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Self-Action of Laser Beams in Semiconductors[*]

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We have derived an expression for the nonlinear dielectric constant due to nonparabolicity of the conduction band and energy-dependent scattering processes in semiconductors under the influence of an external laser beam. The self-action phenomenon due to the combined effect is then studied. The role of linear and nonlinear absorption coefficients is incorporated in the analysis of self-action to show that (i) the former leads to a lower threshold value of the incident power of the beam for self-focusing to occur, and (ii) the latter decreases the self-focusing length, the reduction being appreciable at optical frequencies. Some properties of the self-action phenomenon are then described in practical situations. These considerations are applied in detail to a particular sample of n-InSb, where the absorption threshold is found to be significantly greater than the diffraction threshold and therefore, defocusing is favored. Finally, the possiblity of self-focusing, taking absorption effects into account, is discussed in elemental and group-III-V compound semiconductors—Ge, Si, GaAs, GaSb, GaP, InAs, InSb, InP, and AlSb — for visible He-Ne and infrared Q-switched CO_2 lasers.

I. INTRODUCTION

This paper is concerned with the theory of self-action phenomenon (e.g., focusing, defocusing, trapping, etc.) of laser beams in semiconductors and generalizes the earlier work of Tzoar and Gersten[1] and the present authors[2] to study the role of absorption processes in this phenomenon. It is expected that in semiconductors with the usual electron-hole concentration, the free-carrier effects would be much more important than other effects which give rise to self-action, such as the high-frequency Kerr effect,[3] electrostriction,[4] nonlinear electronic polarization,[5] self-induced thermal effects,[6] etc. In this paper, therefore, we have considered the two relevant mechanisms due to nonparabolicity (NP) of the conduction band[1] and the energy-dependent scattering[2] (EDS). While the NP mechanism is the dominant mechanism in low-band-gap semiconductors (e.g., InSb), the EDS mechanism is dominant in elemental semiconductors (e.g., Ge and Si). However, irrespective of one mechanism being dominant over the other, the EDS mechanism besides giving rise to a nonlinear dielectric constant also accounts for the absorption of the beam inside the sample which has so far been neglected. Thus a complete account of the self-action phenomenon in semiconductors should be made by invoking both these mechanisms together, which is the aim of the present work.

In Sec. II we derive, using an elementary hydrodynamic approach, an expression for the nonlinear dielectric constant taking both the above-described mechanisms of NP and EDS into account. In Sec. III, we include the linear absorption process and evaluate its consequences on self-action. We find that the absorption effects lead to a lower limit on the power of the beam (other than the one required to overcome the diffraction effects). In Sec. IV we include, besides the linear absorption term, the nonlinear absorption term which is quadratic in field. In Sec. V, we have described general properties of the self-action phenomenon in some practical situations. In Sec. VI, we apply these considerations to see if the possibility of self-focusing has any relevance in a particular sample of n-InSb used in the frequency-mixing experiments of Patel *et al.*[7] We find that self-defocusing is favored since the lower limit due to the linear absorption process, introduced in this paper, may be much greater than the incident powers used. In Sec. VII, we have discussed linear absorption effects in relation to some elemental and group-III-V compound semiconductors—Ge, Si, GaAs, GaSb, GaP, InAs, InSb, InP, and AlSb—for visible He-Ne and infrared Q-switched CO_2 lasers. For the former, only GaP appears suitable for self-focusing, while for the latter, free-carrier absorption introduces an upper limit on the carrier concentration and a lower limit on the self-focusing length. In Sec. VIII, a summary of this work is given.

II. NONLINEAR DIELECTRIC CONSTANT

We shall be using the elementary hydrodynamic approach, rather than the rigorous kinetic approach, in evaluating the nonlinear dielectric constant due to the free carriers. To simplify the analysis, we shall also assume that (i) all the carriers have the average energy which in the case of the Maxwellian distribution is $\frac{3}{2}k_0T$ (k_0 is the Boltzmann constant and T is the lattice temperature) and in the case of the Fermi-Dirac distribution $\frac{3}{5}\mathcal{E}_F$ ($\mathcal{E}_F$ is the Fermi energy), that (ii) heating of the electrons and/or lattice by the laser beam is negligible, and that (iii) only fundamental harmonic motion is of interest in this work.

The starting point is the equation of motion of electrons in the presence of an incident laser field $\vec{E}=\vec{E}_0e^{i(\omega t-\vec{k}\cdot\vec{r})}$ and dissipating momentum in collisions with various scattering processes:

$$\hbar\frac{d\vec{K}}{dt}=-e\vec{E}_0e^{i(\omega t-\vec{k}\cdot\vec{r})}-\frac{\hbar\vec{K}}{\langle\tau\rangle}\ , \tag{1}$$

where we have neglected the magnetic field effects. Here $\hbar\vec{K}$ is the average momentum gained by the electrons in the presence of the field, $\langle\tau\rangle$ is the average relaxation time, and the other symbols have their usual meaning. We assume that there exists a power law for the dependence of τ on energy so that

$$\tau=\tau_0(K_E/K_0)^n\ , \tag{2}$$

where $\vec{K}_E=\vec{K}_0+\vec{K}$ is the total wave vector of the electrons in the presence of the laser field, $\vec{K}_0$ is the wave vector in the absence of the field, and τ_0 is the value of τ in the absence of the field. We may recall that $n=+3$ for ionized impurity scattering, $+1$ for acoustic phonon scattering, and -1 for (elastic) polar-optical-phonon scattering. Using in Eq. (2) the approximation that the increase in momentum due to the field is small, we have[8]

$$|K_E|\simeq K_0\left(1+\frac{K\mu}{K_0}+\frac{K^2(1-\mu^2)}{2K_0^2}\right)\ , \tag{3}$$

where μ is the cosine of the angle between the direction of the field and the original direction of motion; the higher-order terms in K/K_0 have been neglected.

