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Theory of Electron-Spin Resonance in Gapless Superconductors

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The transverse dynamic spin susceptibility for conduction electrons in dirty gapless superconductors, in particular, in the vortex state of type-II superconductors, is calculated. It is shown that in the gapless region the dynamic susceptibility consists of two terms, the regular term and the anomalous term. In the low-frequency region of experimental interest, the regular term reduces to the static spin susceptibility, which is determined, for example, by the Knight-shift measurement in superconductors, while the anomalous term has a pole, which is associated with a resonance of the spin of conduction electrons. The resonance linewidth T_2^{-1} is determined from the imaginary part of the resonance frequency. It is shown that T_2^{-1} behaves quite differently in the superconducting state depending on whether T_2^{-1} in the normal state is primarily due to the spin-orbit scattering or due to the exchange scattering from the magnetic impurities.

I. INTRODUCTION

Dynamical spin susceptibility for conduction electrons has been studied extensively both theoretically and experimentally, since it provides useful means to study the interaction between the spin of conduction electrons and the impurities.

On the contrary, there appears no relevant calculation for superconductors, partially because in bulk type-I superconductors magnetic fields are expulsed from inside of the bulk except in the thin-skin layer at the surface, which makes the use of the resonance technique extremely difficult.[1] However, in the case of type-II superconductors in the high-field region or thin films, where magnetic fields are considered almost uniform in the specimen, we expect that the electron-spin-resonance technique can be used to study the spin-scattering mechanism of conduction electrons from magnetic and/or nonmagnetic impurities.

In this work we would like to report the calculation of the dynamical spin susceptibility for conduction electrons in dirty superconductors in high magnetic fields. We consider that the superconductor is either a bulk and in the vortex state or a very thin film so that the magnetic field in the specimen is almost uniform. Furthermore, we assume that the spin-relaxation rate due to the impurities is small.

In most of the calculations, however, we consider a superconducting thin film in the presence of parallel magnetic fields, since in the gapless region the result obtained can be easily generalized to describe the spin susceptibility of the type-II superconductors in the vortex state, if we reinterpret appropriately the pair-breaking parameter in the theory.[2]

It is shown that in the gapless region the dynamical susceptibility has a similar expression as the one in the normal state. Besides the static part, which is related to the g shift of the impurity spin, the dynamical susceptibility has a contribution from the anomalous region which contains a complex pole.[3] The real and the imaginary part of the energy corresponding to the pole is interpreted in terms of the resonance frequency and the linewidth T_2^{-1} for the transverse spin.[3] The linewidth T_2^{-1} behaves quite differently in the superconducting state depending on whether T_2^{-1} is primarily due to the spin-orbit scattering or due to the exchange scattering from magnetic impurities. In the former case, T_2^{-1} decreases rapidly in the superconducting state, while in the latter case T_2^{-1} increases. Therefore the measurement of T_1^{-1} in the superconducting state provides certainly useful means to distinguish the two contributions to T_2^{-1}.

II. FORMULATION

We will recapitulate here some of the properties of a superconducting thin film in a parallel magnetic field,[4] which are necessary for the calculation of the dynamical susceptibility. In the presence of a magnetic field the properties of conductions electrons are described by the Hamiltonian

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}_1+\mathcal{H}_2 , \qquad (1)$$

where

$$\mathcal{H}_0=\sum_\alpha \int \Psi_\alpha^\dagger(\vec{r})\left(-\frac{1}{2m}(\vec{\nabla}-ie\vec{A})^2+\tfrac{1}{2}\omega_e\vec{\sigma}_z\right)\Psi_\alpha(\vec{r})\,d^3r + \omega_s\sum_i \vec{S}_i^z\ , \quad (2)$$

$$\mathcal{H}_I=-|g|\int \Psi_\uparrow^\dagger(\vec{r})\Psi_\downarrow^\dagger(\vec{r})\Psi_\downarrow(\vec{r})\Psi_\uparrow(\vec{r})\,d^3r\ , \quad (3)$$

and

$$\mathcal{H}_2=J\sum_{i\alpha\beta}\Psi_\alpha^\dagger(\vec{r}_i)\vec{S}_i\cdot\vec{\sigma}_{\alpha\beta}\Psi_\beta(\vec{r}_i) + V\sum_{j\alpha}\Psi_\alpha^\dagger(\vec{r}_j)\Psi_\alpha(\vec{r}_j)+iV_{s\cdot o}\,p_0^{-2} \times\sum_{\substack{k,\alpha,\rho\\u,v,w}}\epsilon_{uvw}(\vec{\nabla}_u\Psi_\alpha^\dagger(\vec{r}_k)\sigma_{\alpha\beta}{}^w(\vec{\nabla}_v\Psi_\beta(\vec{r}_k))\ , \quad (4)$$

and $\Psi_\alpha^\dagger(\vec{r})$, $\Psi_\alpha(\vec{r})$ are electron field operators, $\vec{S}_i$ is the spin operator of magnetic impurity at the site (i), $\omega_e=g_e\mu_BH$, $\omega_s=g_s\mu_BH$, and subscripts i, j, and k run over the positions of magnetic impurities, impurities with the ordinary potential and impurities with the spin-orbit potential, and ϵ_{uvw} is the complete antisymmetric tensor.

Here $\mathcal{H}_0$ describes the free motion of conduction electrons and impurity spins in the presence of magnetic field H, which is applied parallel to the Z axis. In contrast to the similar formulation in the normal metal, we have to take into account the orbital effect arising from the magnetic field, which plays an important role in superconductivity. $\mathcal{H}_1$ is the simple BCS interaction, which gives rise to the superconducting pairing below the transition temperature. Lastly, $\mathcal{H}_2$ describes the interaction between conduction electrons and a variety of impurities (magnetic as well as nonmagnetic), which gives rise to not only the relaxation of conduction electrons but also that of the spins carried by conduction electrons. The first term is the exchange interaction arising from the magnetic impurities, while the second and the third terms are the potential and the spin-orbit scattering arising either from the magnetic and/or from the nonmagnetic impurities. In order to describe the spin-orbit scattering, we adopt here the model employed by Abrikosov and Gor'kov[5] previously in their study of the Knight shift in superconductors.

