almost identical lattice constants, $Pr_{0.1}Y_{2.9}ScFe_4O_{12}$ and $Pr_1Y_2Fe_5O_{12}$, have $\Delta H/Pr$ at 77 °K equal to 1.67 $\times 10^3$ and 3.75$\times 10^3$ Oe, respectively.

To discuss the linewidth narrowing for other rare-earth (RE) ions, the narrowing factor is defined as the ratio of ΔH/RE obtained by extrapolating Seiden's[2] low-concentration measurements to $RE_3Fe_5O_{12}$ to the measured ΔH/RE for $(RE)_3Sc_yFe_{5-y}O_{12}$. At 77 °K and $y=1$, the narrowing factor is 21 for Pr as the RE, 9 for Nd, and 3 for Sm and Ho. (At 77 °K these RE garnets all have essentially the same M_s and consequently the relation between ΔH and the damping constant is the same in all cases.[1,2]) These results indicate that the linewidth per ion of Pr and Nd depends substantially more on the host lattice than do the linewidths due to the other RE ions. From the limited data available, it thus appears that K_1 and ΔH are strongly influenced by the host lattice only for Pr and Nd ions.

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Recursion Relations and Fixed Points for Ferromagnets with Long-Range Interactions*

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Recent calculations on ferromagnets with n-component spins and with long-range forces (the interaction between spin decays as $1/r^{d+\sigma}$, where d is the dimensionality of the system and $\sigma > 0$) have given exponents which are apparently discontinuous at $\sigma=2$, i.e., when the transition to short-range interactions is made. By solving Wilson's *exact* recursion relations to order $\epsilon^2(\epsilon=2\sigma-d)$ we show that the exponents η, γ, and φ are continuous functions of σ and that there exists a region of long-range potentials defined by $2>\sigma>2-\eta_{SR}$ where the exponents assume their short-range values.

I. INTRODUCTION

Soon after the discovery of the ϵ expansion[1] for critical exponents of Heisenberg systems with short-range (SR) interactions, Fisher, Ma, and Nickel[2] calculated η and γ for an analogous system in which the exchange parameters decay according to a power law $r^{-(d+\sigma)}$, where d is the dimensionality of the system and $\sigma > 0$.[3] They find, in agreement with a previous calculation by Joyce[4] on the spherical model, that for $\sigma > 2$ the exponents have their SR values but become σ dependent for $\sigma < 2$. In particular the long-range (LR) value of η provided by the ϵ expansion is $\eta = 2-\sigma$ with no corrections to orders ϵ^2 and ϵ^3 (at least). If valid uniformly in ϵ and $\sigma \leq 2$ this would imply that for σ sufficiently close to 2, η would be less than its SR value η_{SR} and there would be a discontinuity at the transition, $\sigma = 2$, between LR and SR forces. It was suggested by Stell,[5] on the contrary, that in this situation η will actually be the greater of the two values $2-\sigma$ and η_{SR}.

In this paper, starting from the exact recursion relations[1,6] we show that, indeed, η does not assume values below η_{SR} and that there exists a region of weakly LR potentials $(2-\sigma<\eta_{SR})$ for which $\eta=\eta_{SR}$. This removes the discontinuity in η at $\sigma=2$. The susceptibility exponent γ and the crossover exponent[7-9] φ are also continuous. The LR ϵ expansion, when evaluated for $\sigma = 2-\eta_{SR}$ gives values of γ and φ which agree with the SR values of these exponents as calculated by Wilson.[1] Therefore, there is no discontinuity to order ϵ^2 but this may well be true to all orders in ϵ.