We can, therefore, rewrite Eq. (1) as

$$\frac{d\vec{K}}{dt}=-\frac{e\vec{E}_0}{\hbar}e^{i(\omega t-\vec{k}\cdot\vec{r})}-\frac{\vec{K}}{\tau_0}\left(1-\frac{nK^2}{6K_0^2}\right), \tag{4}$$

where we have used the average values $\langle\mu\rangle=0$ and $\langle\mu^2\rangle=\frac{2}{3}$. As in Ref. 2, we shall solve the nonlinear differential Eq. (4) by an iteration process up to the first order, realizing that the nonlinear term in (4) is small in our considerations. Thus the solution of Eq. (1) is

$$\vec{K}=-\frac{e\vec{E}_0}{\hbar}\left[\nu_0\left(1-\frac{ne^2E_0^2}{6\hbar^2K_0^2(\omega^2+\nu_0^2)}\right)-i\omega\right]\times\left[\nu_0^2\left(1-\frac{ne^2E_0^2}{3\hbar^2K_0^2(\omega^2+\nu_0^2)}\right)+\omega^2\right]^{-1}\times e^{i(\omega t-\vec{k}\cdot\vec{r})}\ ,\qquad\nu_0=\tau_0^{-1}\ . \tag{5}$$

Equation (5) accounts for the EDS contribution, while the NP contribution arising from the energy dependence of carrier mass has not yet been included. Using the Kane band model[9] and neglecting the spin-orbit splitting energy (which is justified in most of the group-III-V semiconductors), we can express this dependence conveniently as

$$\langle m^*\rangle=m_n(1+2\langle\mathcal{E}\rangle/\mathcal{E}_g), \tag{6}$$

where $\langle m^*\rangle$ is the average mass of the carriers, m_n is the mass at the bottom of the conduction band, and $\mathcal{E}_g$ is the energy band gap. The average energy $\langle\mathcal{E}\rangle$ may be expressed in terms of K_E as[9]

$$\langle\mathcal{E}\rangle+\langle\mathcal{E}\rangle^2/\mathcal{E}_g=\hbar^2\langle K_E\rangle^2/2m_n\ . \tag{7}$$

Thus using Eqs. (6) and (7) we can express the energy dependence of mass on wave number as

$$\langle m^*\rangle\simeq m_n\left(1+\frac{\hbar^2K_0^2}{m_n\mathcal{E}_g}+\frac{e^2E_0^2}{m_n\mathcal{E}_g}\,\frac{1}{\omega^2+\nu_0^2}\right), \tag{8}$$

where we have made use of the inequality that $\hbar^2K_E^2/2m_n\ll\mathcal{E}_g$ (which is valid up to room temperature). Using Eqs. (5) and (8), the imaginary part of the nonlinear conductivity, which gives rise to the real part of the nonlinear dielectric constant due to free carriers, can be written, with the definition $\langle\sigma\rangle \vec{\mathrm{E}} = Ne\hbar\vec{\mathrm{K}}/\langle m^*\rangle$, as

$$\langle\sigma_i\rangle \simeq -i\,\frac{Ne^2}{m_n\omega}\left(1-\frac{\nu_0^2}{\omega^2}+\frac{ne^2E_0^2\nu_0^2}{3\hbar^2K_0^2\omega^2(\omega^2+\nu_0^2)}\right) \times\left(1-\frac{\hbar^2K_0^2}{m_n\mathcal{E}_g}-\frac{e^2E_0^2}{m_n\mathcal{E}_g}\,\frac{1}{\omega^2+\nu_0^2}\right), \tag{9}$$

where N is the carrier concentration. This expression includes the carrier scattering and nonparabolicity effects, but neglects the absorption. Using Eq. (9) and Maxwell's curl equations, it is straightforward to obtain the real part of the dielectric constant as

$$(\epsilon_2)_{\mathrm{NP,EDS}} = -\frac{4\pi Ne^2}{m_n\omega^2}\left[\left(1-\frac{\hbar^2K_0^2}{m_n\mathcal{E}_g}\right)\frac{ne^2\nu_0^2}{3\hbar^2K_0^2\omega^2(\omega^2+\nu_0^2)} - \left(1-\frac{\nu_0^2}{\omega^2}\right)\frac{e^2}{m_n\mathcal{E}_g(\omega^2+\nu_0^2)}\right]. \tag{10}$$

