PHYSICAL REVIEW B VOLUME 8, NUMBER 6 15 SEPTEMBER 1973

Magnetic Field Enhancement of Self-Focusing of Laser Beams in Semiconductors*

M. Jain, J. I. Gersten, and N. Tzoar
Department of Physics, City College of the City University of New York, New York, New York 10031
(Received 26 February 1973)

A theory of self-focusing of laser beams in semiconductors in a homogeneous magnetic field is presented. The focal length and beam profile are calculated as a function of the magnetic field strength and the laser intensity. We find that a much smaller light intensity is needed for self-focusing when the cyclotron frequency approaches the laser frequency.

I. INTRODUCTION

As a consequence of the invention of powerful lasers, a host of nonlinear optical effects have been discovered. One of the most interesting effects is the self-focusing of light beams in matter, an effect which received much attention both theoretically and experimentally.[1] Several mechanisms for self-focusing have been discussed, such as the Kerr effect, electrostriction, thermal perturbation of the sample, nonlinear electronic polarization, and forward stimulated Brillouin scattering. Recently two new mechanisms which exist in semiconductors have been proposed. The first one results from the large nonparabolicity of the conduction electrons for narrow-gap semiconductors.[2] Here the nonlinearity arises from the velocity-dependent mass. The second one makes use of the energy-dependent collision time for the conduction electrons in semiconductors,[3] due to their interaction with longitudinal-optic phonons.

In this paper we discuss the self-focusing phenomenon in a narrow-gap semiconductor embedded in a uniform magnetic field. We take the velocity-dependent mass to be the dominant contributor to the nonlinear dielectric response of the medium. The effect of the magnetic field on the nonlinearity becomes enormous when the laser frequency ω and the cyclotron frequency ω_c are of the same magnitude. Then the electrons and the electric field vector of the circularly polarized light are rotating in space almost coherently and the light accelerates the electrons to a much higher velocity than in the absence of the magnetic field. This results in a large increase in the nonlinear properties of the semiconductors. The main limitation on the electron acceleration is the collision time τ. Here we are limited to the case when ω, ω_c, and $\omega-\omega_c$ are much larger than τ^{-1}. Moreover, the photon mean free path decreases with increasing magnetic field, which we overcome, in part, by decreasing the electron densities. It is the effects on self-focusing resulting from (i) the increase of the nonlinearity and (ii) the decrease of the mean free path, as a function of the magnetic field strength and laser intensity, to which we address ourselves in this paper. We find, however, that for realistic situations, self-focusing in the presence of a magnetic field can be achieved with laser powers one to two orders of magnitude smaller than without the magnetic field.

In Sec. II we derive an expression for the nonlinear dielectric function in the presence of a magnetic field. Section III concerns itself with the self-focusing of a radiation beam and in Sec. IV numerical results and a discussion of these results are presented.

II. CALCULATION OF THE NONLINEAR DIELECTRIC FUNCTION

Consider a semiconductor with electron density n in the conduction band. It is assumed that n is sufficiently small that collective effects may be neglected. The single-particle Hamiltonian for narrow-gap semoconductors formally resembles that of a relativistic electron and is given by[4]

$$H_0=[m^{*2}c^{*4}+c^{*2}p^2]^{1/2}, \tag{1}$$

where $\vec{p}$ is the electron's momentum and $c^*=(E_g/2m^*)^{1/2}$. Here c^* plays the same role in the dynamics that the speed of light c plays in relativistic mechanics. The energy gap has been denoted by E_g and the effective mass near the bottom of the conduction band by m^*. It should be noted that for realistic situations $c^*/c \ll 1$.

We now consider the electron dynamics when the semiconductor is embedded in a uniform magnetic field $\vec{B}_0$ taken to point along the $-z$ direction. In addition, we irradiate the sample with circularly polarized light whose direction of propagation is directed along the magnetic field. The Hamiltonian then becomes

$$H=\{m^{*2}c^{*4}+c^{*2}[\vec{p}+(e/c)\vec{A}]^2\}^{1/2}, \tag{2}$$

where the vector potential is given by

$$\vec{A}=\tfrac{1}{2}\vec{B}_0\times\vec{r}+A_1[\hat{\imath}\cos(\xi-\chi)+\hat{\jmath}\sin(\xi-\chi)], \tag{3}$$

where $\xi=kz-\omega t$ and χ is a constant phase angle. Hamilton's equation then leads to the following equation of motion for the electrons:

