## Comment on "Model for an Exciton Mechanism of Superconductivity"-A Reply\*

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The model considered by Inkson and Anderson does not correspond to that suggested by Allender, Bray, and Bardeen to exhibit excitonic superconductivity at a metal-semiconductor interface. It is necessary for the semiconductor to have peaks in  $\epsilon_2(\vec{q}, \omega)$  at small  $\omega$  for nearly all  $\vec{q}$  rather than only near  $\vec{q} = 0$ .

The authors' main point is that their model<sup>1</sup> and that of Inkson and Anderson<sup>2</sup> describe semiconductors with different physical properties. We agree with the calculations that Inkson and Anderson have made within their model. Their model may be good for many semiconductors and emphasizes the difficulty of finding a suitable material to exhibit an exciton mechanism, but does not rule it out. In their original paper,<sup>1</sup> the authors suggested criteria for finding a suitable material.

In our model, the "exciton" is a polarization of a covalent or resonance bond in a semiconductor; i.e., in coordinate space, an electron is excited from the bonding region of high electron density to the adjacent nonbonding region of lower density. Since the spatial distances involved are quite small  $(\sim 2 \text{ Å})$ , the dominant wave vectors  $\vec{q}$  are large; i.e., umklapp processes are of paramount importance. The metal's electron pairs at the Fermi surface interact by exchanging these large-q virtual "excitons," or really, virtual electron-hole pairs.

A dielectric function describing such a material should have large peaks in  $\epsilon_2(\mathbf{q}, \omega)$  at a frequency  $\omega = \omega_g$ , nominally the average gap frequency, corresponding to the polarization of the covalent bonds. These peaks should be at low frequencies  $\omega \sim \omega_{\pi}$  for nearly all values of  $\overline{q}$ . Further, there should be a large oscillator strength for production of electronhole pairs at  $\omega \sim \omega_g$  for all  $\vec{q}$ . The dielectric function given by Inkson and Anderson in their Eq. (4) does not meet these requirements. It has an exciton peak at  $\omega = \omega_g$  only for  $\mathbf{q} \simeq 0$ . For larger  $\mathbf{q}$ , the peak moves close to  $\omega_{\nu}$ , the plasmon frequency. This has the effect of severely restricting the phase space available for the virtual exchange of excitons with low energies  $(\omega \simeq \omega_{e})$ , the very exchange which is necessary to achieve large values of  $\lambda_{ex}.~$  We note that Inkson and Anderson's model is consistent with the empirical evidence that large values of  $\omega_p^2/\omega_g^2$  (i.e., large  $\lambda_{ex}$ ) imply small local-field effects. Our model, on the other hand, requires both large values of  $\omega_{b}^{2}/\omega_{e}^{2}$  and large local-field

effects.

In our model the production of electron-hole pairs in a metal does not lead to the same electron-exciton interaction as in a semiconductor. In a metal, the frequency of the electron-hole peaks in  $\epsilon_2(\mathbf{q}, \omega)$ increases as the momentum transfer  $\vec{q}$  increases. As already discussed, the electron-hole peaks in  $\epsilon_2(\bar{q},\omega)$  for covalently bonded semiconductors of the type we considered are not far from  $\omega = \omega_{\mathbf{F}}$  for all  $\vec{q}$ . Thus in a metal, the oscillator strength for production of electron-hole pairs, viewed as an average over  $\vec{q}$ , is not peaked in  $\omega$ , in contrast with the situation for a covalent semiconductor. This fact would, for a metal, render impossible the assumption that the exciton energy  $\omega_{\rm NO}$  can be replaced by an average value  $\omega_r$  in our derivation<sup>1</sup> of  $\lambda_{ex}$ .

In summary of our model, the semiconductor is considered, in first approximation, a metal with an electron density equal to that of the valence electrons and with the corresponding screening. Band gaps are then introduced in the vicinity of the Fermi surface to form a covalent or resonancebonded semiconductor, with the electron density increased in the bond directions. It is the effect of the virtual electron-hole pairs produced at these bonds by the electrons tunneling from the metal that is treated in second-order perturbation theory with an appropriately screened interaction, just as in the electron-phonon system. The difference between our results and those of Inkson and Anderson does not arise from any double-counting of exciton processes but from the difference in models.

Miller and co-workers<sup>3</sup> chose PbTe as a promising material to exhibit excitonic effects because of its band structure and the optical-absorption spectrum. According to Lucovsky and White,<sup>4</sup> this and related materials are examples of resonance bonding with large lattice and electronic polarizability. It would be desirable to calculate the dielectric function, its reciprocal, and also localfield effects for PbTe or a similar material.

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- <sup>2</sup>J. C. Inkson and P. W. Anderson, preceding paper, Phys. Rev. B 8, 4429 (1973).
- <sup>3</sup> D. L. Miller, Myron Strongin, O. F. Kammerer, and B. G. Streetman (report of work prior to publication).
- <sup>4</sup>G. Lukovsky and R. M. White, Phys. Rev. B 8, 660 (1973).

<sup>\*</sup>Work supported in part by the U. S. Army Research Office, Durham, N. C., under Contract No. DA-HC04-69-C-0007 and by the National Science Foundation Contract No. GH-33634.
<sup>1</sup>D. Allender, J. Bray, and John Bardeen, Phys. Rev. B 7, 1020 (1973).