It can be easily seen that this expression reduces to the previous results which were obtained earlier. If we let $\mathcal{E}_g \to \infty$, we obtain purely the EDS term, which is the same equation as obtained earlier by us (note, however, a minor error in the numerical coefficient in our earlier paper). On the other hand, if we let $\nu_0 \to 0$, we get the NP term. On comparison with Eq. (15b) of Ref. 1, we find that our averaging procedure slightly overestimates the contribution of NP in that the factor $\frac{3}{4}$ is 1 here. The two mechanisms have been compared numerically in Sec. VI using the parameters of the experiments of Patel *et al.*[7]

III. ACCOUNT OF LINEAR ABSORPTION

In deriving Eq. (9), we have not considered the real part of the conductivity, σ_r, which gives rise to the absorption term. These effects may be taken into account as follows. The intensity P of the beam at a point z (in the direction of propagation) may be expressed as

$$P = P_{\mathrm{in}}e^{-2k_iz}, \tag{11}$$

where P_{in} is the input power and k_i is the absorption coefficient. Using Maxwell's equations, k_i is related to the real and imaginary part of the dielectric constant, ϵ_r and ϵ_i, through

$$k_i \simeq (\omega/2c)(\epsilon_i/\epsilon_r^{1/2}), \tag{12}$$

$$\epsilon_r = \epsilon_L + 4\pi\sigma_i/\omega, \tag{13}$$

$$\epsilon_i = 4\pi\sigma_r/\omega, \tag{14}$$

where ϵ_L is the lattice dielectric constant. The real part of the conductivity, σ_r, can be obtained from Eqs. (5) and (8) using the same procedure as in (9) to give

$$\langle\sigma_r\rangle \simeq \frac{Ne^2\nu_0}{m_n\omega^2}\left(1-\frac{ne^2E_0^2}{6\hbar^2K_0^2\omega^2}\right)\left(1-\frac{\hbar^2K_0^2}{m_n\mathcal{E}_g}-\frac{e^2E_0^2}{m_n\mathcal{E}_g\omega^2}\right). \tag{15}$$

Thus neglecting the terms in Eq. (15), which are proportional to E_0^2, we obtain the linear absorption term k_{i0}:

$$k_{i0} = \frac{\omega}{2c}\,\frac{4\pi Ne^2}{m_n\omega^2}\,\frac{\nu_0}{\omega}\left(1-\frac{\hbar^2K_0^2}{m_n\mathcal{E}_g}\right) \times\frac{1}{[\epsilon_L-(4\pi Ne^2/m_n\omega^2)]^{1/2}}. \tag{16}$$

We shall use, at this stage, the analysis of Akhmanov *et al.*[10] in evaluating the consequences of linear absorption in the self-action phenomenon. The results can be summarized in the following equation:

$$z_f = -(1/k_{i0})\ln(1-k_{i0}R_{nl}), \qquad R_{nl} = \tfrac{1}{2}a\,(2\epsilon_L/\epsilon_2E_0^2)^{1/2}, \tag{17}$$

where R_{nl} is the self-focusing length in the absence of absorption, z_f is the self-focusing length in the presence of absorption (if self-focusing is to occur), and a is the initial radius of the beam. For z_f to exist we require that

$$k_{i0}R_{nl} \le 1. \tag{18}$$

It can be seen immediately from Eq. (18) that taking into account the absorption puts another limit on the minimum power that the laser beam must have in order for focusing to occur, besides the usual threshold put by the diffraction effects. This limit according to Eqs. (17) and (18) is given by

$$E_0^2 \ge a^2\epsilon_Lk_{i0}^2/2\epsilon_2$$

or

$$P_{\mathrm{cr}} = \tfrac{1}{32}(1+\sqrt{\epsilon_L})^2ca^2E_0^2 = [3a^4\epsilon_Lk_{i0}^2(1+\sqrt{\epsilon_L})^2/64\epsilon_2]\ \mathrm{kW}, \tag{19}$$

where P_{cr} is the critical power. Thus, the beam must have initial power equal to or greater than the limit expressed in Eq. (19) in order that focusing be observed. Physically it is obvious since the focusing phenomena and absorption are competitive processes in opposite senses; the former leads to increase in power, while the latter to decrease. Thus if the absorption is high enough (so that before an appreciable focusing is realized, the beam intensity becomes smaller than that required to overcome diffraction) it would be defocused. If we compare the lower threshold due to linear absorption expressed in Eq. (19) with the one due to diffraction divergence,[1,10] we find that the ratio is

$$R = (P_{\mathrm{cr}})_{\mathrm{lin\ abs}}/(P_{\mathrm{cr}})_{\mathrm{diff}} = \tfrac{1}{2}k^2k_{i0}^2a^4, \qquad k = \omega/c$$

where $(P_{\mathrm{cr}})_{\mathrm{lin\ abs}}$ is the critical power due to linear absorption and $(P_{cr})_{diff}$ is the critical power due to diffraction. The main point of interest here is the value of R. If R is sufficiently greater than 1, the lower power threshold for self-focusing due to absorption may be high enough to exceed surface ionization threshold, and thus preclude any possibility of self-focusing. At microwave frequencies, ka^2, which is twice the diffraction length, is quite small; however, the free-carrier absorption coefficient varies at a fast rate with respect to frequency to upset the smallness of diffraction length and in most cases $R \gg 1$. At optical frequencies and for normal dimensions of the beam at the entrance, ka^2 is of the order of 10; thus if the absorption is not too small, R could acquire values much greater than 1. This would depend upon the situations of interest. We shall see in Sec. VI that the lower threshold due to absorption may be higher than the surface ionization threshold in a particular example of n-InSb, and the considerations favor defocusing rather than focusing.

