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Comment on Side-Jump and Side-Slide Mechanisms for Ferromagnetic Hall Effect: A Reply*

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It is shown that the cancellation of the side-jump current, claimed by Smit, does not take place. This nonclassical transverse current is the main contribution to the Hall effect of iron and nickel, and is localized in the scattering centers.

It was recently shown[1-3] that the center of mass of an electron wave packet undergoes an abrupt sideways displacement Δy on scattering by an impurity or phonon in the presence of spin-orbit interaction $(\vec{p}\times\vec{\mu})\cdot\vec{\nabla}V^s(r)/2mce$. This side jump, of order $\Delta y\approx 10^{-10}$ m in a ferromagnetic crystal, is independent of the range or shape of the scattering potential $V^s(r)$ and is the principal cause of the Hall effect of transition-series ferromagnets around room temperature.

In the preceding paper,[4] Smit claims that the side-jump current is canceled by another current, caused by the $-(\vec{p}\times\vec{\mu})\cdot\vec{E}^a/2mc$ interaction, where $\vec{E}^a$ is the applied electric field. However, I will show that there is no such cancellation when the effects of that interaction are treated correctly and completely. I will use here Smit's notation.

The $-(\vec{p}\times\vec{\mu})\cdot\vec{E}^a/2mc$ interaction is a function of $\vec{k}$ (or $\vec{p}$), and therefore may be incorporated into the periodic Hamiltonian H^p, with eigenvalues E_k [similar to Eq. (A1) of Ref. 2]. When calculating, to first order in $\vec{E}^a$, the effect of this interaction on the classical current

$$\vec{j}=\frac{e}{\hbar}\sum_k f_0(E_k)\frac{\partial E_k}{\partial\vec{k}}\ ,$$

it is sufficient to use an equilibrium distribution $f_0(E_k)$, since the usual interaction $-e\vec{E}^a\cdot\vec{r}$ necessary to push the distribution out of equilibrium also contains $\vec{E}^a$. Then $-(\vec{p}\times\vec{\mu})\cdot\vec{E}^a/2mc$ has two effects on $\vec{j}$:

(i) The classical velocity $\partial E_k/\partial\vec{k}$ of an electron is changed for a given $\vec{k}$ (effect considered by Smit). It becomes

$$\frac{\partial E_k}{\partial\vec{k}}=-\frac{i}{\hbar}\langle[\vec{r},H^p]\rangle=-\frac{i}{\hbar}\left\langle\left[\vec{r},\frac{\hbar^2k^2}{2m}-\frac{(\vec{p}\times\vec{\mu})\cdot\vec{E}^a}{2mc}\right]\right\rangle=\frac{\hbar\vec{k}}{m}+\frac{\vec{E}^a\times\vec{\mu}}{2mc}$$

for free electrons.

(ii) The correct equilibrium distribution $f_0(E_k)$ is changed, since E_k is changed for a given $\vec{k}$ (effect forgotten by Smit) and becomes $E_k=\hbar^2k^2/2m-(\vec{p}\times\vec{\mu})\cdot\vec{E}^a/2mc$.

These two effects cancel each other exactly. In fact, the expression $\sum_k f_0(E_k)(\partial E_k/\partial\vec{k})$ vanishes identically for any function E_k. Thus the classical current $\vec{j}$ is zero for an equilibrium Fermi distribution, whether Smit's interaction is switched on or off. There is no "side slide" caused by the applied field $\vec{E}^a$. Therefore the nonclassical "side-jump" current survives, associated with the scattering potential $V^s(r)$.

The questions raised by Smit have already been answered in Appendixes A and B of Ref. 2. Actually, these appendixes concern band electrons rather than the free electrons treated by Smit, and therefore the $-e\vec{E}^a\cdot\langle\vec{q}_k\rangle=e\vec{E}^a\cdot(\vec{k}\times\vec{D})/2$ interaction is introduced there rather than the smaller interaction $-(\vec{p}\times\vec{\mu})\cdot\vec{E}^a/2mc$. But the discussion is essentially the same, because of the formal similarity of these Hamiltonians.

For example, a listing of velocity terms is given in Appendix B of Ref. 2 and shows that all terms cancel each other, except for a nonclassical current localized in the scattering centers (side-jump current) and for the classical skew-scattering term. See Eq. (B8) of Ref. 2.

As shown in Ref. 1, the side-jump mechanism is expected to dominate over the skew-scattering mechanism when the electronic relaxation time τ is relatively short (high temperatures). Therefore, consideration of pure iron data[5] at $100 < T < 843$ K and of dilute iron alloys[5,6] at 300 K is proper. As mentioned in Ref. 1, these data are consistent with the existence of a constant side jump $\Delta y\approx 10^{-10}$ m.

The existence of the side-jump mechanism is also in agreement with the detailed theoretical investigations of Luttinger[7] and of later authors, who find a term proportional to ρ^2 to exist in the coefficient R_s.

The relation of the side-jump current to the Karplus-Luttinger[8] current does not seem to be described correctly by Smit. In a crystal, the electron coordinate may be written $\vec{r}=\vec{R}+\vec{q}$, where $\vec{R}$ is the Wannier coordinate and $\vec{q}$ a periodic dielectric polarization. As discussed in Refs. 1 and 2, the side-jump current is a time variation of $\vec{R}$ and is therefore a part of the commutator $-(i/\hbar)[\vec{R},H]$, where H is the total Hamiltonian. On the other hand, the Karplus-Luttinger current is a time variation of the dielectric polarization $\vec{q}$ and is therefore a part of $-(i/\hbar)[\vec{q}, H]$. A similar confusion between these two types of current also appears in the last equation of Smit's paper.

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