Electric Fields and Currents due to Excess Charges and Dipoles in Insulators

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A general treatment is given of the relationship between internal and external parameters for a charged dielectric in a plane-parallel geometry. The charges induced on the electrodes by surface charges, volume charges, oriented dipoles in the dielectric, and an applied voltage are calculated for contacting or noncontacting electrodes. The fields outside and inside the sample are also calculated. Similarly, the external current is related to the internal conduction and depolarization currents. An analogy is developed between the current flowing through the system in short-circuit conditions and the time derivative of the applied voltage that cancels the external current. Because of its relation to the motion of carriers in the dielectric, the motion of the zero-field plane is investigated under very general conditions.

I. INTRODUCTION

Renewed interest in research on electrically charged dielectrics, or electrets, has made it necessary to use various methods for charge[1-4] and current[5-8] measurements. They all exhibit a common feature: The measured variables are external ones. The charge densities are evaluated from the electric field outside the sample: The currents in the electret, due to the liberation of trapped charges or frozen-in dipoles, are computed from the current measured in an external circuit.

In this paper, we shall derive general relations between the internal parameters and the externally measurable variables. The static fields inside and outside the dielectric sample will be calculated first; then the current in an external circuit will be related to the conduction and depolarization currents in the sample, and to the variation of the applied voltage. The direction of charge-carrier drift depends on the position of the zero-field plane in the dielectric sample: for this reason, a general relation between the motion of the zero-field plane and the external current will finally be derived.

II. STATIC FIELDS

A. Induction Charges

We assume that the dielectric sample, of thickness d_2, exhibits, as shown on Fig. 1, a volume charge density $\rho(z)$, two surface charge densities σ_1 and σ_2, and a volume polarization $P(z)$ due to dipole orientation. The latter term expresses two phenomena: the usual "fast" response of the dielectric, which is characterized by a short relaxation time τ ($\tau \ll 1$ sec), and the "slow" response ($\tau \gg 1$ sec), which results in the so-called heterocharge of the electret. The following calculations will be carried out by describing the fast response by the infinite-frequency dielectric constant ϵ_2, $P(z)$ being the slow polarization, so that we define the induction in the dielectric by

$$D_2(z) = \epsilon_2 E_2(z) + P(z) .$$

The charge distribution induces image charges σ_a and σ_b on two electrodes a and b, separated from the sample by two gas layers 1 and 3, of thickness d_1 and d_3 and permittivity ϵ_1. A static voltage V is applied to the electrodes, one of them, a for instance, being grounded. The cases in which one electrode, or both of them, are in intimate contact with the sample, are particular cases of the previous one. In all of the following calculations, we assume that end effects are negligible, and that all variables depend on z only. The electric fields E_1 and E_3 in layers 1 and 3 are therefore uniform, and the field $E_2(z)$ in the sample is related to the charge density by Poisson's equation:

$$\frac{\partial D_2(z)}{\partial z} = \rho(z) . \tag{1}$$

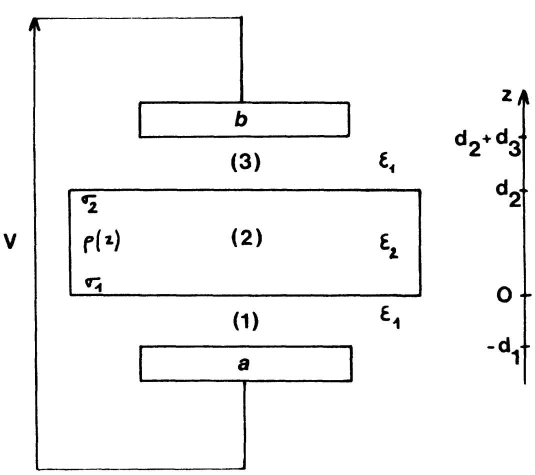

FIG. 1. Charged dielectric (2) with two gas layers (1) and (3).