If we assume that the incident power is greater than that required by Eq. (19), we find that the presence of absorption increases the self-focusing length as can be seen from

$$(Z_f)_{\text{with absorp}}/(Z_f)_{\text{without absorp}} \cong 1+\frac{k_{i0}R_{nl}}{2} \;, \qquad k_{i0}R_{nl} \lesssim 1 \tag{20}$$

where $(Z_f)_{\text{without absorp}} = R_{nl}$. It may be worthwhile mentioning here that in an active medium (where k_{i0} is negative), the self-focusing length would be decreased by the presence of absorption—a situation which is confronted in stimulated Raman scattering.[11] We may also mention that associated with the absorption process is the corresponding heating of the carriers and lattice, which could be a sizeable factor in determining self-action.

IV. ACCOUNT OF NONLINEAR ABSORPTION

We note in Eq. (15) that besides the linear terms in σ_r, there are other terms which give rise to absorption coefficients proportional to the square of the field. If we express the absorption coefficient as $k_i = k_{i0} + k_{i2}E_0^2$, we find that

$$k_{i2} = -\frac{\omega}{2c}\,\frac{4\pi Ne^2}{m_n\omega^2}\,\frac{\nu_0}{\omega}\left[\frac{ne^2}{6\hbar^2K_0^2\omega^2}\left(1-\frac{\hbar^2K_0^2}{m_nE_g}\right) + \frac{e^2}{m_n\mathcal{E}_g\omega^2}\right]\frac{1}{(\epsilon_L - 4\pi Ne^2/m_n\omega^2)^{1/2}} \;. \tag{21}$$

We may point out here that k_{i2} is negative in our example if n is positive (otherwise it depends upon the relative contribution of NP and EDS). Recalling the corresponding equation [Eq. (2.54)] of Akhmanov *et al.*, we can describe the effect of nonlinear absorption on self-action as

$$Z_f = R_{nl} + \frac{\omega}{2c}\,a^2\,\frac{2c\epsilon_r^{1/2}}{\omega\epsilon_2}\,k_{i2} \;, \tag{22}$$

where the approximation implicit in the derivation of this result is that the linear absorption term is such that $k_{i0}R_{nl} \ll 1$. Since k_{i2} is most likely negative in our example, we conclude that, when Eq. (22) is valid, the presence of nonlinear absorption would reduce the self-focusing length. It is interesting to note here that the self-focusing length does not become zero even when the initial power tends to infinity (as it would, without nonlinear absorption), but to a value determined by the diffraction length $R_d = (\omega/2c)a^2$ and the nonlinear properties. Also, there would be a critical power where Z_f can become zero (which is a consequence of the negative nonlinear absorption coefficient). For $z_f = 0$, this is given by

$$P = (1+\epsilon_L^{1/2}/8k_{i2})^2\epsilon_2 c \;. \tag{23}$$

For a typical case of n-InSb (parameters given in Sec. VI), this turns out to be ~ 2×10^6 W/cm^2. Since both the diffraction length and the nonlinear effects adjust in a way so as to increase the effect of nonlinear absorption with increasing frequency of radiation, this effect seems to become significant at optical frequencies (cf. Sec. VI).

V. SELF-ACTION CONSIDERATIONS

The self-focusing length (if it is to occur) can be calculated by following the standard eikonal technique employed by Akhmanov *et al.* or by the variational treatment of Tzoar and Gersten.[1] Taking the diffraction divergence into account, the relevant expression for the self-focusing length[10] is

$$R_{nl}^{\text{diff}} = \frac{R_d}{[(\omega^2a^2/2c^2\epsilon_r)\,\epsilon_2E_0^2 - 1]^{1/2}} \;, \tag{24}$$

where the spatial distribution of the intensity of the beam along the radial axis has been assumed to be Gaussian in nature. It can be seen that the expression for R_{nl}^{diff} reduces to the expression for R_{nl}, if diffraction effects are neglected.

It may be worthwhile to repeat here the consequences of Eq. (24) which are well known in the context of liquids and some insulators but have been overlooked in the case of semiconductors. First, it shows a self-trapped (or waveguide) mode solution when the nonlinear dielectric constant attains a value of

$$(\epsilon_2E_0^2)_{sf} = 2\epsilon_Lc^2/\omega^2a^2 \;. \tag{25}$$

In this condition, the beam would propagate unaffected provided also that the initial phase wave front of the beam is plane. This waveguide propagation mode would occur at a critical intensity of 4×10^6 W/cm^2 for InSb, using the parameters of

Sec. VI. Second, the dependence of the self-focusing length possesses a minimum with respect to the initial beam diameter.[12] By a direct differentiation, it may be calculated that the optimum a for minimum $R_{nl}^{\rm diff}$ is given by

$$a_{\rm opt} = (2c/\omega)(\epsilon_L/\epsilon_2 E_0^2)^{1/2} \tag{26a}$$

and

$$(R_{nl}^{\rm diff})_{\rm min} = (\omega/2c)\,a_{\rm opt}^2\ . \tag{26b}$$

Hence it follows that the inhomogeneities in the intensity distribution of the laser beam within $r \sim a_{\rm opt}$ would become quite important. For n-InSb, the optimal radius and the minimum self-focusing length are ~ 70 μ and 1.5 mm, respectively, for an incident power flux of 10^8 W/cm^2.