Although we are ultimately interested in the electron-spin resonance inthe gapless superconductors, either in the vortex state of dirty type-II superconductors or in thin films in parallel magnetic fields, we assume for the moment that we are dealing with a dirty thin film, where the order parameter is constant all over the specimen. Then the thermal Green' functions for electrons have been already obtained and given in the four-component spinor representation[4,6]:

$$\hat{G}(\vec{p},\omega)=-\frac{1+\rho_3\sigma_3}{2}\ \frac{i\tilde{\omega}_+ +\xi_p\rho_3+\tilde{\Delta}_+\rho_1\sigma_1}{\tilde{\omega}_+^2+\xi_p^2+\tilde{\Delta}_+^2} - \frac{1-\rho_3\sigma_3}{2}\ \frac{i\tilde{\omega}_- +\xi_p\rho_3+\tilde{\Delta}_-\rho_1\sigma_1}{\tilde{\omega}_-^2+\xi_p^2+\tilde{\Delta}_-^2}\ , \quad (5)$$

where

$$\xi_p=(1/2m)p^2-\mu\ , \quad (6)$$

and $\tilde{\omega}_\pm$ and $\tilde{\Delta}_\pm$ satisfy

$$\tilde{\omega}_\pm=\omega\pm iI+\frac{1}{2}\left(\frac{1}{\tau}+a_1+\frac{a_2}{3}\right)\frac{u_\pm}{(1+u_\pm^2)^{1/2}} + \left(\frac{a_2'}{3}+\frac{1}{3\tau_{so}}\right)\frac{u_\mp}{(1+u_\mp^2)^{1/2}}\ ,$$
$$\tilde{\Delta}_\pm=\Delta+\frac{1}{2}\left(\frac{1}{\tau}-a_1-\frac{a_2}{3}\right)\frac{1}{(1+u_\pm^2)^{1/2}} + \left(-\frac{a_2}{3}+\frac{1}{3\tau_{so}}\right)\frac{1}{(1+u_\mp^2)^{1/2}}\ , \quad (7)$$

and

$$u_\pm=\tilde{\omega}_\pm/\tilde{\Delta}_\pm\ ,$$
$$I=\tfrac{1}{2}(\omega_e+c_MJ\langle S_z\rangle)\ ,$$
$$\frac{1}{2\tau}=c_1\pi N(0)|V|^2\ ,$$
$$\frac{1}{2\tau_{so}}=c_2\pi N(0)|V_{so}|^2\int\sin^2\theta\,\frac{d\Omega}{4\pi}\ , \quad (8)$$
$$\tfrac{1}{6}a_2=c_M\pi N(0)|J|^2\langle S_z^2\rangle\ ,$$
$$\tfrac{1}{3}a_2'=c_M\pi N(0)|J|^2[S(S+1)-\langle S_z^2\rangle]\ ,$$

and a_1 is the pair-breaking energy due to the magnetic field. In the case of a thin film of the thickness d in a parallel field, a_1 is calculated as[4]

$$a_1=\tfrac{1}{2}D\langle(2e\vec{A})^2\rangle=\tfrac{1}{6}D(eHd)^2 \quad (9)$$

and D is the diffusion constant. In the above expression $N(0)$ is the density of states of conduction electrons at the Fermi level, and c_M, c_1, and c_2 are the concentration of the magnetic impurities, the impurities giving rise to the potential scattering, and the spin-orbit scattering, respectively. We may refer τ and $3\tau_{so}$ to the electron lifetime and the spin lifetime due to the spin-orbit scattering, respectively, while $6a_2^{-1}$ and $3a_2'^{-1}$ are the lifetimes arising from the exchange scattering (with the parallel spin and with the tranverse spin, respectively).[7] In fact, $3a_2'^{-1}$ may be considered as the spin-scattering time associated with the exchange interaction from the magnetic impurities. Finally ρ_i and σ_i are the Pauli spins operating in the particle-hole space and the ordinary spin space of electrons, respectively.[8]

In the derivation of the Green's function, we assumed that the impurities are distributed completely randomly and that there is no correlation among impurity spins. Furthermore we neglect the dynamical properties of the impurity spins completely and consequently the Kondo effect associated with the impurity spins.

III. TRANSVERSE SUSCEPTIBILITY

The transverse susceptibility is expressed in terms of the retarded product of the electron-spin operators:

$$\chi^{+-}(\omega)=\langle[\sigma_+,\sigma_-]\rangle(0,\omega)\ . \tag{10}$$

The above retarded product is obtained from the corresponding thermal product by analytical continuation. The thermal product is, on the other hand, expressed in terms of the single-particle Green's function, Eq. (5), as

$$\chi^{+-}(-i\omega_\nu)=T\sum_n\int\frac{d^3p}{(2\pi)^3}\ \tfrac{1}{4}\ \mathrm{Tr}[\alpha_+\Lambda_+(\omega_n,\omega_{n+\nu})\times\tilde{G}(\vec{p},\omega_{n+\nu})\alpha_-\tilde{G}(\vec{p},\omega_n)]\ , \tag{11}$$

where $\alpha_\pm$ are the spin operators in the four-component representation,[4,8]

$$\alpha_\pm=\rho_3\sigma_1\pm i\sigma_2\ , \tag{12}$$

and $\alpha_+\Lambda_+$ is the renormalized spin vertex. Λ_+ is calculated by summing up the ladder-type diagrams arising from the impurity scattering (see Appendix A), and we have

$$\Lambda_+=(A+B\rho_1\sigma_2)\mathfrak{D}^{-1}\ , \tag{13}$$

where

$$A=\left[1-\frac{1}{2}\left(\frac{1}{\tau}-a_1+\frac{a_2}{3}-\frac{1}{3\tau_{so}}\right)\times\frac{1}{\Omega}\left(1+\frac{u_1u_2-1}{(1+u_1^2)^{1/2}(1+u_2^2)^{1/2}}\right)\right]\ ,$$

$$B=-\frac{i}{2}\left(\frac{1}{\tau}-a_1+\frac{a_2}{3}-\frac{1}{3\tau_{so}}\right)\frac{1}{\Omega}\ \frac{(u_1+u_2)}{(1+u_1^2)^{1/2}(i+u_2^2)^{1/2}}\ ,$$

$$\mathfrak{D}=1-\frac{1}{\Omega}\left[\frac{1}{\tau}-\frac{1}{3\tau_{so}}-\left(a_1-\frac{a_2}{3}\right)\frac{u_1u_2-1}{(1+u_1^2)^{1/2}(1+u_2^2)^{1/2}}\right]\ ,$$

and

$$\Omega=(\tilde{\omega}_1^2+\tilde{\Delta}_1^2)^{1/2}+(\tilde{\omega}_2^2+\tilde{\Delta}_2^2)^{1/2}\ , \tag{14}$$

$$\tilde{\omega}_1=\tilde{\omega}_{n,+}\ ,\qquad \tilde{\omega}_2=\tilde{\omega}_{n+\nu,-}\ . \tag{15}$$