$$\frac{d}{dt}\frac{m^*\vec{v}}{[1-(v/c^*)^2]^{1/2}} = -e[\vec{E}+(\vec{v}/c)\times\vec{B}], \quad (4)$$

where $\vec{E} = -(1/c)\partial\vec{A}/\partial t$ and $\vec{B} = \nabla\times\vec{A}$. In order to to account for the effects of electron collisions we introduce a phenomonological relaxation term into Eq. (4). Thus we obtain

$$\left(\frac{d}{dt}+\frac{1}{\tau}\right)m^*\vec{v}\Gamma = -e[\vec{E}+(\vec{v}/c)\times\vec{B}], \quad (5)$$

where $\Gamma = [1-(v/c^*)^2]^{-1/2}$. This is a realistic model for the carrier densities and field intensities considered in this paper.

Equation (5) will be solved in the following approximation. We note that the ac magnetic field induces currents which are typically a factor v/c smaller than the currents induced by the ac electric field. Thus we neglect the ac magnetic force and simply replace $\vec{B}$ by $\vec{B}_0$ in Eq. (5). In principle we can go to very large values of B_0 and thus the static field should not be neglected. Hence we obtain the following set of equations:

$$\left(\frac{d}{dt}+\frac{1}{\tau}\right)m^*v_x\Gamma = \frac{e\omega A_1}{c}\sin(\xi-\chi)+\frac{e}{c}B_0 v_y, \quad (6a)$$

$$\left(\frac{d}{dt}+\frac{1}{\tau}\right)m^*v_y\Gamma = \frac{-e\omega A_1}{c}\cos(\xi-\chi)-\frac{e}{c}B_0 v_x, \quad (6b)$$

$$\left(\frac{d}{dt}+\frac{1}{\tau}\right)m^*v_z\Gamma = 0. \quad (6c)$$

In the absence of any perturbation the electrons form a cold plasma so that $\vec{v}=0$. We now investigate the steady-state solution to Eqs. (6a)–(6c). Let $v_x = v_0\cos(\xi-\chi-\varphi)$ and $v_y = v_0\sin(\xi-\chi-\varphi)$. From Eq. (6c) it immediately follows that $v_z=0$. The values of φ and v_0 follow directly upon inserting the above forms into Eqs. (6). They are determined by equating the real and imaginary parts of

$$\omega-\frac{\omega_c}{\Gamma}+\frac{i}{\tau} = \frac{eA_1\omega e^{i\varphi}}{m^*c\Gamma v_0}. \quad (7)$$

We note that $\Gamma = (1-v_0^2/c^{*2})^{-1/2}$ and is independent of time. In the above expression we have introduced the zero-field cyclotron frequency $\omega_c \equiv eB_0/m^*c$. Thus we find

$$\varphi = \tan^{-1}[1/(\omega-\varpi_c)\tau], \quad (8a)$$

$$v_0 = \omega a c^*\varpi_c/\omega_c\Delta, \quad (8b)$$

where we have introduced a dimensionless parameter $(a = eA_1/m^*c^*c)$ describing the strength of the field, a term describing the detuning from resonance

$$\Delta = [(\omega-\varpi_c)^2+1/\tau^2]^{1/2}, \quad (8c)$$

and a renormalized cyclotron frequency

$$\varpi_c = \omega_c(1-v_0^2/c^{*2})^{1/2}. \quad (8d)$$

Note that Eqs. (8b)–(8d) must be solved self-consistently to obtain v_0. In the work that follows we will always work with parameters such that $(\omega-\omega_c)\tau \gg 1$ and $\omega\tau \gg 1$. From Eq. (8d) it also follows that $(\omega-\varpi_c)\tau \gg 1$. Consequently, the phase shift of Eq. (8a) will be small.

The current density associated with the electronic motion is given by

$$\vec{J} = -ne\vec{v} = -nev_0[\hat{i}\cos(\xi-\chi-\varphi)+\hat{j}\sin(\xi-\chi-\varphi)]. \quad (9)$$