Some other interesting consequences, which are not implicit in Eq. (24), arise from the saturation of the dielectric constant as the field intensity goes up during focusing. The saturation phenomenon has been predicted by Tzoar and Gersten, however, without considering their consequences. The important effects stem from the fact that, due to saturation, the rate of self-focusing decreases near the focal region than otherwise. Without going into details of the calculation of the saturation dielectric constant, we would like to mention that the saturation phenomenon leads to a minimum radius to which the beam can be focused. This radius is given by

$$a_{\rm min} = (2c/\omega)(\epsilon_L/\epsilon_{\rm sat})^{1/2}\ , \tag{27}$$

where $\epsilon_{\rm sat}$ is the saturation dielectric constant ($\sim \epsilon_L$). Thus for a 10-μ beam, the minimum radius possible is of the order of 0.003 mm. The saturation effects, therefore, eliminate the singularity at the focus (since the focal region is finite). The saturation effects also lead to an optimal power of the optical waveguide formation, which is given by[3]

$$P_{\rm opt} = 2P_{\rm cr}(1 + \epsilon_{\rm sat}/\epsilon_L)^{1/2}\ . \tag{28}$$

Thus with saturation effects included, the maximum power of the beam is of the order of the critical power required to overcome diffraction, contrary to what one might expect a first sight. It is also interesting to note that the minimum beam radius, given in Eq. (27), leads to the formation of an oscillatory waveguide if the beam has arbitrary but finite divergence at the entrance (i.e., spherical phase wave front rather than plane). In general, for an initially convergent beam, the self-focusing length is decreased.

The above discussion pertains to laser fields which are constant in time (or are of sufficiently long duration). We consider other effects as the pulse duration becomes shorter and shorter for transient effects to play a role. It then becomes increasingly important to take nonstationary self-action processes into account. The slowest mechanism is that due to nonuniform thermal heating (characteristic relaxation time ~ 10^{-1} sec) and the fastest one due to the high-frequency Kerr effect and/or electrostriction. An important consequence of the nonstationary processes to be mentioned here is the "chromatic aberration" and space-time stratification of laser pulses.

VI. APPLICATION TO *n*-InSb

We shall now use these considerations in the frequency mixing experiments of Patel *et al.* which were conducted in various group-III-V semiconductors. We have chosen n-InSb as a typical case. The various parameters to be used in the theory are listed in Table I. Substitution of these parameters in Eq. (10) yields the values

$$(\epsilon_2)_{\rm EDS} \cong -10^{-9}\ \text{cgs esu}\ , \tag{29a}$$

$$(\epsilon_2)_{\rm NP} \simeq 3\times 10^{-7}\ \text{cgs esu}\ . \tag{29b}$$

First, we see that despite the defocusing contribution due to EDS, the NP mechanism leads to a net focusing contribution and exceeds the former roughly by two orders of magnitude. Thus the total contribution of the free carriers to the nonlinear dielectric constant is positive and fairly large (we can compare these figures with the corresponding one for Te based upon other mechanisms which are roughly $\epsilon_2 \sim 10^{-12}$ cgs esu). Using a power density of 10^8 W/cm^2, the self-focusing length, given by Eq. (24), is found to be ~ 4 mm.

It might be expected, as is commonly understood in liquids (CS_2, CCl_4, $C_6H_5NO_2$, etc.), that the pow-

TABLE I. Typical parameters for *n*-InSb.

Sample	n-InSb
Temperature	80 °K
Sample length in the direction of propagation	1 mm
Electron density	6×10^{16} cm^{-3}
Band gap	0.2 eV
Angular frequency	1.78×10^{14} rad/sec
Incident power density	10^3 kW/cm^2
Scattering mechanism	Ionized impurity scattering
Power-law constant	3
Statistics	Degenerate
Effective mass at the conduction-band edge	$\sim 2\times 10^{-29}$ g
Initial beam diameter	Not known (taken typically as 0.5 mm)