Finally, the thermal product is calculated as

$$\chi^{+-}(-i\omega_\nu)=T\sum_n\int\frac{d^3p}{(2\pi)^3}\ [G_+(\vec{p},\omega_n)G_-(\vec{p},\omega_{n+\nu})A\ +F_+(\vec{p},\omega_n)F_-(\vec{p},\omega_{n+\nu})B]\mathfrak{D}^{-1} \tag{16}$$

$$=N(0)\left\{1-\pi T\sum_{n=-\infty}^{\infty}\left(1-\frac{u_1u_2-1}{(1+u_1^2)^{1/2}(2+u_2^2)^{1/2}}\right)\Big/\left[\Omega-\frac{1}{\tau}+\frac{1}{3\tau_{so}}+\left(a_1-\frac{a_2}{3}\right)\frac{u_1u_2-1}{(1+u_1^2)^{1/2}(1+u_2^2)^{1/2}}\right]\right\}, \tag{17}$$

where we have introduced

$$G_\pm(\vec{p},\omega_n)=-\frac{i\tilde{\omega}_{n,\pm}+\xi_p}{\tilde{\omega}_{n,\pm}^2+\xi_p^2+\tilde{\Delta}_{n,\pm}^2}$$

and

$$F_\pm(\vec{p},\omega_n)=-\frac{\Delta_{n,\pm}}{\tilde{\omega}_{n,\pm}^2+\xi_p^2+\tilde{\Delta}_{n,\pm}^2}\ . \tag{18}$$

In the static limit (i.e., $\omega_\nu=0$) and in the limit $I\to 0$ (i.e., the vanishing Zeeman splitting of the conduction electrons), Eq. (22) reduces to

$$\chi^{+-}(0)=N(0)\left(1-2\pi T\sum_{n=0}^{\infty}\left\{1\Big/(1+u^2)\left[\left(\Delta(1+u^2)^{1/2}-\frac{a_1}{1+u^2}-\frac{a_2}{3}\ \frac{(2u^2-1)}{1+u^2}\right)+\frac{2}{3\tau_{so}}\right]\right\}\right)\ , \tag{19}$$

which is nothing but the static-spin susceptibility of the gapless superconductors which appears in the Knight shift, for example.[4,9] [Note that in the limit $I\to 0$, $\chi^{+-}(0)=\chi^{zz}(0)$.]

IV. ELECTRON-SPIN RESONANCE IN GAPLESS SUPERCONDUCTORS

Equation (17) gives a complete expression of the dynamical transverse susceptibility in superconductors valid for all field and temperature regions. However, since the spin-resonance signal becomes extremely small at lower temperatures where the energy gap is finite, because of the reduction in the quasiparticle number above the energy gap, it is extremely difficult to observe the resonance at lower temperatures. In fact, from the experimental point of view, the gapless region is of primary interest. Furthermore, in the gapless region we can derive rather simple expression for the dynamical susceptibility, which can be analyzed in terms of the one in the normal state. Therefore we will limit ourselves to the gapless superconductors in the following study. In the gapless region of a dirty superconductor, we can formally expand the physical quantities in powers of the order parameter Δ, although in some instances such expansions should be handled with caution. It is then convenient to separate the summation over n in Eq. (17) into the regular region which corresponding to $\omega_n\cdot\omega_{n+\nu}>0$ and the anomalous region where $\omega_n\cdot\omega_{n+\nu}<0$ and ω_n and

$\omega_{n+\nu}$ are internal frequencies,

$$\chi^{+-}(-i\omega_\nu)=\chi^{+-}_{\rm reg}(-i\omega_\nu)+\chi^{+-}_{\rm an}(-i\omega_\nu)\,. \qquad (20)$$

In the frequency region of practical interest, where $\omega(\cong I)$, the frequency of the ac field, is much smaller than Δ_{00}, the BCS energy gap at $T=0$ K and $H=0$ (i.e., $\omega/\Delta_{00}\ll 1$), the regular term is essentially given in terms of the static susceptibility and we have

$$\chi^{+-}_{\rm reg}(\omega)\cong\chi^{+-}(0)\,, \qquad (21)$$

as defined in Eq. (19). In order to make further progress, we notice that in the gapless region we can solve Eq. (7) by assuming that

$$u_\pm=X_\pm/\Delta+O(\Delta)\,. \qquad (22)$$

Substituting Eq. (22) into Eq. (7) and collecting the lowest-order terms in Δ, we have

$$\omega\pm iI=X_\pm-\left(a_1+\frac{a_2}{3}+\frac{a_2'}{3}+\frac{1}{3\tau_{\rm so}}\right)+\left(\frac{1}{3\tau_{\rm so}}-\frac{a_2'}{3}\right)\frac{X_\pm}{X_\mp} \qquad (23)$$

or

$$(X_\pm)^{-1}=\frac{1}{2}\left[\left(1+\frac{I\pm ib}{I'}\right)\frac{1}{\omega\pm iI'+a}+\left(1-\frac{I\pm ib}{I'}\right)\frac{1}{\omega\mp iI'+a}\right]\,, \qquad (24)$$

where

$$a=a_1+\tfrac{1}{3}(a_2+a_2'+1/\tau_{\rm so})\,,\qquad b=\tfrac{1}{3}(1/\tau_{\rm so}-a_2')\,,$$

and

$$I'=(I^2-b^2)^{1/2}\,.$$

Now in terms of $X_\pm$ we express Eq. (21) as

$$\chi^{+-}_{\rm reg}(\omega)\cong 1-2\pi T\sum_{n=0}^{\infty}\frac{\Delta^2}{X_n^2(X_n-\frac{2}{3}a_2+2/3\tau_{\rm so})}$$

$$\cong 1+\frac{\Delta^2}{2(2\pi T)^2}\,\psi^{(2)}\left(\frac{1}{2}+\frac{a_1}{2\pi T}\right)\,, \qquad (25)$$

where $\psi^{(2)}(z)$ is the tetragamma function. Here we neglected small quantities $a_2/2\pi T_{c0}$ and $1/3\pi\tau_{\rm so}T_{c0}$ in the derivation. As expected then $\chi^{+-}_{\rm reg}(\omega)$ is identical to the static spin susceptibility for a superconducting thin film obtained previously by Fulde and Maki.[9]