For $z=0$ and $z=d_2$, the latter equation can be written as

$$-\epsilon_1 E_1 + D_2(0) = \sigma_1 , \quad (2)$$

$$\epsilon_1 E_3 - D_2(d_2) = \sigma_2 . \quad (3)$$

The necessary boundary conditions are given by the applied voltage V

$$V = -E_1 d_1 - \int_0^{d_2} E_2(z)\,dz - E_3 d_3 . \quad (4)$$

Suppose first that the dielectric sample is not charged; the surface charge σ_a is due to the applied voltage only, and is easily found to be given by

$$L\sigma_a/\epsilon_1 = -V,$$

where

$$L = d_1 + (\epsilon_1/\epsilon_2)d_2 + d_3 .$$

Suppose now that the sample is charged, and that the electrodes are short circuited; the charges on the electrodes are determined by the charge densities on the sample and the geometry of the system: We may define a voltage V_0 by

$$L(\sigma_a/\epsilon_1) = V_0 .$$

The superposition of these two equilibrium states yields

$$L(\sigma_a/\epsilon_1) = V_0 - V . \quad (5)$$

V_0 is the equivalent voltage of the electret. It is the voltage that must be applied to the capacitor in order to cancel the surface charges on electrode a. Relation (5) is valid whether the electrodes are in contact with the sample or not, with the appropriate changes in the definition of L. The voltage V_0 may be calculated from Eqs. (1)–(4), making use of the fact that the quantity $\int_0^{d_2}[\int_0^z \rho(u)\,du]\,dz$ may be integrated by parts:

$$\int_0^{d_2}\left[\int_0^z \rho(u)\,du\right]dz = d_2\int_0^{d_2}\rho(z)\,dz - \int_0^{d_2} z\rho(z)\,dz .$$

The first term on the right-hand side of the latter relation is proportional to the total volume charge; the second term involves the dipole moment of the distribution. When gas layers 1 and 3 are present, the equivalent voltage is found to be given by

$$V_0 = -\left(\frac{d_2 - \langle z\rangle}{\epsilon_2} + \frac{d_3}{\epsilon_1}\right)\frac{Q}{S} - \frac{d_3}{\epsilon_1}\sigma_2 - \left(\frac{d_2}{\epsilon_2} + \frac{d_3}{\epsilon_1}\right)\sigma_1 + \frac{1}{\epsilon_2}\int_0^{d_2} P(z)\,dz . \quad (6)$$

Q is the total volume charge and S is the area of the sample. The quantity $\langle z\rangle$ is defined as

$$\langle z\rangle = \int_0^{d_2} z\rho(z)\,dz \Big/ \int_0^{d_2}\rho(z)\,dz .$$

When the injected charges are of one polarity, and are continuously distributed in one region of the sample, $\langle z\rangle$ is the mean penetration depth of these charges. On the contrary, if the dielectric exhibits several nonoverlapping distributions of charges, then $\langle z\rangle$ may be split into several terms $\langle z_1\rangle$, $\langle z_2\rangle$, etc., each of which is the mean position of one of the charge distributions. A typical case is that of an electret charged positively on one side and negatively on the other side, under near-surface trapping conditions. Relation (6) is similar to that derived by other authors[9] for a nonpolar substance $[P(z)=0]$. When one of the electrodes is in contact with the sample (for instance, if it has been evaporated on the dielectric), two possibilities arise:

(i) The dielectric under investigation has not been charged on the metallized side. Such is the case if the sample is charged by discharge in the air gap separating the electrode from the sample. Then the equivalent voltage is obtained from relation (6) by making the gap thickness and the corresponding surface charge zero.

