find the shear constant K_t.

III. SPECIMENS AND PROCEDURE

Two films were prepared by evaporating pure aluminum from an alumina-coated dimpled boat. Fire-polished Corning 7059 slides were placed 15 cm above the boat, and held nominally at room temperature. Specific details such as the oxygen pressure, deposition rate, and parameters of the films's low-temperature properties are given in Table I.

The granular aluminum films so obtained have the unusual property of appearing to be nearly homogeneous as far as the vortices are concerned. For illustration, note that for our 110-Å film, the mean grain size of 62 Å is much smaller than the temperature-dependent coherence distance, which varies between 1000 and 2000 Å over the range where data were taken. Consequently, the so-called pinning forces associated with structural inhomogeneity are quite small and coherent motion of the vortex lattice is possible.

Magnetic-field-induced resistance transitions were made after the edges of the specimen were defined by scribing with a sharpened needle. x-y recordings of the detected (410-Hz) ac voltage as a function of a perpendicularly applied swept magnetic field show a trace whose shape is independent of the current density for $J \lesssim 20$ A/cm^2. The transition curves have a rather large curvature located where the resistance attains about 90% of its normal value, and this maximum curvature position has been taken as the value of the upper critical field H_{c2}. In a few cases a reproducible sharp step was observed at H_{c2} when H was swept downward from the normal state. By making plots of H_{c2} vs T, a linear extrapolation to zero field was used to find T_c, and the slope $-dH_{c2}/dT$. Our values of the normal resistivity ρ_n and T_c fall on a curve of ρ_n vs T_c, which was plotted by Deutscher *et al.*[11]

Knowledge of the $\kappa(T_c)$ for our films is an es-

TABLE I. Parameters characterizing the preparation and low-temperature properties of two oxygen-doped aluminum films.

d (Å)	110	200
P_{O_2} (μ torr)	100	60
Rate (Å/sec)	1.6	1.3
$-dH_{c2}/dT$ (Oe/K)	632	550
ρ_n ($\mu\Omega$ cm)	11.5	6.7
T_c (K)	1.75	1.53
l (Å)	38	44
κ (T_c)[a]	3.1, 2.7, 3.2	2.7, 2.5, 1.9
Grain size[b] (Å)	62	93

[a]According to methods a, b, and c, respectively, as given in Sec. III of text.

[b]Estimated mean grain size, as taken from Fig. 8 of Ref. 11.

sential part of quantitative comparison of experiment with the theory. Its values were ascertained according to the following three methods.

(a) An effective mean free path l is derived from the H_{c2} measurements using the following relations:

$$H_{c2}=\varphi_0/2\pi\xi^2(T) \quad (11)$$

and

$$\xi^2(T)=0.72\xi_0 l(1-T/T_c)^{-1}, \quad (12)$$

where ξ_0 is taken as the coherence distance for pure aluminum (16 000 Å) multiplied by the critical temperature for pure aluminum (1.19 K) divided by the T_c of the film. Lastly, using the pure aluminum value of 157 Å for the zero-temperature London penetration depth $\lambda_L(0)$, κ is determined from

$$\kappa(T_c)=0.75\lambda_L(0)/l. \quad (13)$$

(b) The film's $H_c(0)$ is extrapolated by multiplying the pure aluminum value of 99 Oe by $T_c/1.19$ K. With a parabolic fit to the temperature dependence of $H_c(T)$, one has

$$\kappa(T_c)=\frac{-dH_{c2}/dT\,|_{T_c}}{2\sqrt{2}\,H_c(0)}. \quad (14)$$

(c) The Gorkov relation defines κ in terms of the normal resistivity[12]

$$\kappa(T_c)=\kappa_0+7.5\times10^3\rho_n\gamma^{1/2}, \quad (15)$$

with $\kappa_0=0.010$ and $\gamma=1.36\times10^3$ erg/cm^{3}K^2 for pure aluminum.

As can be seen from the results, which are listed in Table I, these three methods do not always agree with each other. For the purposes of calculating theoretical values for the shear constant, we have arbitrarily selected method a, prejudiced by the fact that the best over-all agreement with the theory is obtained. For impure superconductors, the distinction between κ_1 and κ_2 need not be accounted for, since $\kappa_1\approx\kappa_2$.[13] We therefore drop use of a subscript elsewhere in this paper. For its temperature dependence, κ is assumed to be proportional to the ratio of H_{c2} and the parabolic function $[1-(T/T_c)^2]$, which fits $H_c(T)$.