Now we will examine the anomalous contribution which contains the basic information on the electron-spin resonance. Making use of Eqs. (22) and (24), $\chi^{+-}_{\rm an}(-i\omega_\nu)$ in the gapless region is expressed as

$$\chi^{+-}_{\rm an}(-i\omega_\nu)=-\pi TN(0)\sum_{n=0}^{\nu-1}\left[2-\Delta^2\left(\frac{1}{2X_1^2}+\frac{1}{2X_2^2}-\frac{1}{X_1X_2}\right)\right]D^{-1}(\omega_n,\,\omega_{\nu-n})\,, \qquad (26)$$

where

$$D(\omega_n,\,\omega_{\nu-n})=\omega_\nu+2iI+\frac{4}{3\tau_{\rm so}}+\tfrac{2}{3}(a_2+a_2')+\Delta^2\left[\frac{\omega_n+iI+a-\frac{2}{3}a_2}{2X_1^2}+\frac{\omega_{\nu-n}+iI+a-\frac{2}{3}a_2}{2X_2^2}-\left(\frac{a_2'}{3}+\frac{1}{3\tau_{\rm so}}\right)\left(\frac{1}{2X_1'^2}+\frac{1}{2X_2'^2}\right)-(a_1-\tfrac{1}{3}a_2)\frac{1}{X_1X_2}\right]\,, \qquad (27)$$

and

$$X_1=X_+(\omega_n)\,,\quad X_1'=X_-(\omega_n)\,,$$

$$X_2=|X_-(\omega_{n-\nu})|=X_+(\omega_{\nu-n})\,,$$

$$X_2'=|X_+(\omega_{n-\nu})|=X_-(\omega_{\nu-n})\,.$$

Therefore, in Eq. (26) both the numerator and the denominator are expanded in powers of Δ^2. When $4/3\tau_{\rm so}+\frac{2}{3}(a_2+a_2')$ is not extremely small, we can expand the denominator in Δ^2. [Note that contrary to the regular term, we have to keep the terms like $4/3\tau_{\rm so}$ and $\frac{2}{3}(a_2+a_2')$ in the denominator of the anomalous term.] Then the summation over n is somewhat tedious but easily carried out and we have (see Appendix B for details)

$$\chi^{+-}_{\rm an}(-i\omega_\nu)=-N(0)\left[\frac{1}{\omega_\nu+2iI+T_{2n}^{-1}}\left(\omega_\nu+\frac{\Delta^2}{2\pi T}\left\{\frac{1}{2}\left[\psi^{(1)}\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho\right)-\psi^{(1)}(\tfrac{1}{2}+\rho)\right]+\frac{2\pi T}{\omega_\nu+2iI+2a}\left[\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho\right)-\psi(\tfrac{1}{2}+\rho)\right]\right\}\right)\right.$$
$$\left.-\frac{\Delta^2}{\omega_\nu+2iI+T_{2n}^{-1}}\left\{\frac{\omega_\nu+2iI+\frac{4}{3}(a_2+a_2')}{\omega_\nu+2iI+2a}\left[\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho\right)-\psi(\tfrac{1}{2}+\rho)\right]+\frac{1}{6\pi T}\left(2a_2+a_2'+\frac{1}{\tau_{\rm so}}\right)\left[\psi^{(1)}\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho\right)-\psi^{(1)}(\tfrac{1}{2}+\rho)\right]\right\}\right]\,, \qquad (28)$$

where $\rho=a/2\pi T$;

$$T_{2n}^{-1}=4/3\tau_{\rm so}+\tfrac{2}{3}\,(a_2+a_2'),$$

the linewidth in the normal state; and $\psi(z)$ and $\psi^{(1)}(z)$ are the digamma and the trigamma functions. Here we assumed that I, $1/\tau_{\rm so}$, a_2, and a_2' are much smaller than Δ_{00} (or T_{c0}) as before. Finally, $\chi_{an}^{+-}(\omega)$ is determined by analytical continuation from $\chi_{an}^{+-}(-i\omega_\nu)$. In the present case this is achieved simply replacing ω_ν by $-i\omega$. Furthermore, noting that the frequency $\omega(\cong 2I)$ is much smaller than T_{c0}, we have

$$\chi_{an}^{+-}(\omega)=i\omega N(0)\left(1+\frac{\Delta^2}{2(2\pi T)^2}\times[\rho^{-1}\psi^{(1)}(\tfrac{1}{2}+\rho)+\psi^{(2)}(\tfrac{1}{2}+\rho)]\right)U(\omega)^{-1}, \quad (29)$$

where

$$U(\omega)=-i(\omega-2I)\left(1+\frac{\Delta^2}{2(2\pi T)^2}\rho^{-1}\psi^{(1)}(\tfrac{1}{2}+\rho)\right)+T_{2n}^{-1}+\tfrac{4}{3}(a_2+a_2')\frac{\Delta^2}{2(2\pi T)^2}\rho^{-1}\psi^{(1)}(\tfrac{1}{2}+\rho)+\tfrac{2}{3}\left(2a_2+a_2'+\frac{1}{\tau_{so}}\right)\frac{\Delta^2}{2(2\pi T)^2}\psi^{(2)}(\tfrac{1}{2}+\rho). \quad (30)$$

The electron-spin-resonance frequency is determined from the pole of Eq. (29):

$$U(\omega)=0. \quad (31)$$

We note first of all that the resonant frequency is given by $\omega_{res}=2I$ as in the normal state. On the other hand, the linewidth is now given by

$$T_2^{-1}=\left[T_{2n}^{-1}+\tfrac{4}{3}(a_2+a_2')\frac{\Delta^2}{2(2\pi T)^2}\rho^{-1}\psi^{(1)}(\tfrac{1}{2}+\rho)+\frac{2}{3}\left(2a_2+a_2'+\frac{1}{\tau_{so}}\right)\frac{\Delta^2}{2(2\pi T)^2}\psi^{(2)}(\tfrac{1}{2}+\rho)\right]\times\left(1+\frac{\Delta^2}{2(2\pi T)^2}\rho^{-1}\psi^{(1)}(\tfrac{1}{2}+\rho)\right)^{-1}, \quad (32)$$

where T_{2n} is defined already after Eq. (28).