erful light beams from Q-switched lasers should be self-focused. In semiconductors, where the nonlinear dielectric constant is about 10^2– 10^4 times higher than in the above-mentioned liquids, we expect to observe a strong self-focusing effect within a sample of normal dimensions. However, using Eq. (19), we find that the minimum power density threshold required for focusing is quite high. Using the well-known formulation of ionized impurity scattering and the parameters described above, we find that the relaxation time corresponding to the average energy of the carriers is approximately 5×10^{-13} sec. Since this value of the relaxation time looks unusually small, we also calculated it from the experimental measurements[13] of electron mobility in the particular sample, which is $\sim5\times10^4$ $\text{cm}^2/\text{V sec}$. Using an average relation $\langle\mu\rangle = e\langle\tau\rangle/\langle m^*\rangle$, and the above value of mobility $\langle\mu\rangle$, we arrive at a value of $\langle\tau\rangle$ sufficiently close to the value calculated from the scattering formulation. We gained further confidence in this value of $\langle\tau\rangle$ from an experimental measurement of energy relaxation time done by Heinrich *et al.*[14] in gallium antimonide in relatively purer samples. They have reported values of the order of 10^{-12} sec assuming dominance of polar-optical-phonon scattering. Since our sample is highly impure, we can expect that relaxation time of carriers would be smaller than this measured value. Using this value of τ_0, we find using Eq. (16) that the absorption length due to scattering losses[15] is ~ 2 mm. We also checked the absorption length deduced indirectly from the reflectance measurements in n-InSb in the range of dominance of free-carrier absorption.[16] In the measurements of Kurnick and Powell, the absorption length at an electron concentration three times greater than our sample is found to be 1 mm at $T = 78\,^\circ\text{K}$ and $\lambda = 10\ \mu$. We feel in the light of these measurements that the absorption coefficient of 5 cm^{-1} in the sample in question is not unrealistic. From Eq. (19), it then follows that

$$P_{\text{cr}} \sim (2\times10^5/a^2)\,\text{W/cm}^2\ . \qquad (30)$$

Using the beam radius of the order of 0.25 mm we find that P_{cr} is $\sim 3.3\times10^6$ W/cm^2, which is much higher than the power used in the experiments.[7] Thus we find that, with the inclusion of the linear dissipative process, self-defocusing is favored rather than self-focusing. We note that the minimum power density required for self-focusing exceeds the surface ionization threshold which is $\sim 3\times10^6$ W/cm^2 for InSb; thus self-focusing is precluded by the surface ionization process even if we reach power densities beyond the threshold. This is quite in contrast with the conjectures made when the absorption process is neglected. The threshold due to diffraction using the same parameters as listed before is $\sim2\times10^6$ W/cm^2, and it seems that self-focusing can be achieved, if one so wishes, by working in the power density range above the diffraction threshold but below the surface ionization threshold. Our analysis denies such a possibility in the n-InSb sample considered here. Of course, if we choose the parameters in a different range (especially purer samples and photon energy less than the minimum characteristic energy required for quantum excitations of carriers by photons), the absorption threshold may be less than the surface ionization threshold. However, irrespective of the details of one particular sample discussed here, one does get an indication that it is not unlikely to find situations where lower limit due to absorption could be very high. In general, we feel that the linear absorption term should not be overlooked before drawing conclusions about the focusing and defocusing of the beam.

The effect of the nonlinear absorption can also be evaluated using these parameters and Eqs. (19)–(21). Thus Eq. (22) can be written in our present example as

$$Z_f = R_{nl} - 0.28R_d\,, \quad R_d = 1.87\,\text{cm}. \qquad (31)$$

Since the nonlinear absorption coefficient is negative, it turns out that its inclusion reduces the self-focusing length. This conclusion is, however, based on a positive value of n, and it is possible, in principle, that for elastic optical-phonon scattering it may result in an increase. Since the diffraction length can be compared to the self-focusing length (at optical frequencies), we conclude from the present example that the nonlinear absorption term should be considered in self-action phenomenon.

VII. APPLICATION TO ELEMENTAL AND GROUP-III-V SEMICONDUCTORS

In this section, we wish to apply the considerations developed in previous sections to elemental semiconductors Ge and Si and to compound semiconductors belonging to the III-V group, viz., GaAs, GaSb, GaP, InAs, InSb, InP, and AlSb. The choice of the above materials is arbitrary but is dictated by recent progress in the understanding of these compounds and by the large amount of information which is now available. Similar considerations can be extended to groups II-VI and IV-VI, and to the recently developed mixed crystals and ternary compounds. We did not undertake calculation of these groups of compound semiconductors due to the lack of relevant information.

As has been mentioned in Sec. III, absorption of the laser beam in the sample leads to a decrease in the power which may then be insufficient to achieve self-focusing. Thus the considerations in this section are mainly those of the linear absorption coefficient and its connection to self-action through Eqs. (16) and (19).

We note that for self-focusing to occur, the absorption threshold P_{cr}, given in Eq. (19), must not exceed the surface ionization threshold. Since P_{cr} increases with increasing carrier concentration ($\propto N^2$, if the absorption coefficient is linear in N), the mechanism of free-carrier absorption puts an upper limit on the carrier concentration for self-focusing to occur. On the other hand, since we cannot arbitrarily increase N, and expect self-focusing, the nonlinear dielectric constant, $\epsilon_2 E_0^2$ attains a maximum value corresponding to maximum N and consequently results in a minimum of self-focusing length.