II. RECURSION RELATIONS

We shall discuss a classical generalized n-component Heisenberg model[1,6,8] of dimensionality d

which is described by the Hamiltonian

$$\frac{H}{kT} \equiv -\mathcal{H}[\sigma_i(q)] = \tfrac{1}{2}\sum_{i=1}^{n}\int \frac{d^dq}{(2\pi)^d}\,(aq^2+bq^\sigma+r)S_i(\vec{q})S_i(-\vec{q}) + u\sum_{i,j=1}^{n}\iiint \frac{d^dq_1}{(2\pi)^d}\frac{d^dq_2}{(2\pi)^d}\frac{d^dq_3}{(2\pi)^d} \times S_i(\vec{q}_1)S_i(\vec{q}_2)S_j(\vec{q}_3)S_j(-\vec{q}_1-\vec{q}_2-\vec{q}_3)\ , \quad (1)$$

a and b are positive. A cutoff $q \equiv |\vec{q}\,| < 1$ is understood in all momentum-space integrations. The variable $S_i(\vec{q})$ is a Fourier component of the magnetization field, the two-spin interactions aq^2 and bq^σ correspond to the SR and LR (decaying as $r^{-(d+\sigma)}$ in coordinate space) interactions, respectively. The renormalization operation is defined in the following way[10,11]: Put

$$\mathcal{H}'[S_i'(\vec{q}\,')] = \ln \mathrm{Tr}^{>} e^{\mathcal{H}[S_i(\vec{q})]}\ , \quad (2)$$

where $\mathrm{Tr}^{>}$ stands for functional integral over all Fourier components $S_i(\vec{q})$ with $\frac{1}{2} < q < 1$. The result of such a partial integration is a functional of $S_i(\vec{q})$ with $q < \frac{1}{2}$. To bring the new Hamiltonian $\mathcal{H}'$ to a form as close as possible to the original one the momenta and spins are rescaled.

$$\vec{q}\,' = 2\vec{q}, \quad S_i'(\vec{q}\,') = \zeta^{-1}S_i(q), \quad q < \tfrac{1}{2} \quad (3)$$

where the factor ζ is left arbitrary for a moment.

Explicit equations for $\mathcal{H}'$ can be obtained by expanding the exponential in (2) with respect to the parameter u, which is assumed small.[1,6] The resulting integrals are then easily performed. The new Hamiltonian $\mathcal{H}'$ is written again in the form of (1) with some new constants a', b', r', u'. For the two-spin part we obtain

$$a'q^2 + b'q^\sigma + r' = \zeta^2 2^{-d}\left(a\frac{q^2}{4} + b\frac{q^\sigma}{2^\sigma} + r + 4(n+2)u\int^{>}\frac{d^dp}{(2\pi)^d}\frac{1}{ap^2+bp^\sigma+r} - 32(n+2)u^2\int^{>}\frac{d^dp}{(2\pi)^d}\int^{>}\frac{d^dp_1}{(2\pi)^d}\frac{1}{ap^2+bp^\sigma+r} \times \frac{1}{ap_1^2+bp_1^\sigma+r}\,\frac{1}{a(\frac{1}{2}\vec{q}+\vec{p}+\vec{p}_1)^2 + b|\frac{1}{2}\vec{q}+\vec{p}+\vec{p}_1|^\sigma + r}\right)\ , \quad (4)$$

where $\int^{>}$ denotes integration over the domain $1 > |\vec{p}\,| > \frac{1}{2}$. In the quartic term of the new Hamiltonian the coefficient is

$$u' = \zeta^4\, 2^{-3d}\left(u - 4(n+8)u^2\int^{>}\frac{1}{(ap^2+bp^\sigma+r)^2}\frac{d^dp}{(2\pi)^d}\right). \quad (5)$$

At the critical temperature the Hamiltonian will, after many renormalization operations, approach the fixed point of the recursion relations (4) and (5). In order to arrive at a fixed point, however, the spin rescaling factor ζ must be given an appropriate value. Then the value of η can be deduced by comparing the scaling properties of the spin-spin correlation function expressed in terms of η and ζ, respectively, namely,

$$G(k;\,T_c) \equiv \frac{\langle S_i(\vec{k})\,S_i(\vec{k}')\rangle}{\delta(\vec{k}+\vec{k}')} = 2^{2-\eta}\,G(2k;\,T_c)\ . \quad (6)$$

On the other hand, since $G(k;\,T_c)$ has dimensions of length to the power d multiplied by spin squared, we get

$$G(k;\,T_c) = \zeta^2\,2^{-d}\,G(2k;\,T_c)\ . \quad (7)$$

Equations (6) and (7) imply[1,10]

$$\zeta^2 = 2^{d+2-\eta}\ . \quad (8)$$

The problem thus reduces to finding the value of ζ associated with the most stable fixed point of the recursion relations (4) and (5).