One notes that from Eqs. (8a)–(8d), v_0 depends on the field strength α in a highly nonlinear way. Thus Eq. (9) describes the nonlinear response of the electrons to the external field. This current acts as the source term in the wave equation

$$\nabla^2\vec{A}-\frac{\epsilon_L}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\frac{4\pi}{c}\vec{J}, \quad (10)$$

where ϵ_L is the lattice dielectric constant. Let us now examine the plane-wave solution to Eq. (10). Since the current lags behind the vector potential by a phase angle φ, there will be attenuation of the wave as it propagates through the medium. As a first approximation one can regard A_1 as being attenuated exponentially with damping constant β. Thus, upon inserting Eq. (3) into Eq. (10) we obtain the following for β and k:

$$(\beta^2-k^2+\epsilon_L\omega^2/c^2)^2+4\beta^2k^2 = (4\pi m e v_0/cA_1)^2, \quad (11a)$$

$$\frac{2\beta k}{\beta^2-k^2+\epsilon_L\omega^2/c^2} = \tan\varphi. \quad (11b)$$

Solving the above two equations simultaneously for β gives

$$\beta^2 = \frac{\omega^2\epsilon_L}{2c^2}\left\{-1+\left(1-\frac{\varpi_c}{\omega}\right)\frac{\omega_p^2}{\Delta^2}\frac{\varpi_c}{\omega_c}+\left[1-2\left(1-\frac{\varpi_c}{\omega}\right)\frac{\omega_p^2}{\Delta^2}\frac{\varpi_c}{\omega_c}+\frac{\omega_p^4}{\Delta^2\omega^2}\frac{\varpi_c^2}{\omega_c^2}\right]^{1/2}\right\}. \quad (11c)$$

If we work within the restrictions outlined above so that φ will be small we can neglect the effect of attenuation. Then we introduce the dielectric constant $\epsilon = k^2c^2/\omega^2$ and find

$$\epsilon = \epsilon_L\left(1-\frac{\omega_p^2}{\omega^2}\frac{\omega}{\Delta}\frac{\varpi_c}{\omega_c}\right), \quad (12)$$

where the plasma frequency is defined by

$$\omega_p = (4\pi ne^2/m^*\epsilon_L)^{1/2}. \quad (13)$$

It is instructive to make a power-series expansion of Eq. (12) in powers of the nonlinear parameter α. One obtains

$$\epsilon = \epsilon_L\left[1-\frac{\omega_p^2/\omega^2}{1-\omega_c/\omega}+\frac{a^2}{2}\frac{\omega_p^2}{\omega^2}\left(\frac{1}{1-\omega_c/\omega}\right)^4+O(a^4)\right]. \quad (14)$$

We note the dramatic enhancement of the quadratic term as the magnetic field is made large enough so that ω_c approaches ω.

To understand the origin of this enhancement we go back to the case of zero magnetic field. Here the nonlinearity results in the modulation of the mass, i.e., $m \Rightarrow m(1+\frac{1}{2}a^2)$. Since ω_p^2 is proportional to m^{-1} we obtain $(\epsilon/\epsilon_L) = 1 - (\omega_p^2/\omega^2)(1-\frac{1}{2}a^2)$, as it should be. For finite magnetic fields we obtain

$$m \Rightarrow m[1+\tfrac{1}{2}a^2(1-\omega_c/\omega)^{-2}] \; .$$

Thus our model of the mass modulation results in the following dielectric function:

$$\frac{\epsilon}{\epsilon_L} = 1 - \omega_p^2\left[\omega\left(\omega - \frac{\omega_c}{1+\frac{1}{2}\alpha^2(1-\omega_c/\omega)^{-2}}\right)\times[1+\tfrac{1}{2}\alpha^2(1-\omega_c/\omega)^{-2}]\right]^{-1} . \qquad (14')$$

Rewrite Eq. (14′) as

$$\frac{\epsilon}{\epsilon_L} = 1 - \omega_p^2\left(\omega^2\left(1-\frac{\omega_c}{\omega}\right)\times\left\{1+\frac{\omega_c/\omega}{1-\omega_c/\omega}\left[1-\left(1+\frac{\frac{1}{2}\alpha^2}{(1-\omega_c/\omega)^2}\right)^{-1}\right]\right\}\times[1+\tfrac{1}{2}\alpha^2(1-\omega_c/\omega)^{-2}]\right)^{-1} \qquad (14'')$$

and expand Eq. (14″) to first order in a^2; thus we recover our results of Eq. (14).

III. SELF-FOCUSING EQUATIONS

The nonlinearity of the dielectric constant has a profound effect on the propagation of electromagnetic radiation through the semiconductor. In a laser beam, in general, the intensity is not uniform along the profile of the beam, being maximum at the center and tapering off to zero along the edges. Therefore the nonlinearity of ϵ results in presenting different dielectric constants to different parts of the beam, thereby causing self-refraction. As long as the dielectric constant increases with increasing field the result will be a refraction of the beam into the region where the intensity is greatest—thus giving the self-focusing effect.