A. Critical Concentration for Self-Focusing

The critical concentration N_{cr} can be calculated by requiring that P_{cr} [cf. Eq. (19)] be less than or at the most equal to the surface ionization threshold. The latter arises due to valence-band–conduction-band transfer induced by impacts of accelerated primary free carriers. It can be shown from Eq. (5) that

$$P_{\mathrm{ion}}=(1+\epsilon_L^{1/2})^2 c\langle m^*\rangle\omega^2\mathcal{E}_g/16\pi e^2\ , \tag{32}$$

where P_{ion} is the ionization power density. It may be mentioned that at temperatures above the liquid-nitrogen temperature (77 °K) impact ionization of impurities would not be relevant in the previously mentioned samples. We shall assume that the free-carrier absorption coefficient increases linearly with increasing carrier concentration (although slight deviations are expected to occur in nonparabolic semiconductors). Combining Eqs. (19) and (32) and using the condition $P_{\mathrm{cr}}=P_{\mathrm{ion}}$, we obtain

$$N_{\mathrm{cr}}=(2\omega/ea\beta)\,(\epsilon_2\langle m^*\rangle\mathcal{E}_g/\epsilon_L)^{1/2}\ , \tag{33}$$

where $\beta=k_{i0}/N$ is a constant to be taken from experimental measurements on free-carrier absorption. This is a general result in terms of the nonlinear dielectric constant coefficient ϵ_2 and can be further simplified for parabolic and nonparabolic semiconductors.

For the group-III-V compound semiconductors, which are nonparabolic, we replace ϵ_2 in Eq. (33) by $(\epsilon_2)_{\mathrm{NP}}$ assuming the nonparabolicity mechanism to be dominant (which is especially true in small-band-gap semiconductors InSb, InAs, and GaSb). Even if they are comparable (for large-band-gap ones), the error involved in replacing ϵ_2 by $(\epsilon_2)_{\mathrm{NP}}$ in (33) would be by a multiplicative factor of $\sqrt{2}$. Thus from the second term of Eqs. (10) and (33):

$$N_{\mathrm{cr}}=\frac{16\pi e^2}{a^2\epsilon_L\beta^2\langle m^*\rangle\omega^2}\ , \tag{34}$$

for nonparabolic semiconductors. This result shows that the critical concentration increases quadratically with decreasing wavelength and decreasing beam radius. It also increases with increasing effective mass of the carriers (note that β is roughly inversely proportional to $\langle m^*\rangle$ and ω^{-2}).

For the elemental semiconductors, which are known to be parabolic even near the edges of the Brillouin zone, only $(\epsilon_2)_{\mathrm{EDS}}$ would exist. Thus replacing ϵ_2 in Eq. (33) by $(\epsilon_2)_{\mathrm{EDS}}$ [cf. first term of Eq. (10)], we obtain

$$N_{\mathrm{cr}}=16\pi|n|e^2\nu_0^2\mathcal{E}_g/9a^2\epsilon_L\beta^2\langle m^*\rangle\omega^4 k_0 T\ , \tag{35}$$

where we have used $\hbar^2K_0^2=3\langle m^*\rangle k_0 T$, assuming nondegenerate distribution of carriers. However, rather than using Eq. (35) to calculate N_{cr} for parabolic semiconductors, where uncertainties in the estimation of ν_0 could be important, it is advantageous to get rid of this parameter by substituting for β from Eq. (16). This yields

$$N_{\mathrm{cr}}=4|n|\,\mathcal{E}_g c^2\langle m^*\rangle/9a^2\pi e^2 k_0 T \tag{36}$$

for parabolic semiconductors. This result, on the other hand, shows that the critical concentration is independent of wavelength, but now depends upon the band gap and sample temperature.

B. Minimum Self-Focusing Length

The minimum self-focusing length $(R_{nl})_{\mathrm{min}}$ can be calculated by using Eqs. (34) or (36) for the carrier concentration in Eq. (10) and the maximum field intensity corresponding to the ionization threshold [see Eq. (32)]. Neglecting diffraction effects [omitting 1 in Eq. (24)], it is straightforward to obtain the following result:

$$(R_{nl})_{\mathrm{min}}=\frac{a\omega}{4e}\left(\frac{\epsilon_L\langle m^*\rangle}{\pi N_{\mathrm{cr}}}\right)^{1/2} \tag{37a}$$

for nonparabolic semiconductors and

$$(R_{nl})_{\mathrm{min}}=\frac{3a\omega^2}{4e}\left(\frac{\epsilon_L\langle m^*\rangle k_0 T}{\pi\mathcal{E}_g N_{\mathrm{cr}}|n|\,\nu_0^2}\right)^{1/2} \tag{37b}$$

for parabolic semiconductors. The corresponding power density to achieve minimum self-focusing length is the surface ionization power density P_{ion}. The concept of minimum self-focusing length was introduced by Tzoar and Gersten[1] by imposing an upper limit on the power density ($\leq P_{\mathrm{ion}}$); however, in their case, the carrier concentration can be arbitrarily increased.

To apply Eqs. (34), (36), and (37) to real systems numerically, we have considered in particular the propagation of a visible He-Ne laser ($\lambda=0.63\ \mu$) and the infrared Q-switched CO_2 laser ($\lambda=9.6$ and $10.6\ \mu$). We have collected the absorption data (in the fundamental and free-carrier absorption regime) for the various samples at two wavelengths (~ 0.6 and 10 μ), which are deduced from the reflectance and transmission measurements. Since the photon energy corresponding to 0.6 μ (2.06 eV) is greater than the fundamental absorption edge of all samples except gallium phosphide, only GaP is suitable for self-focusing of the visible laser. In the case of other semiconductors, the absorption coefficient falls in the range of 10^3–10^5 cm^{-1}, and as we have seen in Sec. VI, it will render the lower threshold due to absorption much higher than the ionization threshold. At $\lambda \sim 10$ μ, the photon energy (0.12 eV) is too small to induce any quantum excitations (see Sec. VI); at this wavelength only the conventional intraband free-carrier absorption is important.[17] From the measurements performed on various samples, we have collected the coefficients β and listed them in Table II.