Interference-effect measurements entail applying a fixed, small rf current and a swept dc current in a fixed perpendicular magnetic field and at a fixed temperature. Details of the apparatus have been given before.[9] Since the in-phase component of the rf voltage is required, the path length of the reference leg to the rf mixer is adjusted for zero time delay using a 10-nsec pulse and a fast oscilloscope. For rf frequencies below about 50 MHz, the phase shifts in the circuit have negligible effect, and the qualitative shape of the transition steps is frequency independent. When higher rf frequencies are used, the circuit phase shifts are cancelled by readjusting the reference path length. For most of the data, recordings of the derivative of the detected rf voltage, obtained by a small modulation of the dc current versus the dc-electric-field display peaks which are nearly symmetric in the vicinity of the transition field and which have asymptotes approaching a common base line both below and above the transition. These features found at the lower frequencies, where the phase is accurately known, were used as criteria for a correct setting of the phase at the higher frequencies. Phase corrections were unnecessary for measurements made at the lowest magnetic fields, where the lower frequencies were employed.

Measurements of the shape of the transition peak were made relative to the base line, which incidentally is not at true zero–the idealization of the theory. Our procedure for obtaining experimental values for the width δx_1 consists of taking the ratio of the full width at the relative half-maximum to the transition field of the $n=1$ peak. Since a Lorentzian seemed to give a good fit in most cases, we have taken $z_1=1$ in the analysis. At low magnetic field, where weak signals are incurred, a larger modulating current was used to improve the signal to noise ratio. Here, a small correction ($\lesssim 15\%$) was applied in computing δx_1.

There are a number of sources of uncertainty in this experiment. At low magnetic fields, random noise on the order of 20% of the relative peak height, for a measurement interval of 20 min, is fairly typical. Errors of judgment enter at low magnetic field because the base line for measuring the peak width departs from a horizontal line. At high fields, the resistivity depends upon the measuring current as the dc I-V characteristic shows noticeable curvature. For all our measurements we have taken ρ_f to be given by the tangent to the I-V curve at the transition point. In some cases, one sees an extra transition peak associated with the other high symmetry orientation of the moving vortex lattice (a rotation through 30°). Here, a value for ΔE_{11} is adduced from a Lorentzian fit, by taking $\sqrt{2}$ times the full width at two-thirds relative maximum, a procedure that avoids overlap effects.

Transition curves were made for various values of the independently variable quantities H, T, and f. For the 110-Å film, 200 curves were recorded, and for the 200-Å film, 75 were recorded. With the latter, only the region $B\ll H_{c2}$ was investigated in any detail.

IV. EXPERIMENTAL RESULTS

As a preliminary test, we verify the prediction that dynamic fluctuation effects influence the widths at low velocities, or equivalently, at low rf fre-

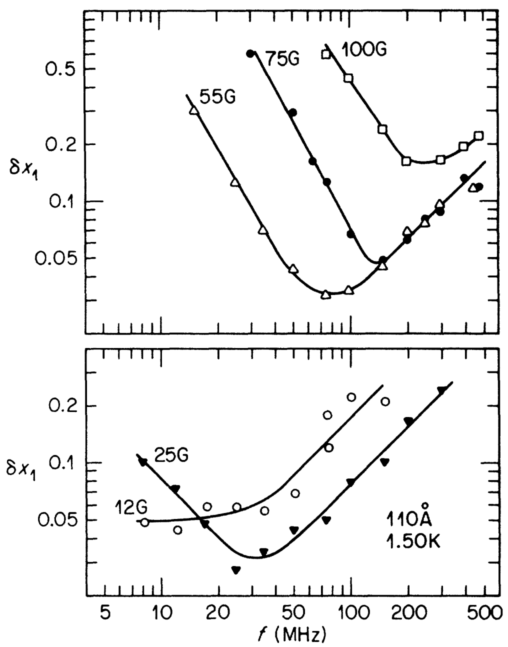

FIG. 1. Measured widths of the $n=1$ interference transition for five applied magnetic field values plotted as a function of the frequency of the rf current. Data for the two lowest fields are plotted separately for clarity.

quencies. In Fig. 1 a plot of our measurements of δx_1 as a function of f is shown for several applied transverse fields. One clearly has two branches, or regions, where δx_1 is either an increasing or decreasing function of f. The right-hand branch, for which δx_1 is linear in f, is characterized also by an $n=1$ transition which is much broader than the $n>1$ transitions. These features are in agreement with the theory [Eq. (8)]. Quantitative agreement of the low-frequency branch with Eq. (10) is not found. The especially rapid dependence of the magnitude of δx_1 upon magnetic field was not expected. Moreover, the predicted magnitudes are as much as a factor of 10 too large, based upon a substitution of measured values of J_p into Eq. (10). For the remaining part of this paper, data which were analyzed according to Eq. (8) to obtain K_t fall on the high-frequency branch.