Equation (32) may be rewritten

$$T_2^{-1}=\frac{4}{3\tau_{so}}\left(1-\frac{\Delta^2}{2(2\pi T)^2}[\rho^{-1}\psi^{(1)}(\tfrac{1}{2}+\rho)-\tfrac{1}{2}\psi^{(2)}(\tfrac{1}{2}+\rho)]\right)+\tfrac{2}{3}a_2\left(1+\frac{\Delta^2}{2(2\pi T)^2}[\rho^{-1}\psi^{(1)}(\tfrac{1}{2}+\rho)+2\psi^{(2)}(\tfrac{1}{2}+\rho)]\right)+\tfrac{2}{3}a_2'\left(1+\frac{\Delta^2}{2(2\pi T)^2}[\rho^{-1}\psi^{(1)}(\tfrac{1}{2}+\rho)+\psi^{(2)}(\tfrac{1}{2}+\rho)]\right), \quad (33)$$

where

$$a_2=(3/\tau_s)[\langle S_z^2\rangle/S(S+1)]$$

$$a_2'=(3/2\tau_s)[1-\langle S_z^2\rangle/S(S+1)],$$

and τ_s^{-1} is the electron lifetime due to the exchange scattering, when the impurity spin has no net polarization. From Eq. (28) we conclude that the resonance linewidth becomes narrower in the superconducting state, when T_{2n}^{-1} is primarily due to the spin-orbit scattering, while the linewidth is broadened in the superconducting state when T_{2n}^{-1} is primarily due to the exchange scattering from magnetic impurities.

We note also the residue of the pole is given by

$$R=\left(1+\frac{\Delta^2}{2(2\pi T)^2}[\rho^{-1}\psi^{(1)}(\tfrac{1}{2}+\rho)+\psi^{(2)}(\tfrac{1}{2}+\rho)]\right)\times\left(1+\frac{\Delta^2}{2(2\pi T)^2}\rho^{-1}\psi^{(1)}(\tfrac{1}{2}+\rho)\right)^{-1}\cong 1+\frac{\Delta^2}{2(2\pi T)^2}\psi^{(2)}(\tfrac{1}{2}+\rho)=\frac{\chi_s(0)}{\chi_n}, \quad (34)$$

which describes the strength of the resonance signal. R decreases monotonically in the superconducting state.

So far we limit our consideration to the case of a superconducting thin film in parallel magnetic fields. From an experimental point of view, it is more interesting to study the electron-spin resonance in bulk type-II superconductors in the vortex state. In fact, Eqs. (25) and (29) describe the static part and the anomalous part of the dynamical spin susceptibility for conduction electrons in dirty type-II superconductors, if we reinterpret some of the parameters involved. First, the pair-breaking energy a_1 is given by[10]

$$a_1=DeH. \quad (35)$$

Second, the order parameter in the vortex state in the vicinity of the upper critical field is expressed as[10]

$$\Delta^2=\frac{eT}{\sigma}\frac{H_{c2}(T)-H}{[2\kappa_2^2(t)-1]\beta_A+n}[\psi^{(1)}(\tfrac{1}{2}+\rho)]^{-1}, \quad (36)$$

where $\beta_A=1.16$, $\sigma=\tau_{tr}e^2N/m$ is the conductivity in the normal state, $\kappa_2(t)$ is the second Ginzburg-Landau parameter,[10,11] n is the demagnetization factor,[12] and H is the external magnetic field. Substituting Eq. (36) into Eq. (25), we have

$$\frac{\Delta g_s}{\Delta g_n}=1+\frac{1}{4\pi\sigma D}\frac{1-H/H_{c2}(T)}{[2\kappa_2^2(t)-1]\beta_A+n}\rho\frac{\psi^{(2)}(\frac{1}{2}+\rho)}{\psi^{(1)}(\frac{1}{2}+\rho)}, \quad (37)$$

where Δg_n and Δg_s are the shift in the g factor for the impurity spin in the normal state and in the superconducting state, respectively. Since $\psi^{(2)}(\frac{1}{2}+\rho)<0$, we expect that the g shift decreases in the superconducting state. In the vortex state of type-II superconductors, Eq. (33) is rewritten

$$T_2^{-1}=\frac{4}{3\tau_{so}}\left[1-\frac{1}{4\pi\sigma D}\frac{1-H/H_{c2}(T)}{(2\kappa_2^2(t)-1)\beta_A+n}\left(1-\tfrac{1}{2}\rho\frac{\psi^{(2)}(\frac{1}{2}+\rho)}{\psi^{(1)}(\frac{1}{2}+\rho)}\right)\right]$$

$$+\tfrac{2}{3}a_2\left[1+\frac{1}{4\pi\sigma D}\frac{1-H/H_{c2}(T)}{[2\kappa_2^2(t)-1]\beta_A+n}\left(1+2\rho\frac{\psi^{(2)}(\frac{1}{2}+\rho)}{\psi^{(1)}(\frac{1}{2}+\rho)}\right)\right]$$
$$+\tfrac{2}{3}a_2'\left[1+\frac{1}{4\pi\sigma D}\frac{1-H/H_{2c}(T)}{[2\kappa_2^2(t)-1]\beta_A+n}\left(1+\rho\frac{\psi^{(2)}(\frac{1}{2}+\rho)}{\psi^{(1)}(\frac{1}{2}+\rho)}\right)\right]. \quad (38)$$

We expressed so far the linewidth in the superconducting state as a function of magnetic field H. However, since the experiment is usually carried out with a fixed magnetic field in variable temperatures, it will be more convenient to express Eq. (38) as a function of temperature. In a constant magnetic field we have then

$$T_2^{-1}=\frac{4}{3\tau_{so}}\left(1-\frac{1}{4\pi\sigma D}\frac{1-T/T_c(H)}{[2\kappa_2^2(t)-1]\beta_A+n}S_1(t)\right)+\frac{4}{3\tau_s}\left(1+\frac{1}{4\pi\sigma D}\frac{1-T/T_c(H)}{[2\kappa_2^2(t)-1]\beta_A+n}S_2(t)\right) \quad (39)$$

and

$$T_2^{-1}=\frac{4}{3\tau_{so}}\left(1-\frac{1}{4\pi\sigma D}\frac{1-T/T_c(H)}{[2\kappa_2^2(t)-1]\beta_A+n}S_1(t)\right)+\frac{4}{3\tau_s'}\left(1+\frac{1}{4\pi\alpha D}\frac{1-T/T_c(H)}{[2\kappa_2^2(t)-1]\beta_A+n}S_3(t)\right) \quad (40)$$

for the randomly oriented impurity spins and for the completely polarized impurity spins, respectively, where

$$S_1(t)=\{[\rho\psi^{(1)}(\tfrac{1}{2}+\rho)]^{-1}-1\}\left(1-\frac{\rho}{2}\frac{\psi^{(2)}(\frac{1}{2}+\rho)}{\psi^{(1)}(\frac{1}{2}+\rho)}\right),$$
$$S_2(t)=\{[\rho\psi^{(1)}(\tfrac{1}{2}+\rho)]^{-1}-1\}\left(1+\tfrac{3}{2}\rho\frac{\psi^{(2)}(\frac{1}{2}+\rho)}{\psi^{(1)}(\frac{1}{2}+\rho)}\right),$$
$$S_3(t)=\{[\rho\psi^{(1)}(\tfrac{1}{2}+\rho)]^{-1}-1\}\left(1+2\rho\frac{\psi^{(2)}(\frac{1}{2}+\rho)}{\psi^{(1)}(\frac{1}{2}+\rho)}\right), \quad (41)$$