Wilson and Fisher[1] showed that in the case of SR forces and for the dimensionality d close to 4 the fixed-point values r^* and u^* are of order $\epsilon = 4-d$, a^* is a fixed positive constant and $b^* = 0$. Similar results hold for the model with LR forces[2] where the role of dimension 4 is taken over by 2σ ($\epsilon = 2\sigma - d$). At the fixed point one has $r^* = O(\epsilon)$, $u^* = O(\epsilon)$, $b^* =$ constant, and $a^* = 0$ to order ϵ. We shall see below that a^* is not zero to order ϵ^2. That is the reason why we put the SR forces explicitly into the Hamiltonian (1). Even if we start with $a = 0$, SR forces appear after the first renormalization operation and will compete with the LR part of the interaction in determining the critical behavior. If the LR decay is slow so that $2-\sigma$ is not small, the LR term wins and the exponents are the same as if the SR term is ignored. However, the situation is different when ϵ^2 and $2-\sigma$ are of the same order of magnitude, or if $2-\sigma$ is much smaller than ϵ^2. This is just the relevant region when we study the limit $\sigma \to 2$.

III. FIXED POINTS

In evaluating (5) to order ϵ^2 we can neglect r in the denominator of the propagator because, at the fixed point, r is of order ϵ and by neglecting it we introduce an error of order ϵ in a term which is already of order ϵ^2. Furthermore, $2-\sigma$ will also be assumed to be of order ϵ or smaller, so that we can replace $p^\sigma = p^2[1+(\sigma-2)\ln p+\cdots]$ by p^2 since p does not become small in the integration region. The fixed point value of u is thus seen to be

$$u^* = \frac{2\pi^2}{n+8}\,\epsilon(a^*+b^*)^2 \quad . \tag{9}$$

To obtain the values of a^* and b^* from (4) we must evaluate the coefficients of q^2 and q^σ in the second integral. (The first integral is q independent and serves only to renormalize the critical temperature.) Let us assume for a moment that the LR interactions are going to determine the exponents so that ζ must be chosen to keep b constant, namely, $\zeta^2 = 2^{d+\sigma}$, which corresponds to $\eta = 2-\sigma$. Now we will find the fixed point corresponding to this choice of ζ and examine its properties.

Using the result (A5) of the Appendix, the recursion relation (4) becomes

$$a' = a - (2-\sigma)a\ln 2 + \frac{n+2}{2(n+8)^2}\,\epsilon^2\,(a+b)\ln 2\ ,$$
$$b' = b \quad . \tag{10}$$

The fixed point is

$$a^* = \frac{\eta_{SR}\, b^*}{2-\sigma-\eta_{SR}}\quad , \tag{11}$$

where b^* is arbitrary positive and fixed, and

$$\eta_{SR} = \frac{n+2}{2(n+8)^2}\,\epsilon^2 \quad . \tag{12}$$

Let us examine two cases. First let $2-\sigma > \eta_{SR}$. Then it follows from (10) that $|a'-a^*| < |a-a^*|$, which means that the Hamiltonian approaches the fixed point, which, in this case, is stable. Second, if $2-\sigma<\eta_{SR}$, then a^* is negative for b^* positive. On the other hand, starting with a positive a, Eq. (10) implies $a'>a$, so that the new Hamiltonian diverges from the fixed point. Thus we conclude, that our choice of ζ was good for $2-\sigma>\eta_{SR}$, but fails for $2-\sigma<\eta_{SR}$. Then we must redefine ζ as

$$\zeta^2 = 2^{d+2-\eta_{SR}} \quad . \tag{13}$$

This corresponds to $\eta = \eta_{SR}$. Now the fixed point is

$$b^* = 0,\quad u^* = \frac{2\pi^2}{n+8}\,\epsilon\, a^{*2}\quad , \tag{14}$$

where a^* is arbitrary positive, but fixed. Thus we recover the familiar fixed point of the Heisenberg model with SR interactions. Indeed, η_{SR} is identical to η as found previously.[12]

The results are summarized schematically in Fig. 1: There exists a region of LR potentials defined by $2-\sigma<\eta_{SR}$ in which the Heisenberg model has the critical behavior of the SR system. The discontinuity of η is removed.