The two extreme regions of the mixed state $B \ll H_{c2}$ and $B \lesssim H_{c2}$ are discussed separately in what follows. In the low-field region, the lowest field at which data could be taken is limited, in principle, by the facts that the transition width increases with decreasing B and that obviously one must always have $\delta x_1 < 1$ in order to observe a transition. Although one can take advantage of the fact that the widths decrease with decreasing frequency, there is a lower-frequency limit, also, dictated by the condition that small velocities be avoided. Hence, in practical terms, we find that the dc current should obey the inequality

$$J - J_p > J_p \tag{16}$$

at the transition point. According to the low-field dc resistance measurements on our films, we have $J_p \propto B^{-1/2}$ and $\rho_f \propto B$. Thus, from Eq. (9) it follows that the lowest frequency is actually independent of B, being on the order of 10 MHz at 1.5 K. From Eqs. (5) and (8), the corresponding minimum B is 10^{-1} G. However, the minimum B during the course of these measurements was actually limited to 1 G owing to a lack of definition in the base line and therefore in the measurement of the width.

As a consequence of the above results, we find that the very-low-field region given by $b \gg \Lambda$, for which we expect $c_{66} \propto B^{1/2}$ [cf. Eq. (6)], is not accessible to the present experiments on very thin films. That is, the crossover point between "low" and "very-low" fields, where $b \sim \Lambda$ corresponds to $B \sim 10^{-3}$ G at 1.5 K, which is much smaller than the practical lower limit.

Data taken at low magnetic fields were analyzed according to Eq. (8), and then K_t was plotted as a function of B. Since there was a small amount of B dependence, the data were extrapolated to find the $B=0$ intercept, following the interpretation that Eq. (5) applies, where K_t is expected to be independent of B at low fields. For the 110-Å film, zero-field intercepts are plotted in Fig. 2 against the reduced temperature function $(1 - T/T_c)$. Each point of Fig. 2 originates from a plot of K_t vs B having between 5 and 10 data points. Flags on the data points enclose an estimated 90% confidence interval, assuming the original data points are a sample from a normal distribution. For comparison with the theory, values for K_t were computed

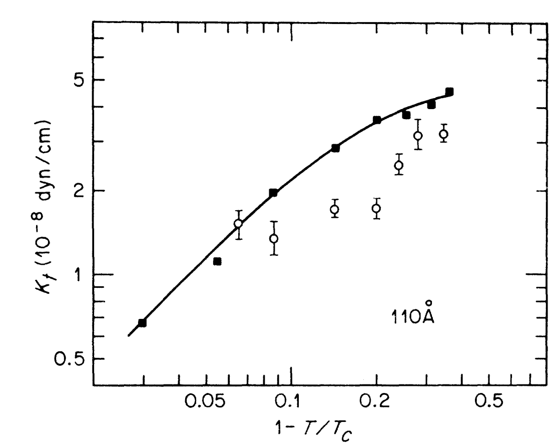

FIG. 2. Measured (open circles) and calculated (squares and the smooth curve fit) values of the shear constant plotted against the reduced temperature function.

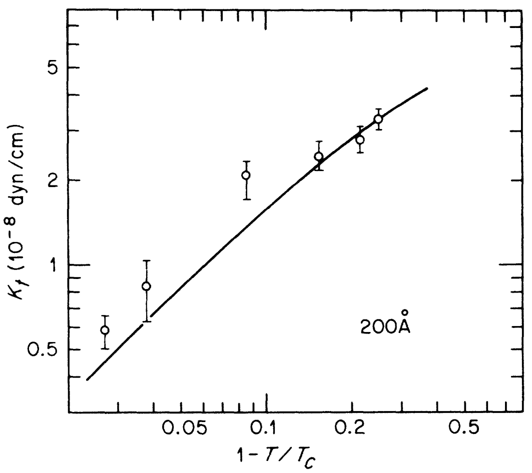

FIG. 3. Measured (open circles) and calculated (curve) shear constant vs reduced temperature function.