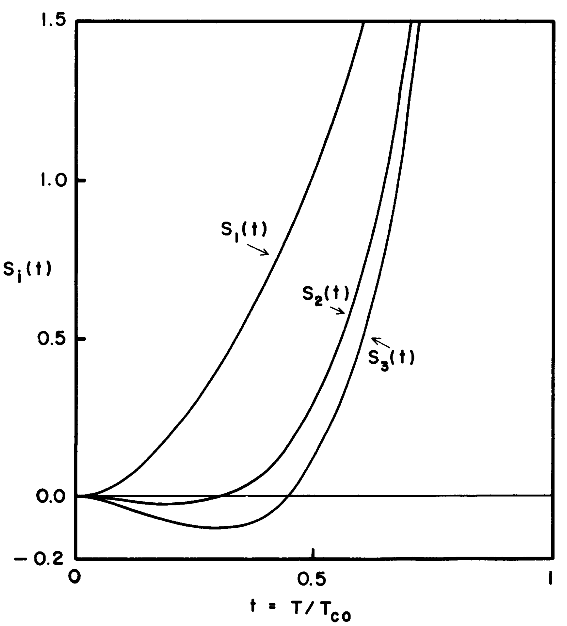

FIG. 1. Universal functions $S_1(t)$, $S_2(t)$, and $S_3(t)$ which appear in the expression of the resonance linewidth T_2^{-1} in gapless superconductors are shown as functions of the reduced temperature $T_c(H)/T_{c0}$.

and $\tau_s'^{-1}=[S\tau_s^{-1}/(S+1)]$ and $T_c(H)$ is the transition temperature in a magnetic field H. The universal functions $S_1(t)$, $S_2(t)$, and $S_3(t)$ are shown in Fig. 1 as functions of the reduced temperature $t=T_c(H)/T_{c0}$ and T_{c0} is the transition temperature in the absence of field. All $S_1(t)$, $S_2(t)$, and $S_3(t)$ are positive except $S_2(t)$ and $S_3(t)$ at lower temperatures, which implies that the linewidth becomes narrower when T_{2n}^{-1} is primarily due to the spin-orbit scattering, while it broadens when T_{2n}^{-1} is primarily due to the exchange scattering as stated before. Furthermore, the second term in Eq. (40) is exactly what one expects from the detailed balance[3] between the decays of the polarization of the spins of the conduction electrons and those of the impurities, since the relaxation rate of the impurity spins in the superconducting state is expressed in terms of the same function as that for the nuclear spins.[4,13]

V. CONCLUDING REMARKS

We have calculated here the dynamical transverse spin susceptibility for conduction electrons in dirty gapless superconductors, in particular, in the vortex state of type-II superconductors. We find that in the gapless superconductors the dynamical spin susceptibility has a resonance pole similar to the one in the normal metals, although the residue of the pole decreases rapidly in the superconducting state. The linewidth T_2^{-1} of the resonance in the superconducting state behaves quite differently depending on whether the width is primarily due to the spin-orbit scattering or due to the exchange scattering from magnetic impurities. In the former case T_2^{-1} decreases rapidly in the superconducting state, while in the latter case T_2^{-1} increases rapidly as the temperature decreases below the transition temperature. Therefore the electron-spin-resonance measurement in the superconducting state provides certainly useful information about the origins of the spin-scattering mechanism of conduction electrons.

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APPENDIX A: DETERMINATION OF THE RENORMALIZED SPIN VORTEX

Making use of the standard technique used in the impurity-scattering problem, the renormalized spin vortex is expressed in terms of the sum of the ladder-type impurity corrections. This summation can be formally carried out by solving the Bethe-Saltpeter equation for the renormalized vertex $\alpha_+\Lambda_+$

$$\alpha_+\Lambda_+ = \alpha_+ + \frac{1}{2\tau\Omega}\rho_3\left(\alpha_+\Lambda_+ - \frac{(u_1 - i\rho_1\sigma_2)\,\alpha_+\Lambda_+(u_2 - i\rho_1\sigma_2)}{(1+u_1^2)^{1/2}(1+u_2^2)^{1/2}}\right)\rho_3 + \frac{1}{6\tau_{so}\Omega}\sigma_3\left(\alpha_+\Lambda_+ - \frac{(u_1 - i\rho_1\sigma_2)\,\alpha_+\Lambda_+(u_2 - i\rho_1\sigma_2)}{(1+u_1^2)^{1/2}(1+u_2^2)^{1/2}}\right)\sigma_3$$

$$+\frac{a_2}{6\Omega}\sigma_3\rho_3\left(\alpha_+\Lambda_+ - \frac{(u_1 - i\rho_1\sigma_2)\alpha_+\Lambda_+(u_2 - i\rho_1\sigma_2)}{(1+u_1^2)^{1/2}(1+u_2^2)^{1/2}}\right)\sigma_3\rho_3 + \frac{1}{3\tau_{so}\Omega'}\sigma_2\rho_3\left(\alpha_+\Lambda_+ - \frac{(u_1' - i\rho_1\sigma_2)\alpha_+\Lambda_+(u_2' - i\rho_1\sigma_2)}{(1+u_1'^2)^{1/2}(1+u_2'^2)^{1/2}}\right)\sigma_2\rho_3$$

$$+\frac{a_3'}{3\Omega'}\sigma_2\left(\alpha_+\Lambda_+ - \frac{(u_1' - i\rho_1\sigma_2)\,\alpha_+\Lambda_+(u_2' - i\rho_1\sigma_2)}{(1+u_1'^2)^{1/2}(1+u_2'^2)^{1/2}}\right)\sigma_2\ , \qquad \text{(A1)}$$

where

$$\Omega = (\tilde{\omega}_{n,+}^2 + \Delta_{n,+}^2)^{1/2} + (\tilde{\omega}_{n+\nu,-}^2 + \tilde{\Delta}_{n+\nu,-})^{1/2}\ , \qquad \Omega' = (\tilde{\omega}_{n,-}^2 + \tilde{\Delta}_{n,-}^2)^{1/2} + (\tilde{\omega}_{n+\nu,+}^2 + \tilde{\Delta}_{n+\nu,+}^2)^{1/2} \qquad \text{(A2)}$$

and u_1, u_1', u_2, and u_2' refer to $\tilde{\omega}_{n,+}$, $\tilde{\omega}_{n,-}$, $\tilde{\omega}_{n+\nu,-}$, and $\tilde{\omega}_{n+\nu,+}$, respectively. Here we neglected the pair-breaking term arising from the magnetic field for simplicity.

