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Side-Jump and Side-Slide Mechanisms for Ferromagnetic Hall Effect*

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It is shown that the "side-jump" $\Delta\vec{r}=\vec{p}\times\vec{\mu}/2mce$, caused by spin-orbit interaction, cannot give rise to a transverse velocity in the steady state, in which $\langle dp/dt\rangle=0$.

In two recent papers[1,2] the anomalous Hall effect in ferromagnetics is ascribed to nonclassical side jumps of the electrons during collisions. In this paper it will be shown that any such effect is compensated by the action of the applied electric field E^a (side-slide mechanism).

As in Ref. 2, we shall first consider the direct contribution of the spin-orbit interaction

$$H^{so}=-(\vec{p}\times\vec{\mu})\cdot\vec{E}/2mc=(\vec{E}\times\vec{\mu})\cdot\vec{p}/2mc \qquad (1)$$

to the velocity operator

$$\vec{v}^{so}=-i\hbar^{-1}[\vec{r},H^{so}]=\vec{E}\times\vec{\mu}/2mc \qquad (2)$$

In the steady state the average value of the total electric field $E=E^a+E^s+E^p$ acting on each electron is zero, because $e\langle E\rangle=\langle\dot{p}\rangle=0$. Here $eE^{s,p}=-\nabla V^{s,p}$, where $V^{s,p}$ are the scattering and periodic potentials, respectively. From the first part of Eq. (1) we see that a moving spin acquires an electric dipole moment $\vec{\mu}^e=\vec{p}\times\vec{\mu}/2mc=e\Delta\vec{r}$, which $\Delta\vec{r}$ is just the side jump of Ref. 2, Eq. (10); then $\vec{v}=d(\Delta\vec{r})/dt$ also gives (2). Thus by only considering the effect of V^s, Lyo and Holstein[2] and, in a less accurate way, Berger,[1] in fact, calculated the scattering term omitted by Karplus and Luttinger[3] but, in turn, neglect the applied-electric-field term of these authors.

A formal proof of $\langle E\rangle=0$ in (2) for Bloch electrons goes briefly as follows:

$$E_{nn}=E^a+\sum_{n'}(E^p_{nn'}H^a_{n'n}/d_{nn'}+E^s_{nn'}V^s_{n'n}/d_{nn'}+\text{c. c.})$$
$$+\sum_{n'}\sum_{n''}(E^p_{nn'}V^s_{n'n''}V^s_{n''n}/d_{nn'}d_{nn''}+\text{cycl.}), \qquad (3)$$

where $d_{nn'}=\epsilon_n-\epsilon_{n'}+is$ $(s\to 0^+)$ and n includes both k and the band index; $\vec{k}$ is conserved for E^p and for $H^a=-eE^ax$. With $eE^p_{nn'}=(\epsilon_n-\epsilon_{n'})(\partial/\partial x)_{nn'}$ we can apply closure when correcting for $(\partial/\partial x)_{nn}$, thereby introducing $\partial\epsilon/\partial k_x$. The closure parts of the H^a terms cancel E^a, and those of the third-order terms in (3) cancel the second-order ones with V^s, leaving for each band

$$eE_{kk}=\frac{m}{\hbar^2}\left[\frac{\partial^2\epsilon_k}{\partial k_x^2}eE^a-2\pi\sum_{k'}\left(\frac{\partial\epsilon_k}{\partial k_x}-\frac{\partial\epsilon_{k'}}{\partial k'_x}\right)|V^s_{kk'}|^2\delta(\epsilon_k-\epsilon_{k'})\right]. \qquad (4)$$

When inserting the V^s part of (4) for free electrons into (2), Eq. (7) of Ref. 2 is reproduced.

We need $\sum_k E_{kk}f_k$; in the ensuing double summation in (4) we use the identity $\sum_k\sum_{k'}(a_k-a_{k'})f_k=\sum_k\sum_{k'}a_k(f_k-f_{k'})$ so that we can employ the Boltzmann equation

$$-2\pi\sum_{k'}|V^s_{kk'}|^2(f_k-f_{k'})\delta(\epsilon_k-\epsilon_{k'})=\left(\frac{\partial f_k}{\partial k_x}\right)eE^a,$$

yielding with (4),

$$\sum_k E_{kk}f_k=\sum_k\left[\left(\frac{\partial^2\epsilon}{\partial k_x^2}\right)f+\left(\frac{\partial\epsilon}{\partial k_x}\right)\left(\frac{\partial f}{\partial k_x}\right)\right]\frac{mE^a}{\hbar^2}=0.$$

Indirectly, H^{so} influences $\vec{v}$ because $\nabla_k\times\vec{q}(k)\neq 0$, where $\vec{q}=i\int u_n^*\nabla_k u_n\,d\tau$ takes the place of $\Delta\vec{r}$ above, but is larger by a factor of $\approx 10^4$. We now obtain in the same approximation as above,

$$v_y^{so} = \left[\frac{\partial q_x}{\partial k_y} - \left(\frac{\partial q_x}{\partial k_y} - \frac{\partial q_y}{\partial k_x} \right) - \frac{\partial q_y}{\partial k_x} \right] eE^a/\hbar = 0 \ ,$$

where the first term results from the redistribution of the electrons due to the change in energy $-eE^a q_x$; the second term is caused by E^a directly, and the third one by collisions (V^s).

Recently, the spin Hall effect has been separated off in two differently doped samples of InSb[4] and held as evidence for the side-jump mechanism, which, however, should be compensated by the side-slide mechanism. It seems that the two data obtained will allow for alternate interpretations.

The only surviving mechanism, so far, is that of skew scattering,[5] predicting $R_s \propto \rho$ for impurity scattering when the concentration is varied. This relation has recently been verified experimentally by Fert and Jaoul[6] on a series of Ni alloys at low T. The often-quoted Fe alloys[7] are measured at room temperature, and are irrelevant for this test.

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[1]L. Berger, Phys. Rev. B 2, 4559 (1970); Phys. Rev. B 5, 1862 (1972).

[2]S. K. Lyo and T. Holstein, Phys. Rev. Lett. 29, 423 (1972).

[3]R. Karplus and J. M. Luttinger, Phys. Rev. 95, 1154 (1954).

[4]J. N. Chazalviel and I. Solomon, Phys. Rev. Lett. 29, 1676 (1972).

[5]J. Smit, Physica (Utr.) 24, 39 (1958).

[6]A. Fert and O. Jaoul, Phys. Rev. Lett. 28, 303 (1972).

[7]W. Jellinghaus and M. P. DeAndres, Ann. Phys. (N.Y.) 7, 189 (1961).