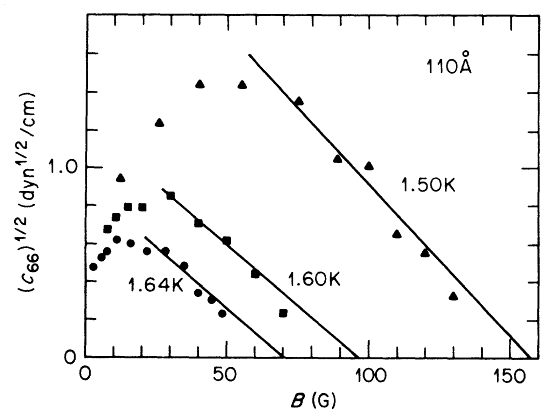

FIG. 4. Square root of the shear modulus and a high field linear fit vs applied magnetic field.

from Eq. (5) using critical-field data and our calculated temperature-dependent κ. A smooth curve is drawn through the theory points, closed squares, in Fig. 2. The above procedures were repeated for the 200-Å film, and those results are plotted in Fig. 3. Agreement with the theoretically expected temperature dependence is judged to be satisfactory. As indicated by our earlier discussion, the theory curves are subject to appreciable systematic error from uncertainty in knowledge of $\kappa(T_c)$, although the temperature dependence should be correct. The origin of the systematic deviation of the middle points of Fig. 2 is unknown.

It has been conjectured by Horn and Parks that the driving force for vortex motion in thin films could be substantially lower than it is in the bulk, because ρ_f for a thin film is generally much smaller than the theoretical value of $\rho_n B/H_{c2}(0)$.[14] Our results for K_t are obtained from a theory which makes explicit use of the driving force. Since we used empirical values of ρ_f in the foregoing analysis, we would conclude that our results do not support their hypothesis.

While collecting data on the resistivity, measurements of the pinning current J_p were also obtained. At low B, J_p varies as $B^{-1/2}$, and just below H_{c2} it has a maximum related to the "peak effect."[15] This behavior is understandable if one considers the theoretical expression[10] for J_p which contains a prefactor K_t^{-1}, and the theoretical expression for the shear modulus, which vanishes at H_{c2}.[8] Thus, as the shear modulus goes to zero, larger local fluctuations of the vortex lattice are expected, and these in turn would enhance J_p.[16]

For our 110-Å film at temperatures above 1.3 K, the resistance transitions do not show any peak-effect structure near H_{c2}. With a smaller grain size, the pinning forces are correspondingly smaller than for the 200-Å film. It therefore was possible to take data for K_t in high magnetic field and at high temperatures. These were converted to $c_{66}^{1/2}$ using Eq. (4). Results for several high temperatures are plotted in Fig. 4. The linear fits near H_{c2} are drawn through the critical-field points determined from the resistance transitions. Although the Labusch expression (7) would not necessarily apply to a thin film, the magnetic field dependence is in agreement with that theory. However, the magnitudes of the slopes do not, as evident from the comparisons in Table II. Again, there is probably agreement to within the uncertainty in our knowledge of κ. Also included in Fig. 4 are low-field points where c_{66} becomes linear in B.

In this paper we have used the dynamic quantum interference effect technique to indirectly measure essentially a static property, the shear constants of the vortex lattice in a thin film. Recent theoretical considerations by Schmid and Hauger have shown that one must take care in considering measurements made at low velocity, where the fluctuations themselves can cause an anomalous increase in the transition widths. Insofar as the theory accounts for these effects, it indicates that

TABLE II. Comparison of experimental slopes, from Fig. 4, and theoretical slopes, from Eq. (7), for $(c_{66})^{1/2}$ near H_{c2}. 110-Å film.

T (K)	$-\left.\frac{d(c_{66})^{1/2}}{dB}\right\|_{H_{c2}}$ Experiment	Theory
1.50	0.016	0.026
1.60	0.013	0.027
1.64	0.013	0.027

other broadening mechanisms can lead to an underestimate of K_t or c_{66}. It is believed the criterion that the results for K_t should be independent of the measurement frequency has been largely met for our data. Qualitatively, the very existence of steplike transitions is interpreted as a confirmation of the existence of a shear elastic modulus in thin films as pointed out by Schmid.[4] (Our experiments are not sensitive to compressional modes.) Moreover, quantitative results for the shear modulus are in satisfactory agreement with the theory for a thin film at low fields,[2,4] where $c_{66} \propto B$ [Eqs. (4) and (5)], and with a theory for the bulk at high fields,[8] where $c_{66} \propto (H_{c2} - B)^2$ [Eq. (7)].

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