The results of this investigation for various samples are summarized in Table II. For the group-III-V compounds, Eq. (34) is used, while for elemental semiconductors Eq. (36) is used. The critical concentrations, in general, are of the order of 5×10^{16} cm^{-3} with the exception of InP and GaAs where they are an order of magnitude higher. From these concentrations, the maximum value of the coefficient ϵ_2 is calculated for various samples and is then used to calculate the diffraction threshold also listed in Table II. The corresponding values of minimum self-focusing length are also given therein. The results show that for elemental semiconductors, self-focusing is not possible since the diffraction threshold exceeds the ionization threshold. For other samples, minimum self-focusing length is of the order of a few mm. These results are intended to correspond to room-temperature situations; however, it is felt that changes in temperature should not produce appreciable changes, unless close to liquid-helium temperature (~ 4 °K). In all of these calculations, we have assumed a beam radius of 0.25 mm (corresponding to a focal area of 10^{-3} cm^2); appropriate modifications should be made through Eqs. (34)–(37), if a different radius is to be used. Final remarks as to whether or not self-focusing can be achieved in each individual sample for the two types of lasers can also be seen in Table II.

We hope this would stimulate experimental realization of the self-action effect in semiconductors.

VIII. SUMMARY

The following summarizes the present study: (i) An expression for the nonlinear contribution to the dielectric constant due to free carriers has been derived taking both the NP and EDS mechanisms into account. (ii) Both linear and nonlinear absorption processes have been taken into account and its consequences on self-action in semicon-

TABLE II. Linear absorption effects discussed in Sec. III applied to elemental and group-III-V compound semiconductors. For details of critical carrier concentration and minimum self-focusing length, see text of Sec. VII.

Sample	Fundamental absorption edge (eV)	Visible He-Ne laser ~0.6μ or 2.06 eV				Q-switched CO_2 laser ~10μ or 0.12 eV					
		Diffraction threshold (W/cm^2)	Ionization threshold (W/cm^2)	Absorption coefficient cm^{-1}	Possibility of self-focusing	Diffraction threshold (W/cm^2)	Ionization threshold (W/cm^2)	$\beta = k_{t0}/N$ (cm^2)	Critical concentration (cm^{-3})	Minimum self-focusing length (mm)	Possibility of self-focusing
Ge	0.75	1.1×10^{18}	8.3×10^{12}	⋯	no	1.4×10^{13}	3×10^{10}	⋯	10^{16}	⋯	no
Si	1.11	2.6×10^{18}	2.1×10^{13}	⋯	no	3.4×10^{13}	7.7×10^{10}	⋯	2×10^{16}	⋯	no
GaAs	1.39	2.2×10^{9}	7.6×10^{12}	10^3–10^5	no	8×10^{6}	2.7×10^{10}	3×10^{-17} [a]	7×10^{17}	1.4	yes
GaSb	0.72	$\sim10^{10}$	2.9×10^{12}	10^3–10^5	no	3.6×10^{7}	$\sim10^{10}$	7×10^{-17} [b]	5×10^{16}	1.6	yes
GaP	2.15	2.6×10^{12}	5×10^{12}	1–10	yes	9.5×10^{9}	1.8×10^{11}	10^{-16} [c]	2×10^{16}	6.4	yes
InAs	0.32	4.4×10^{8}	5.1×10^{11}	10^3–10^5	no	1.6×10^{6}	1.8×10^{9}	8×10^{-17} [d]	8×10^{16}	1.0	yes
InSb	0.22	4.9×10^{8}	2.5×10^{11}	10^3–10^5	no	1.7×10^{6}	3×10^{8}	10^{-16} [e]	4×10^{16}	1.1	yes
InP	1.26	7.8×10^{9}	8.2×10^{12}	10^3–10^5	no	2.8×10^{7}	2.9×10^{10}	5×10^{-17} [f]	3×10^{17}	1.2	yes
AlSb	1.33	6.4×10^{11}	2.5×10^{13}	10^3–10^5	no	2.3×10^{9}	9.2×10^{10}	10^{-16} [g]	3×10^{16}	4.4	yes

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ductors are evaluated. It is seen that the linear absorption process puts a minimum limit on the power of the beam (other than the one required to overcome diffraction divergence) for self-focusing to occur. (iii) Some properties of the self-action phenomenon have been discussed in practical situations. Using the experimental parameters of Patel *et al.* we have shown that the threshold value of the power for focusing due to absorption exceeds the value due to diffraction effects. (iv) Linear absorption effects are discussed in some elemental and compound semiconductors—Ge, Si, GaAs, GaSb, GaP, InAs, InSb, InP, and AlSb—for visible He-Ne and infrared CO_2 lasers, and the possibility of self-focusing in individual samples is deduced. The mechanism of free-carrier absorption introduces an upper limit on the carrier concentration and a lower limit on the self-focusing length.

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