In general there will be a competition between diffraction effects due to the confinement of the beam to a finite width, and a focusing effect due to the nonlinearity of the medium. The latter effect must dominate for self-focusing to occur. This will happen when the power exceeds some critical power.

We now apply the variational approach developed in Ref. 1 to derive the self-focusing equations for the case of a monochromatic circularity polarized wave. Thus we assume that $\vec{A}$ is of the form

$$\vec{A} = A[\hat{\imath}\cos(kz-\omega t-ks)+\hat{\jmath}\sin(kz-\omega t-ks)] \; , \qquad (15)$$

where the amplitude A and the eikonal s are taken to be slowly varying functions of z. Since in self-focusing one finds rather narrow necks in the region of the focal length, we allow A and s to vary rapidly in the radial direction. Furthermore azimuthal symmetry is assumed. The effect of the magnetic field is completely buried in the dielectric constant ϵ. We will work in the domain where the focal length is much smaller than the attenuation length $1/\beta$. Then the Ohmic losses may be ignored to a first approximation and attenuation may be disregarded. The theory is therefore limited to the short-distance propagation of the beam through the medium.

The analysis proceeds in much the same way as in Ref. 1, where the case of linearly polarized light was considered. We start with a variational principle involving the electromagnetic Lagrangian density and make an adiabatic elimination of the time variable. We assume azimuthal symmetry and take A and s to be slowly varying in the z direction, but not necessarily in the radial direction R. Upon expressing the amplitude and eikonal as

$$E_0 = (E_i/f)\,e^{-[R/af(z)]^2}$$

and

$$s = \varphi(z) + \tfrac{1}{2}R^2\beta(z) \; ,$$

we are able to carry out the integration over the radial and azimuthal directions. Here E_0 is the electric field strength, a is the initial beam radius, E_i is the central field strength, and φ, β, and f are dependent on z. The resulting variational principle formally resembles Hamilton's principle and we find that the evolution of f, the dimensionless beam radius, with the variable z is governed by a Hamiltonian

$$H = \frac{\epsilon_L E_i^2 a^2}{32}\left[\left(a\frac{df}{dz}\right)^2 + \left(\frac{1}{afk}\right)^2 - \frac{\epsilon-\epsilon_L}{\epsilon_L}\right]. \qquad (16)$$

As mentioned in Ref. 1 a simple analog exists between self-focusing and the motion of a particle in a central force field, as is obvious from the form of Eq. (16). The effective potential for motion in the f direction can thus be written

$$V_{eff} = \frac{\epsilon_L E_i^2 a^2}{32}\left[\left(\frac{1}{afk}\right)^2 + \frac{\omega_p^2}{\omega^2}\frac{\omega}{\Delta}\frac{\overline{\omega}_c}{\omega_c}\right] , \qquad (17)$$

where we have employed Eq. (12). We note that $\overline{\omega}_c$ and Δ depends on α^2, which is taken to be

$$\alpha^2 = \frac{1}{2}\left(\frac{eE_i}{m^*c^*\omega f}\right)^2 , \qquad (18)$$

as in Ref. 1. Inspection of the expression for V_{eff} shows that as f decreases the first term increases while the second term decreases. Eventually, at small f, the first term completely dominates. This is the region where diffraction effects are very important. At larger f refraction effects, represented by the second term in Eq. (17) are more important. The beam starts out initially with beam radius $f=1$ at $z=0$. If a minimum in V_{eff} exists in the region $0<f<1$, then focusing will occur. The condition that there exist a minimum will correspond to the existence of a critical power P_{cr} for self-focusing to occur. It is clear from Eq. (17) that the critical power will now be dependent on the magnetic field in some nontrivial fashion.