(ii) The sample has been charged before evaporating the electrode, or, after being metallized, by charge bombardment and injection. Then V_0 remains unchanged if electrode a is in contact with the dielectric, and, if electrode b is in contact with the sample, becomes

$$V_0 = -\frac{d_2 - \langle z\rangle}{\epsilon_2}\frac{Q}{S} - \frac{d_2\sigma_1}{\epsilon_2} + \frac{1}{\epsilon_2}\int_0^{d_2} P(z)\,dz .$$

If both sides are metallized, the latter relation still holds. In all cases, the induction charge on electrode b is the opposite of the sum of the induction charge on a and the total charge trapped in the sample:

$$L(\sigma_b/\epsilon_1) = V - V_0 - (L/\epsilon_1)(\sigma_1 + \sigma_2 + Q/S).$$

B. External Fields

In the general case, the external field created by the electret is related to the induction charge on electrode a by

$$E_1 = \sigma_a/\epsilon_1$$

so that

$$LE_1 = V_0 - V . \quad (7)$$

This relation holds only if there is a gap between electrode a and the sample.

Similarly, if electrode b is separated from the sample, the resulting field in layer 3 is

$$LE_3 = V_0 - V + (L/\epsilon_1)(\sigma_1 + \sigma_2 + Q/S) .$$

Consequently the equivalent voltage of the electret is the voltage that must be applied to the capacitor in order to cancel the field in gap 1. This property is used as a method of measuring surface charge densities.[1]

C. Internal Fields

The knowledge of the internal field is essential for the study of stimulated currents, for it determines the direction of drift of the free charges. Figure 2 shows two examples of field distributions.

(a) The sample exhibits a surface density σ_1 and a uniform negative volume density ρ to a depth a.

(b) The sample exhibits a surface density σ_1, a uniform negative volume density ρ to a depth a, and a uniform positive volume density ρ' to a depth a'.

The value of $E_2(z)$ obviously depends on the applied voltage: A variation of V results in a vertical translation of the curves. Consequently, for suitable values of the applied voltage, one or several zero-field planes or domains will exist; in such a case, some of the released charges will drift towards the nearer electrode, others towards the further one. The influence of the zero-field plane has been investigated in one particular case by other authors[10,11] and will be derived in a general way in Sec. III.

III. STIMULATED CONDUCTION AND DEPOLARIZATION CURRENTS

A. General Relations

We assume that a conduction current $j(z,t)$ exists, resulting from the drift of released charges in the electric field due to the space charge and to the applied voltage. The volume polarization $P(z,t)$ is also assumed to vary because of the liberation of frozen-in dipoles.

Assuming air gaps 1 and 3 to be perfectly insulating, and electrodes a and b to be perfectly conducting, the conservation of the total current is expressed by

$$J(t)=\epsilon_1\frac{dE_1(t)}{dt}=j(z,t)+\frac{\partial D_2(z,t)}{\partial t}=\epsilon_1\frac{dE_3(t)}{dt}\ , \qquad (8)$$

where $J(t)$ is the current flowing in the external circuit, and $D_2(z,t)$ the induction in the sample as previously defined. Integrating relation (8) over the whole thickness of the sample and using relation (4), one obtains

$$J=\frac{\epsilon_1}{L\epsilon_2}\left(\int_0^{d_2}j(z,t)\,dz+\int_0^{d_2}\frac{\partial P(z,t)}{\partial t}\,dz-\epsilon_2\frac{dV}{dt}\right). \qquad (9)$$

This relation is still valid if one gas layer, or both of them, has zero thickness. It does not depend on the nature of the contact—Ohmic or blocking—between the electrodes and the samples. When both electrodes are in contact with the dielectric plate and are short-circuited (which is a very usual experimental situation), then

$$J(t)=\frac{1}{d_2}\left(\int_0^{d_2}j(z,t)\,dz+\int_0^{d_2}\frac{\partial P(z,t)}{\partial t}\,dz\right). \qquad (10)$$