The above equation for $\alpha_+\Lambda_+$ is easily solved, making use of the *Ansatz*

$$\Lambda_+ = \Lambda_{(1)} + \Lambda_{(2)}\rho_1\sigma_2 \qquad \text{(A3)}$$

Substituting (A3) into (A1) and comparing the coefficient of 1 and $\rho_1\sigma_2$, we find

$$\left\{1 - \left[\frac{1}{2\Omega}\left(\frac{1}{\tau} + \frac{1}{3\tau_{so}} + \frac{a_2}{3}\right)\left(1 - \frac{u_1u_2 - 1}{(1+u_1^2)^{1/2}(1+u_2^2)^{1/2}}\right) - \frac{1}{\Omega'}\left(\frac{1}{3\tau_{so}} + \frac{a_2'}{3}\right)\left(1 - \frac{u_1'u_2' - 1}{(1+u_1'^2)^{1/2}(1+u_2'^2)^{1/2}}\right)\right]\right\}\Lambda_{(1)}$$

$$-i\left[\frac{1}{2\Omega}\left(\frac{1}{\tau} + \frac{1}{3\tau_{so}} + \frac{a_2}{3}\right)\frac{u_1 + u_2}{(1+u_1^2)^{1/2}(1+u_2^2)^{1/2}} + \frac{1}{\Omega'}\left(\frac{1}{3\tau_{so}} + \frac{a_2}{3}\right)\frac{u_1' + u_2'}{(1+u_1'^2)^{1/2}(1+u_2'^2)^{1/2}}\right]\Lambda_{(2)} = 1\ , \qquad \text{(A4)}$$

$$-i\left[\frac{1}{2\Omega}\left(\frac{1}{\tau} + \frac{1}{3\tau_{so}} - \frac{a_2}{3}\right)\frac{u_1 + u_2}{(1+u_1^2)^{1/2}(1+u_2^2)^{1/2}} - \frac{1}{\Omega'}\left(\frac{1}{3\tau_{so}} - \frac{a_2'}{3}\right)\frac{u_1' + u_2'}{(1+u_1'^2)^{1/2}(1+u_2'^2)^{1/2}}\right]\Lambda_{(1)}$$

$$+\left\{1 - \left[\frac{1}{2\Omega}\left(\frac{1}{\tau} + \frac{1}{3\tau_{so}} - \frac{a_2}{3}\right)\left(1 + \frac{u_1u_2 - 1}{(1+u_1^2)^{1/2}(1+u_2^2)^{1/2}}\right) - \frac{1}{\Omega'}\left(\frac{1}{3\tau_{so}} - \frac{a_2'}{3}\right)\left(1 + \frac{u_1'u_2' - 1}{(1+u_1'^2)^{1/2}(1+u_2'^2)^{1/2}}\right)\right]\right\}\Lambda_{(2)} = 0\ . \qquad \text{(A5)}$$

Finally, the solution of (A4) and (A5) is given by Eq. (12) in the text.

APPENDIX B: CALCULATION OF ANOMALOUS TERM

Expanding Eq. (26) in powers of Δ^2, we have

$$\chi_{an}^{+-}(-i\omega_\nu) = \frac{-2\pi TN(0)}{\omega_\nu + 2iI + T_{2n}^{-1}}\sum_{n=0}^{\nu-1}\left\{1 - \frac{\Delta^2}{2}\left(\frac{1}{2X_1^2} + \frac{1}{2X_2^2} - \frac{1}{X_1X_2}\right) - \frac{\Delta^2}{\omega_\nu + 2iI + T_{2n}^{-1}}\right.$$

$$\left.\times\left[\frac{\omega_n + iI + a - \frac{2}{3}a_2}{2X_1^2} + \frac{\omega_{\nu-n} + iI + a - \frac{2}{3}a_2}{2X_2^2} - \left(\frac{a_2'}{3} + \frac{1}{3\tau_{so}}\right)\left(\frac{1}{2X_1'^2} + \frac{1}{2X_2'^2}\right) - \left(a_1 - \frac{a_2}{3}\right)\frac{1}{X_1X_2}\right]\right\} \qquad \text{(B1)}$$

$$= -\frac{N(0)}{\omega_\nu + 2iI + T_{2n}^{-1}}\left(\omega_\nu - \frac{\Delta^2}{2}\left[2K_1(\omega_\nu) - K_2(\omega_\nu)\right] - \frac{\Delta^2}{\omega_\nu + 2iI + T_{2n}^{-1}}\left[2K_3(\omega_\nu) - (a_1 - \tfrac{1}{3}a_2)K_2(\omega_\nu)\right]\right)\ , \qquad \text{(B2)}$$

where

$$K_1(\omega_\nu)=2\pi T\sum_{n=0}^{\nu-1}\frac{1}{2X_1^2}\ ,\quad K_2(\omega_\nu)=2\pi T\sum_{n=0}^{\nu-1}\frac{1}{X_1X_2}\ ,\quad K_3(\omega_\nu)=2\pi T\sum_{n=0}^{\nu-1}\left(\frac{\omega_n+iI+a-\frac{2}{3}a_2}{2X_1^2}-\frac{\frac{1}{3}a_2'+1/3\tau_{so}}{2X_1'^2}\right) \tag{B3}$$

and

$$T_{2n}^{-1}=\frac{4}{3\tau_{so}}+\tfrac{2}{3}(a_2+a_2')\ . \tag{B4}$$

Now let us first consider $K_1(\omega_2)$ which is calculated as

$$K_1(\omega_\nu)=\pi T\sum_{n=0}^{\nu-1}\frac{1}{4}\left[\left(1+\frac{I+ib}{I'}\right)\frac{1}{\omega_n+iI'+a}+\left(1-\frac{I+ib}{I'}\right)\frac{1}{\omega_n-iI'+a}\right]^2$$