Before going on to discuss the numerical computation of the various quantities it will be helpful to derive analytic formulas for the low-field limit. Thus we will utilize Eq. (14) and write the effective potential as

$$V_{eff} \approx \frac{\epsilon_L E_i^2 a^2}{32}\left[\frac{\omega_p^2/\omega^2}{1-\omega_c/\omega}+\left(\frac{1}{afk}\right)^2 - \tfrac{1}{2}a^2\frac{\omega_p^2}{\omega^2}\left(\frac{1}{1-\omega_c/\omega}\right)^4\right]. \quad (19)$$

The chief drawback of this approximation is that it predicts the beam radius will shrink to zero when self-focusing occurs. In reality the higher-order terms in α cause a saturation effect which, when combined with the diffraction effect, limits the ultimate beam radius. Thus from Eqs. (18) and (19) we predict a critical power corresponding to a central field intensity

$$E_i^{cr} = \frac{\sqrt{2}\, m^* c^* \omega^2}{eak\omega_p}\left(1-\frac{\omega_c}{\omega}\right)^2 . \quad (20)$$

We therefore expect a marked deviation of the critical power from what it would be in the absence of a magnetic field, varying roughly as the fourth power of $(\omega-\omega_c)/\omega$. Consequently, we expect to find that the critical power can be depressed by several orders of magnitude without much difficulty.

IV. RESULTS AND DISCUSSION

In the previous sections we derived an expression for the nonlinear dielectric constant for a semiconductor in a dc magnetic field. We found that a substantial enhancement of the nonlinear behavior is possible when the cyclotron frequency is close to the frequency of the incident radiation. This enhancement, coupled with the fact that even without a magnetic field the nonlinear behavior is strong, produces the largest nonlinear dielectric behavior known.

The mechanism for the enhancement is simple. In the absence of a magnetic field the electrons are constantly accelerated to some peak velocity given by $eE/m^*\omega$, where E is the electric field. The electrons are then decelerated until they move with this speed in the opposite direction. The process repeats itself periodically. Since the nonlinearity is essentially due to a quasirelativistic effect, a measure of the nonlinearity of the system is provided by $eE/m^*\omega c^* = \alpha$. In the presence of a magnetic field the electrons are moving along the cyclotron orbits. As the velocity of the electron is bent in the opposite direction by the magnetic field, the polarization of the incident light is also pointing in the opposite direction. Thus the light can coherently accelerate the electron for many cycles. In the steady state, the electron is driven at the frequency of the incident radiation and its velocity is finally 90° out of phase with the electrical force; so the field stops doing work on the electrons. The peak velocity is largely enhanced, however. The chief limitation on the electron acceleration is the collision time τ. For this reason we never allowed ω_c to approach too closely to ω.

Using the expressions and formalism derived in previous sections, we now obtain results for a case of practical interest. Our calculations are done for the semiconductor InSb.[5] The relevant parameters are $m^* = \frac{1}{60} m_e$ where m_e is the free-electron mass, $E_g = 0.234$ eV, and $\epsilon_L = 16$. This yields $c^* = 1.11\times 10^8$ cm/sec, which is $\frac{1}{270}$ the velocity of light.

The radiation frequency was taken to correspond to the CO_2 laser, $\omega = 1.742\times 10^{14}$ rad/sec. The plasma frequency was chosen to be such that $\omega_p^2 = \omega^2/10^3$, corresponding to a carrier concentration of $n = 2.55\times 10^{15}$ cm^{-3}. Under these conditions the free-carrier absorption is very small, especially if the experiment is performed around liquid-nitrogen temperature. Also, since $\omega \gg \omega_p$ one may neglect cooperative plasma effects. For these pa-

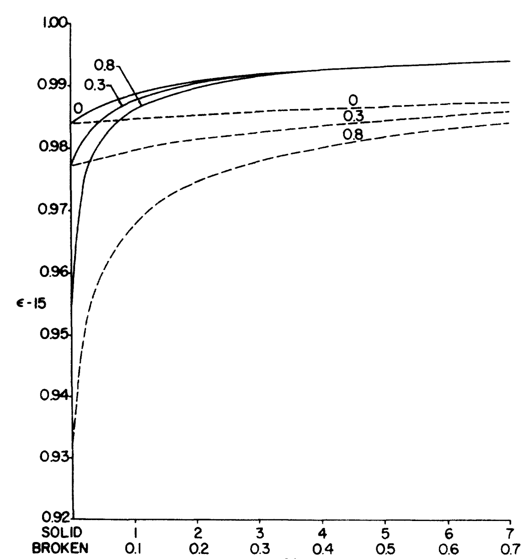

FIG. 1. Nonlinear dielectric function as a function of power parameter γ for $\omega_p/\omega = 0,\ 0.3,\ 0.8$.

rameters the cold-dilute-plasma approximation is well justified. In our calculations we will never let the cyclotron frequency exceed 0.8ω. This value is away from resonance and corresponds to a magnetic field of 132 kG, a value obtainable in the laboratory.