In the general case, the third term in relation (9) just means that a current flows in the external circuit when the applied voltage varies, which results in a variation of the field E_1 outside the electret; this field also varies because the distribution of charges and the dipole orientation are changed. Consequently, the applied voltage may be varied so as to cancel the change in E_1 due to the internal currents; in such a case, the variation of V must be an image of the charge flow and depolarization in the dielectric. It will also result in canceling the external current, since the displacement current in layer 1 is zero. Relation (9) leads to

$$\left[\frac{\partial V}{\partial t}\right]_{J=0}=\frac{1}{\epsilon_2}\left(\int_0^{d_2}j(z,t)\,dz+\int_0^{d_2}\frac{\partial P(z,t)}{\partial t}dz\right),$$

which, as expected, is very similar to relation (10). It shows that the measurement of V in such conditions may give the same information as the measurement of J between shorted electrodes. If the sample does not exhibit any heterocharge, then the external current is

$$J=\frac{\epsilon_1}{L\epsilon_2}\left(\int_0^{d_2}j(z,t)\,dz-\epsilon_2\frac{dV}{dt}\right)$$

and

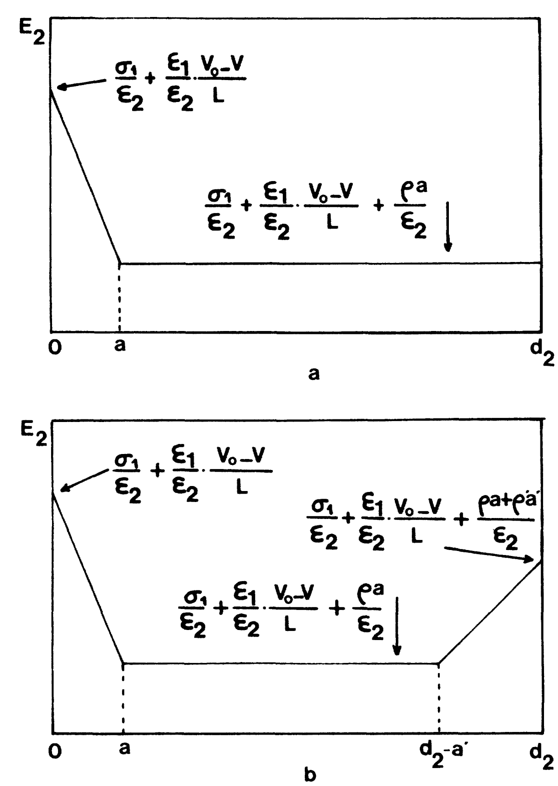

FIG. 2. Example of field distributions in the dielectric.

$$\left[\frac{\partial V}{\partial t}\right]_{J=0}=\frac{1}{\epsilon_2}\int_0^{d_2} j(z,t)\,dz\ . \tag{11}$$

B. Determination of Mean Penetration Depth

A method for measuring the penetration depth of injected charges has already been proposed.[12] It is based on the comparison between the induction charge and the total charge released in a stimulated discharge of the dielectric, that is,

$$q=\int_0^\infty J(t)\,dt\ .$$

We have seen that $J(t)$ and $[\partial V/\partial t]_{J=0}$ are equivalent. We will derive the mean penetration depth of the charges in terms of the latter quantity. One has

$$\int_0^\infty \left[\frac{\partial V}{\partial t}\right]_{J=0} dt = V(\infty)-V(0).$$

The current flowing in the sample is related to the evolution of the space charge by the equation of charge conservation:

$$\frac{\partial j(z,t)}{\partial z}=-\frac{\partial \rho(z,t)}{\partial t}\quad .$$

If we assume that the sample is purely homocharged, we can make use of relation (11), which leads to

$$V(\infty)-V(0)=\frac{d_2}{\epsilon_2}\int_0^\infty j(0,t)\,dt-\frac{d_2}{\epsilon_2}\int_0^\infty \frac{\partial}{\partial t}\frac{Q(t)}{S}\,dt + \frac{1}{\epsilon_2}\int_0^\infty \frac{\partial}{\partial t}\left(\frac{Q(t)\langle z(t)\rangle}{S}\right)dt\ .$$

The first term may be interpreted either as the total charge flowing from the electret into electrode a, if the latter makes an Ohmic contact with the sample, or as the variation of the surface charge σ_1[if the electrode is separated from the sample, then $j(0,t)=d\sigma_1/dt$].