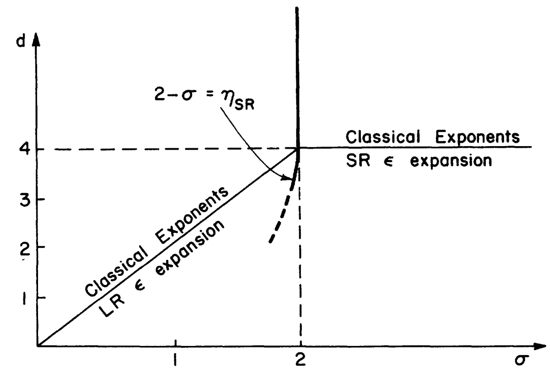

FIG. 1. Domains of LR and SR exponents.

For $2-\sigma<\eta_{SR}$ there is no LR contribution to the spin-spin interaction at the fixed point, $b^*=0$, so that *all* exponents assume their SR values. On the other hand, when we evaluate the LR expressions[2,9] for the susceptibility exponent γ and the crossover exponent φ at $\sigma = 2-\eta_{SR}$ by expanding around the point $\sigma=2$ we find that they are identical with the SR expressions of Wilson[1] to order ϵ^2.

The LR ϵ expansion is valid all the way to the transition line $2-\sigma=\eta_{SR}$. However, the ϵ expansion works only if one knows beforehand what are the thermodynamically relevant variables of the system. On the LR side of the transition line a is irrelevant but on the SR side it becomes relevant and the ϵ expansion must be reorganized. Even though the graph expansion does not tell which variables are relevant, a wrong choice will produce certain warnings. For example, in the expansion of the self-energy of the spin-spin correlation function terms of the form $k^\sigma(1-k^{2-\sigma})/(\sigma-2)$ appear. For $2-\sigma=0(\epsilon^2)$ and $\epsilon\ln k \ll 1$ (expansion parameter must be small) the above expression is indistinguishable from $k^\sigma \ln k$. In this situation we cannot assert with confidence that there are no corrections to η and have to go back to the more fundamental recursion relations to check the stability of the fixed point.

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APPENDIX

Let us consider the integral

$$\int^{>}\frac{d^dp_1}{(2\pi)^d}\int^{>}\frac{d^dp}{(2\pi)^d}\,\frac{1}{ap_1^2+bp_1^\sigma}\,\frac{1}{ap^2+bp^\sigma}\times\frac{1}{a(\tfrac12\vec q+\vec p_1+\vec p)^2+b\,|\tfrac12\vec q+\vec p_1+\vec p\,|^\sigma} \tag{A1}$$

for $q\ll 1$. We are interested in the contributions to this integral which are proportional to q^2 and q^σ.

First we show that there is no contribution of the form q^σ. In the region $|\vec{p}_1+\vec{p}|>q$ the integrand can be expanded in a Taylor series in q; the region which is important for the possible contribution q^σ is a narrow strip $|\vec{p}+\vec{p}_1|<q$ which is schematically shown in Fig. 2. Since q may be arbitrarily small, we may assume $aq^2 \ll bq^\sigma$ and it will be sufficient to consider the case $a=0$. Now, if we extend the integration region of $\vec{p}$ to the whole space it does not affect the contribution q^σ because for $p<\frac{1}{2}$ and for $p>1$ the integrand can again be expanded in q, except in the corner regions such as T (see Fig. 2), whose effect is, however, negligible. In other words, the possible term q^σ is not changed on removing the restriction $\frac{1}{2}<p<1$ because the portions of the strip $|p+p_1|<q$ contained in the new region are the same as in the original one. (The constant in the value of the integral is, of course, changed.) We thus can consider