In Fig. 1 we present the dielectric constant ϵ as a function of the dimensionless power parameter

$$\gamma = \frac{1}{2}\left(\frac{eE_i}{m^* c^* \omega}\right)^2$$

for several values of the cyclotron frequency ω_c. Comparing the curves in which $\omega_c = 0.3\omega$ and $\omega_c = 0.8\omega$ with the curve for which $\omega_c = 0$ we note the large effect of the magnetic field on the nonlinearity of the dielectric constant. This is readily explained by the factor $(1-\omega_c/\omega)^{-4}$ in Eq. (14). The numerical results plotted in Fig. 1 also display the saturation of ϵ when γ becomes large.

We now employ this dielectric constant in calculating the self-focusing of the beam. In Fig. 2 we plot the effective potential V_{eff} of Eq. (17) as a function of the dimensionless beam radius f. The initial beam radius was taken to be 0.0054 cm, corresponding to five vacuum wavelengths. The potential curves are presented for several values of ω_c/ω and several values of γ. Exploiting the

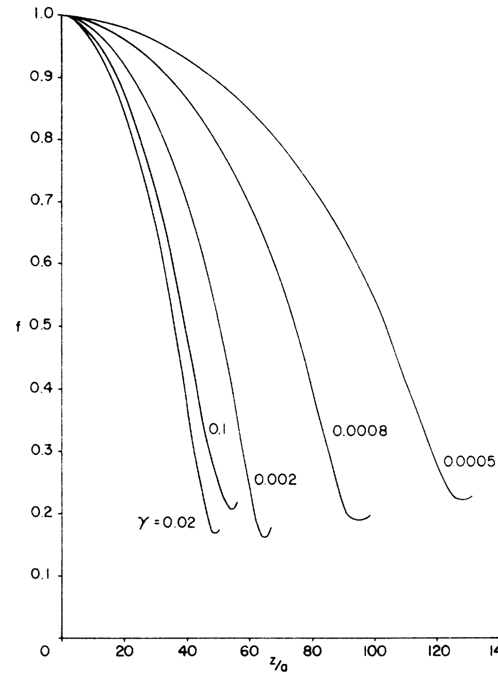

FIG. 3. Dimensionless beam radius f as a function of dimensionless distance z/a along the beam for $\omega_c/\omega = 0.8$ and for various γ values.

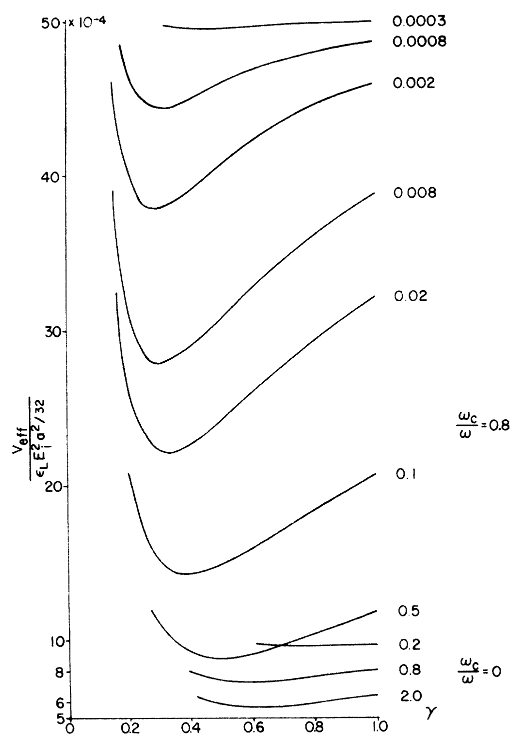

FIG. 2. Effective potential as a function of the dimensionless width f of the beam. Curves are drawn for various γ values for $\omega_c/\omega = 0, 0.8$.

analog with central force motion, we can think of the "time" that it takes for a particle to go from $f = 1$ to the other intersection with the V_{eff} curve as the self-focusing distance. The focal spot radius corresponds to the value of f at this intersection. Thus for those curves in which the magnetic field is absent we find that the beam shrinks to roughly $f = 0.4$. On the other hand, for large magnetic field f shrinks to about $f = 0.2$. Along with this fivefold shrinkage in beam radius is associated a 25-fold increase in central beam intensity. The relative insensitivity of the minimum beam radius to power or magnetic field variation points to the fact that it is essentially determined by diffractive rather than refractive effects.