$$=\frac{\pi T}{2}\sum_{n=0}^{\nu-1}\left\{\left(1+\frac{I}{I'}\right)\left(1+\frac{ib}{I'}\right)\frac{1}{(\omega_n+iI'+a)^2}+\left(1-\frac{I}{I'}\right)\left(1-\frac{ib}{I'}\right)\frac{1}{(\omega_n-iI'+a)^2}-\frac{2iIb}{I'^2}\frac{1}{(\omega_n+iI'+a)(\omega_n-iI'+a)}\right\}$$

$$=\frac{1}{4(2\pi T)}\left\{\left(1+\frac{I}{I'}\right)\left(1+\frac{ib}{I'}\right)\left[\psi^{(1)}(\tfrac{1}{2}+\rho_+)-\psi^{(1)}(\tfrac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_+)\right]\right.$$
$$+\left(1-\frac{I}{I'}\right)\left(1-\frac{ib}{I'}\right)\left[\psi^{(1)}(\tfrac{1}{2}+\rho_-)-\psi^{(1)}\left(\tfrac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_-\right)\right]$$
$$\left.+\frac{Ib}{I'^3}(2\pi T)\left[\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_+\right)-\psi(\tfrac{1}{2}+\rho_+)+\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_-\right)-\psi(\tfrac{1}{2}+\rho_-)\right]\right\}\ , \tag{B5}$$

where

$$\rho_\pm=(a\pm iI')/2\pi T\ . \tag{B6}$$

Taking into account the fact that $I'/2\pi T_{00}$, $b/2\pi T_{c0}$, $1/3\tau_{so}\pi T$, and $a_3/2\pi T_\omega$ are smaller than 1, we reduce (B5) to

$$K_1(\omega_\nu)\cong\frac{1}{2(2\pi T)}\left[\psi^{(1)}(\tfrac{1}{2}+\rho)-\psi^{(1)}\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho\right)\right]\ , \tag{B7}$$

where

$$\rho=a/2\pi T\ . \tag{B8}$$

$K_2(\omega_\nu)$ and $K_3(\omega_\nu)$ are computed similarly, and we have

$$K_2(\omega_\nu)=\pi T\sum_{n=0}^{\nu-1}\left[\left(1+\frac{I}{I'}\right)\left(1+\frac{ib}{I'}\right)\frac{1}{(\omega_n+iI'+a)(\omega_{\nu-n}+iI'+a)}+\left(1-\frac{I}{I'}\right)\left(1-\frac{ib}{I'}\right)\frac{1}{(\omega_n-iI'+a)(\omega_{\nu-n}-iI'+a)}\right.$$
$$\left.-\frac{iIb}{I'^2}\left(\frac{1}{\omega_n+iI'+a}\,\frac{1}{\omega_{\nu-n}-iI'+a}+\frac{1}{\omega_n-iI'+a}\,\frac{1}{\omega_{\nu-n}+iI'+a}\right)\right] \tag{B9}$$

$$=\left(1+\frac{I}{I'}\right)\left(1+\frac{ib}{I'}\right)\frac{1}{\omega_\nu+2(iI'+a)}\left[\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_+\right)-\psi(\tfrac{1}{2}+\rho_+)\right]$$
$$+\left(1-\frac{I}{I'}\right)\left(1-\frac{ib}{I'}\right)\frac{1}{\omega_\nu+2(-iI'+a)}\left[\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_-\right)-\psi(\tfrac{1}{2}+\rho_-)\right]$$
$$-\frac{iIb}{I'^2(\omega_\nu+2a)}\left[\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_+\right)-\psi(\tfrac{1}{2}+\rho_+)+\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_-\right)-\psi(\tfrac{1}{2}+\rho_-)\right] \tag{B10}$$

$$\cong\frac{\omega_\nu}{2\pi T}\psi^{(1)}(\tfrac{1}{2}+\rho)\left(\frac{2(1+ib/I)}{\omega_\nu+2(iI'+a)}-\frac{2iIb}{I'^2}\,\frac{1}{\omega_\nu+2a}\right) \tag{B11}$$

and

$$K_3(\omega_\nu)=\frac{\pi T}{4}\sum_{n=0}^{\nu-1}\left\{(\omega_n+iI+a-\tfrac{2}{3}a_2)\left[\left(1+\frac{I+ib}{I'}\right)\frac{1}{\omega_n+iI'+a}+\left(1-\frac{I+ib}{I'}\right)\frac{1}{\omega_n-iI'+a}\right]^2\right.$$

$$\left.-\left(\frac{a_2'}{3}+\frac{1}{3\tau_{so}}\right)\left[\left(1-\frac{I-ib}{I'}\right)\frac{1}{\omega_n+iI'+a}+\left(1+\frac{I-ib}{I'}\right)\frac{1}{\omega_n-iI'+a}\right]^2\right\} \quad \text{(B12)}$$

$$\cong\frac{1}{4}\left\{\left(1+\frac{I+ib}{I'}+\frac{iI^2b}{I'^3}\right)\left[\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_+\right)-\psi(\tfrac{1}{2}+\rho_+)\right]\right.$$

$$+\left(1-\frac{I+ib}{I'}-\frac{iI^2b}{I'^3}\right)\left[\psi\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_-\right)-\psi(\tfrac{1}{2}+\rho_-)\right.$$

$$+\frac{1}{2\pi T}\left(1+\frac{ib}{I'}\right)\left[\tfrac{2}{3}a_2\left(1+\frac{I}{I'}\right)+\left(\frac{a_2'}{3}+\frac{1}{3\tau_{so}}\right)\left(1-\frac{I}{I'}\right)\right]\left[\psi^{(1)}\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_+\right)-\psi^{(1)}(\tfrac{1}{2}+\rho_+)\right]$$

$$\left.+\frac{1}{2\pi T}\left(1-\frac{ib}{I'}\right)\left[\tfrac{2}{3}a_2\left(1-\frac{I}{I'}\right)+\left(\frac{a_2'}{3}+\frac{1}{3\tau_{so}}\right)\left(1+\frac{I}{I'}\right)\right]\left[\psi^{(1)}\left(\frac{1}{2}+\frac{\omega_\nu}{2\pi T}+\rho_-\right)-\psi^{(1)}(\tfrac{1}{2}+\rho_-)\right]\right\}$$

$$\cong\frac{\omega_\nu}{2(2\pi T)}\left(\psi^{(1)}(\tfrac{1}{2}+\rho)+\frac{2a_2+a_2'+1/\tau_{so}}{6\pi T}\psi^{(2)}(\tfrac{1}{2}+\rho)\right). \quad \text{(B13)}$$

Substituting (A10), (A12), and (A6), we get Eq. (28) in the text.

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