$$\int_{\frac{1}{2}<p_1<1} \frac{d^d p_1}{(2\pi)^d} \frac{1}{p_1^\sigma} \int_{\text{all space}} \frac{d^d p}{(2\pi)^d} \frac{1}{p^\sigma} \frac{1}{|\frac{1}{2}\vec{q}+\vec{p}_1+\vec{p}|^\sigma}$$

$$= \text{const} \int_{\frac{1}{2}<p_1<1} \frac{d^d p_1}{(2\pi)^d} \frac{1}{p_1^\sigma} |\tfrac{1}{2}\vec{q}+\vec{p}_1|^{d-2\sigma} . \quad \text{(A2)}$$

Since $q \ll 1$ the integrand can be expanded and, clearly, there will be no term q^σ in the value of the integral. The only term which is of interest to us is $c(\sigma)q^2$.

This establishes the fact which is crucial for the subsequent calculation, namely, that *the dependence of the integral* (A1) *on* σ *is contained only in the coefficients of the expansion in integral powers of* q. This means that for $2-\sigma=0(\epsilon)$ we may replace the exponent σ in the integrand by 2, introducing an error of the order ϵ in the coefficient of q^2 which can be neglected. The problem thus reduces to the calculation of the integral ($d=4$)

$$I=\int^{>} \frac{d^4 p_1}{16\pi^4} \int^{>} \frac{d^4 p}{16\pi^4} \frac{1}{p_1^2} \frac{1}{p^2} \frac{1}{(\frac{1}{2}\vec{q}+\vec{p}_1+\vec{p})^2} . \quad \text{(A3)}$$

Introducing $\vec{k}=\frac{1}{2}\vec{q}+\vec{p}_1$, and integrating over the angle between $\vec{k}$ and $\vec{p}$ we get

$$I=\frac{1}{32\pi^2}\int^{>} \frac{d^4 p_1}{(2\pi)^4} \int_{1/2}^{1} \frac{2p\, dp}{p^2} \left[k^2+p^2-|k^2-p^2|\right]$$

$$=I_1+I_2+I_3 ,$$

where I_1, I_2, and I_3 are contributions from the regions where $\frac{1}{2}<k<1$, $k<\frac{1}{2}$, and $k>1$, respectively.

A straightforward calculation gives

$$I_1=\text{const}-\frac{1}{4}\frac{1}{(16\pi^2)^2} q^2 \ln 2 + O(q^4) . \quad \text{(A4)}$$

Now let us discuss I_3. The integration region is shown in Fig. 3. Since $q \ll 1$, $k \approx 1$, $p \approx 1$ and we get

$$I_3=\frac{1}{32\pi^2}\int_{p_1(k^2>1)} \frac{1}{p_1^2}\frac{1}{k^2}\int_{1/2}^{1} 4p\, dp$$

$$\approx \frac{3}{64\pi^2}|\vec{q}|+O(|\vec{q}|^3) .$$

Similar result holds for I_2. We see that the contribution from near the boundaries of momentum shells depends nonanalytically on q^2. This is a consequence of the sharp boundaries assumed for the momentum shells. Since q^2 terms do not arise from the integrals I_2 and I_3 these may be ignored for our purposes. However, a term proportional to $|\vec{q}|$ in the two-spin interaction is a nuisance, because it is thermodynamically relevant and will destabilize the fixed point. Such interfering nonanalytic terms can be removed more systematically in several ways.[13] For example, one can put similar terms in the original Hamiltonian and require that their coefficient remains bounded. Alternatively