The critical power can be found by plotting the effective potential curves for decreasing values of γ. We find, for example, that for $\omega_c = 0.8\omega$ and $\gamma = 0.0003$ self-focusing is still possible. This corresponds to an input power of only 8 W. The beam focuses to a radius of $f = 0.329$ in a focal length of $224a = 12.1$ mm. The critical power lies just slightly below 8 W, where the focal length becomes infinite.

The beam radius as a function of propagation distance is presented in Fig. 3 for several values of γ at $\omega_c/\omega = 0.8$. As one might expect, an increase in γ leads to a more precipitous focusing and a shorter focal length. For comparison's sake

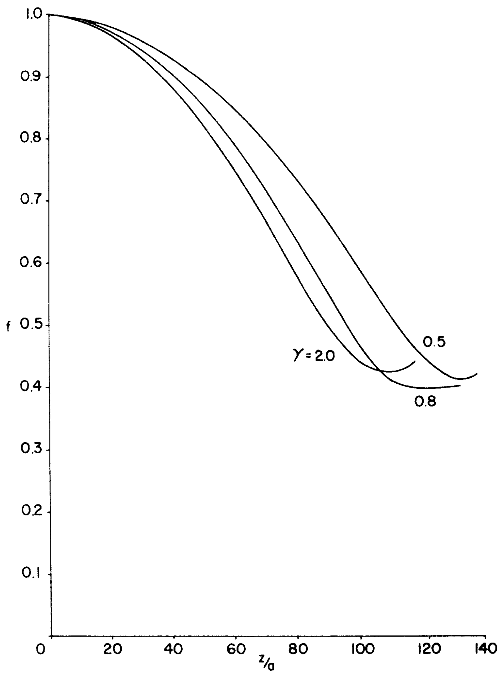

FIG. 4. Dimensionless beam radius f as a function of dimensionless distance z/a along the beam for $\omega_c/\omega = 0$ for various γ values.

we illustrate in Fig. 4 a similar set of curves for the case of no magnetic field. We note that much more power is required to obtain the same focal lengths than when the magnetic field is present. We notice that there is a tendency for the beam to bounce back after the focal point. Thus in the absence of attenuation the beam will want to focus itself periodically in a period $2z_f$. The effect of attenuation is to cause z_f to change as a function of z and ultimately to become infinite. It is worthwhile comparing numerically the with-field case with the no-field case. Without the B field, to obtain a focal length $z = 6$ mm requires $\gamma = 1.0$. From Fig. 5 (see below) we see that in the presence of the B field the same focal length is obtained with $\gamma = 0.0006$.

In the above calculations, the attenuation of the beam as it passes through the semiconductor has been neglected. Thus we limit ourselves to the case of a long photon mean free path. To justify this procedure, we compute some values for the attenuation length. This is done by using Eq. (11c) for β^2 obtained earlier. It is to be noted that β is a function of the power γ. As the beam is focused, the power along the beam changes because of the change in f, thus giving variation in the values of β along the beam. At the initial stages of contraction of the beam a rather large β value is obtained, but as the beam contracts, this value is rapidly depressed. By "averaging" β over f from $f = 1.0$ to the focal spot radius, and then calculating β^{-1}, we can get a reasonable estimate for the attenuation length. This attenuation length is plotted as a function of the power γ in Fig. 5. For comparison, the focal length z_f is also plotted on the same graph. As the attenuation length is found to be strongly dependent on n, the results are obtained for $n = 2.55 \times 10^{15}$ cm^{-3} and also for $n = 6.4 \times 10^{15}$ cm^{-3}. It is seen that for the first case, for large values of γ, the attenuation length is at least one order of magnitude larger than z_f, while even for small γ, $\beta^{-1} \gg z_f$, thus justifying the neglect of attenuation in our calculations for z_f. For the larger n, the results for z_f are really valid only for large γ values where $\beta^{-1} \gg z_f$. Thus the increased effect of attenuation precludes the possibility of obtaining magnetic field enhancement in samples with $n \geq 2 \times 10^{16}$ cm^{-3}, for InSb samples.