Assuming that one type of charge only is present in the sample, and that the electric field and the trapping conditions are such that all the charges will drift towards electrode a, and reach it, then

$$Q(0)=S(\epsilon_2/d_2)\,[V(0)-V(\infty)-V_0(0)]$$

and

$$\langle z(0)\rangle = d_2\,\frac{V(\infty)-V(0)}{V(\infty)-V(0)+V_0(0)}\quad .$$

To derive these relations, an additional assumption has to be made, namely, that the initial charge is essentially a volume charge. If the surface charge is not negligible, or if the existence of a zero-field plane prevents some of the charges from drifting towards electrode a, the above expressions are approximate and yield only an upper limit of the mean penetration depth.

C. Motion of Zero-Field Plane

We are assuming now that the applied voltage is such that one (or several) zero-field plane exists. We will consider, for simplicity, the case of one zero-field plane of abscissa $z_0(t)$,

$$E_2(z_0)=0\ .$$

After Poisson's equation, the internal displacement at any plane z may be expressed as

$$D_2(z,t)=\int_{z_0}^{z}\rho(z,t)\,dz\ . \tag{12}$$

The continuity equation relates the external current to the conduction and displacement currents:

$$J(t)=j(z,t)+\frac{\partial D_2(z,t)}{\partial t}\quad . \tag{13}$$

Substituting relation (12) into (13) gives

$$J(t)=j(z,t)-\frac{dz_0}{dt}\rho(z_0,t)+\int_{z_0}^{z}\frac{\partial\rho(u,t)}{\partial t}du\ .$$

The last term is related to the conduction current by

$$\int_{z_0}^{z}\frac{\partial\rho(u,t)}{\partial t}du=j(z_0,t)-j(z,t)\quad ,$$

and consequently

$$J(t)=j\,(z,t)-\frac{dz_0}{dt}\rho(z_0,t)\quad .$$

This relation is valid whether the electrodes are in contact with the sample or not, whether a voltage is applied or not, and whether heterocharges are present or not.

If we further assume that the diffusion current is negligible as compared to the conduction current, then

$$J(t)=-\frac{dz_0}{dt}\rho(z_0,t)\quad .$$

It can be easily shown that, if several zero-field planes are present, the latter relation is valid for all of them.

When a stimulated discharge is performed under short-circuit conditions, the zero-field plane lies approximately half-way in the charge distribution, so that approximately equal quantities of charges drift in two opposite directions. Under slow retrapping conditions, both electrodes are reached by the charges, so that the external current is not measurable; consequently, stimulated discharges can give information under fast retrapping conditions only.[13]

On the contrary, if the stimulated discharge is made at constant E_1, the external current is zero, so that the zero-field plane does not move; since its initial position depends on the applied voltage, the latter may be so chosen as to make most of the charges drift towards the nearer electrode, and this condition will be automatically maintained throughout the discharge. Consequently, measurements can be made in all trapping conditions.

IV. CONCLUSION

General relations between internal and external variables have been derived for charged dielectrics. The charges induced on nearby electrodes have been related to the charge densities in the sample and to the applied field with contacting or noncontacting electrodes; a general definition of the equivalent voltage of the electret has been given. Similarly, the current flowing in the external circuit has been related to the conduction and depolarization currents, and to the applied voltage, with contacting or noncontacting electrodes. These relations, and the subsequent ones, are valid if the conduction current is due to the space charge only, or if it is also due to Ohmic conduction. An analogy has been developed between the current flowing through the system in short-circuit conditions and the time derivative of the voltage that must be applied to the system to maintain the external field constant. The possibility of measuring the mean penetration depth of the charges using this analogy has been outlined. Finally, the external current has been related to the motion of the zero-field plane by a simple, general relation, showing that the zero-field plane does not move during a zero-external-current discharge, which conveys much versatility to the method.

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