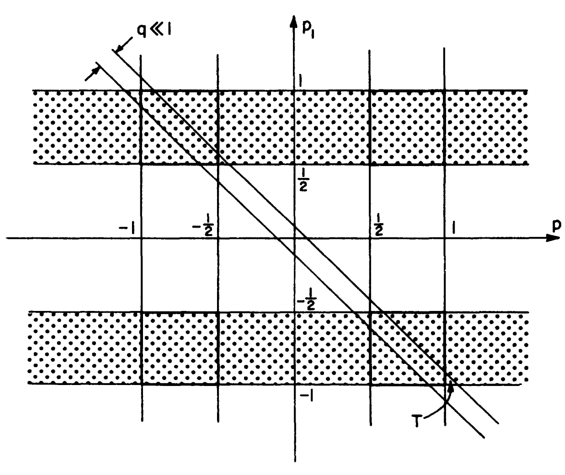

FIG. 2. Integration regions in the momentum space.

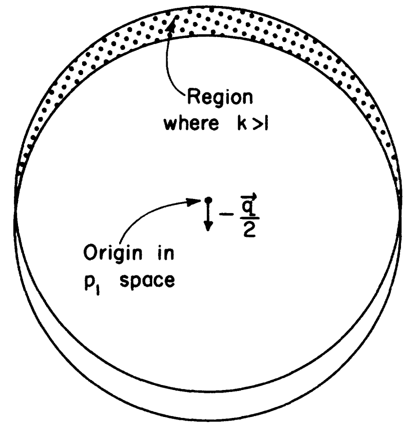

FIG. 3. Integration region for I_3.

one might start with a Hamiltonian with an analytic cut off and preintegrate it over the region $1 < k < \infty$. We shall not go further into this problem here.

The final result for (A1) is thus

$$\text{const} - \frac{1}{4}\frac{1}{(a+b)^3}\frac{1}{(16\pi^2)^2}\, q^2 \ln 2 + O(q^4) + O(2-\sigma) + \text{terms}(q, q^3, \text{ etc.}) \quad \text{(A5)}$$

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Spin-Wave Analysis of the Quadratic-Layer Antiferromagnets K_2NiF_4, K_2MnF_4, and Rb_2MnF_4

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In this and the following two papers, low-temperature spin-wave properties of quadratic-layer antiferromagnets having the K_2NiF_4 structure are reported and analyzed in detail. Here we present the results of a least-squares adjustment of spin-wave theory to the temperature variation of the sublattice magnetization in the compounds K_2NiF_4, K_2MnF_4, and Rb_2MnF_4, as reflected by ^{19}F NMR frequency measurements in zero field. Lowest-order temperature-dependent and temperature-independent corrections to simple spin-wave theory, as formulated by Oguchi, are included in the analysis. The free parameters of the fits are taken to be the exchange coupling, the zero-temperature spin-wave gap energy, and the zero-temperature ^{19}F NMR frequency. Our conclusions are as follows. Spin-wave theory accounts for the sublattice magnetization of these compounds up to somewhat less than one-half the Néel temperature, with the temperature-dependent corrections yielding less than 20% improvement in the range of fit for the Mn^{2+} compounds and a negligible improvement for K_2NiF_4. The breakdown of spin-wave theory is clearly not ascribable to spin-wave interaction effects and is apparently caused by excitations of a fundamentally different nature. Exchange values obtained are in excellent agreement with data from neutron and susceptibility measurements. The "effective" spin-wave-energy-gap values obtained give some evidence for interplanar exchange coupling between second-neighbor planes, yielding upper limits for such coupling of a few parts in 10^4 of the primary exchange. Earlier conclusions regarding the large zero-point spin reduction in K_2NiF_4 are refined here, giving a result slightly larger than but within error limits of the spin-wave-theory value (17.7%).

I. INTRODUCTION

The isomorphic compounds K_2NiF_4, K_2MnF_4 and Rb_2MnF_4, whose magnetic properties were extensively investigated by Breed and co-workers,[1] appear to be almost ideal two-dimensional (2D) antiferromagnets. The large separation between the planes, in which the magnetic ions form a quadratic lattice, and the symmetry relations between these planes of ions combine to make interplanar interactions between the magnetic ions extremely weak. In sharp contrast to a material such as $CrBr_3$, where the ratio of intraplanar to interplanar exchange is quite large, but of order 10, it is,