Figure 5 also shows that there is a marked decrease in z_f with γ only for small γ. For large γ, there is practically no variation in z_f at all; in fact, we detect a small increase in z_f with increasing intensity. This can be understood by examining the effective-potential curves in Fig. 2. One notes

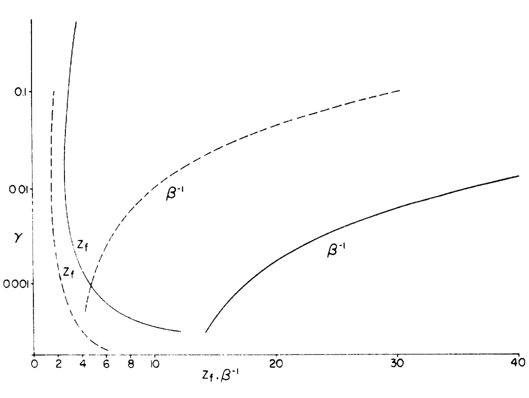

FIG. 5. Focal length z_f and attenuation length β^{-1} as a function of power parameter γ. Solid line, $n = 2.55 \times 10^{15}$ cm^{-3}; dashed line, $n = 6.4 \times 10^{15}$ cm^{-3}.

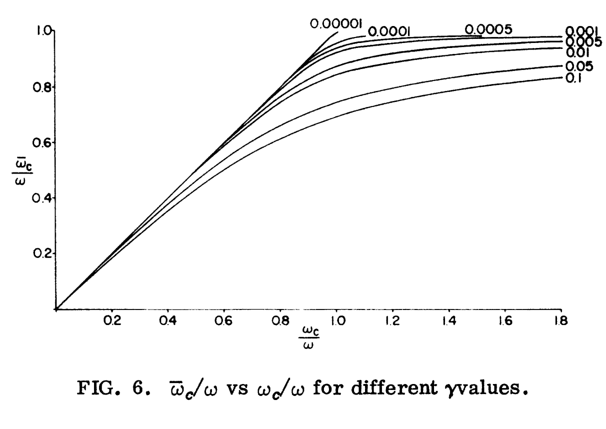

FIG. 6. $\bar{\omega}_c/\omega$ vs ω_c/ω for different γ values.

that for a strong magnetic field (e.g., $\omega_c/\omega = 0.8$), the main effect of increasing the power γ is to vertically displace the potential curve. Since the focusing length is analogous to the time required to go from a maximum orbit radius to a minimum orbit radius, and this is unaffected by a vertical translation of the potential curve, we can understand the insensitivity of z_f with γ. The only reason that we have to work with moderate powers, instead of weak powers, in to overcome the attenuation effects.

In this paper we have shown that there is a resonant enhancement of the self-focusing effect. This resonance occurs between ω and $\bar{\omega}_c$. The chief manifestation of this resonance is observed in Figs. 3 and 4, where one sees a sharp initial variation of beam radius with propagation distance when the magnetic field is present. It should be noted that $\bar{\omega}_c$ is power dependent and, consequently, a detuning from resonance can occur if the strength of the beam gets considerable, as when the beam focuses. In Fig. 6, we present $\bar{\omega}_c/\omega$ as a function of ω_c/ω for various values of γ. At small intensities, the two quantities are essentially the same, as expected. At large γ, however, there is a considerable shift.

---

*Research sponsored by the U.S. Air Force Office of Scientific Research, Air Force System Command, under AFOSR Grant No. 71-1978.

[1]N. F. Pilipetskii and A. R. Rustamov, Zh. Eksp. Teor. Fiz. Pis'ma Red. **2**, 88 (1965) [JETP Lett. **2**, 55 (1965)]; E. Garmire, R. Chiao, and C. Townes, Phys. Rev. Lett. **16**, 347 (1966); S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Usp. Fiz. Nauk **93**, 19 (1967) [Sov. Phys.-Usp. **10**, 609 (1968)]; M. Maier, G. Wendl, and W. Kaiser, Phys. Rev. Lett. **24**, 352 (1970); M. M. T. Loy and Y. R. Shen, Phys. Rev. Lett. **14**, 380 (1969); D. Grishkowsky, Phys. Rev. Lett. **24**, 866 (1970).

[2]N. Tzoar and J. I. Gersten, Phys. Rev. Lett. **26**, 1634 (1971); and Phys. Rev. B **4**, 3540 (1971).

[3]P. K. Dubey and V. V. Paranjape, Phys. Rev. B **6**, 1321 (1972); M. S. Sodha, D. P. Tewari, Jyoti Kamal, and V. K. Tripathi, J. Appl. Phys. **43**, 3736 (1972).

[4]E. O. Kane, J. Phys. Chem. Solids **1**, 249 (1957).

[5]C. Hilsum and A. C. Rose, *Semiconducting III-V Compounds* (Pergamon, New York, 1961); *Semiconductors and Semimetals*, edited by R. K. Willardson and A. C. Beer (Academic, New York, 1966).